FULLY DEVELOPED PIPE FLOW

In this lesson, we will:

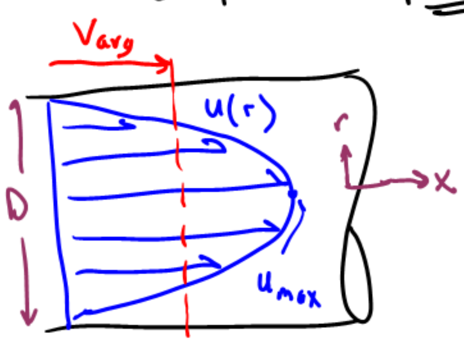
- Discuss what happens beyond the entrance region: **Fully Developed Pipe Flow**
- Discuss differences between laminar and turbulent fully developed pipe flow, such as velocity profile, wall shear stress, and pressure drop
- Do an example problem

Comparison of Laminar and Turbulent Fully Developed Pipe Flow

• Velocity Profiles

LAMINAR

- Can solve exactly
- Flow is steady
- Velocity profile is parabolic



$$\boxed{V_{avg} = \frac{1}{2} u_{max}}$$

$$\boxed{u(r) = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right)} \;\star$$

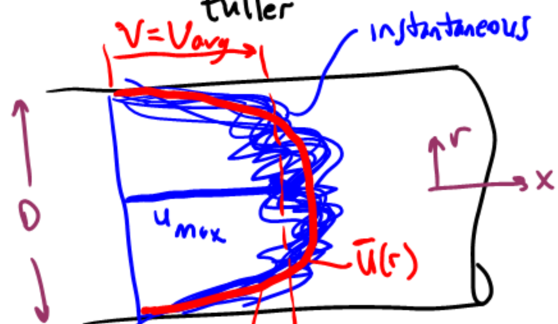
V

TURBULENT

- Cannot solve exactly
- Unsteady (3-D swirling eddies)

[But steady in the mean]

- mean velocity profile is "fuller"



★ mean velocity profile

$$\boxed{V_{avg} \approx 85\% \text{ of } u_{max}}$$

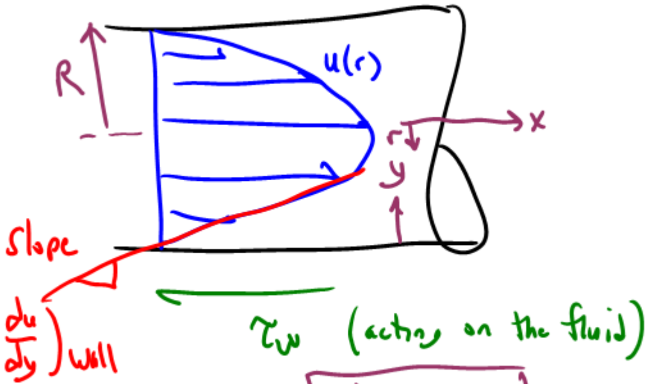
No analytical solution

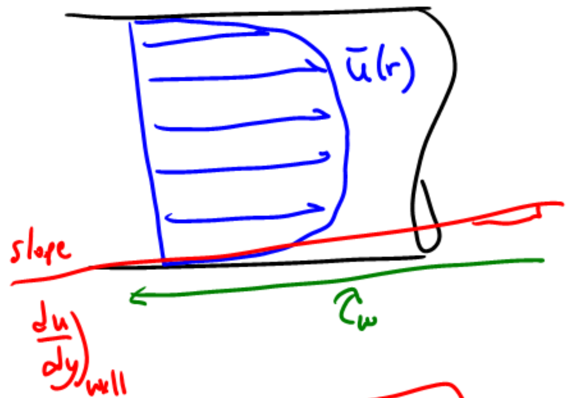
See text for some empirical eqs

- Power law
- Log law

LAMINAR | TURBULENT

recall, $\tau = \mu \frac{du}{dy}$ for simple shear flows





$y = R - r \rightarrow \boxed{\frac{du}{dy} = -\frac{du}{dr}}$

Let $\boxed{\tau_w = \mu \left| \frac{du}{dr} \right| \text{ @ the wall}}$

$\boxed{\tau_{w_{turb}} > \tau_{w_{lam}}}$

TURBULENT PIPE FLOW HAS A LARGER PRESSURE DROP FOR THE SAME FLOW RATE

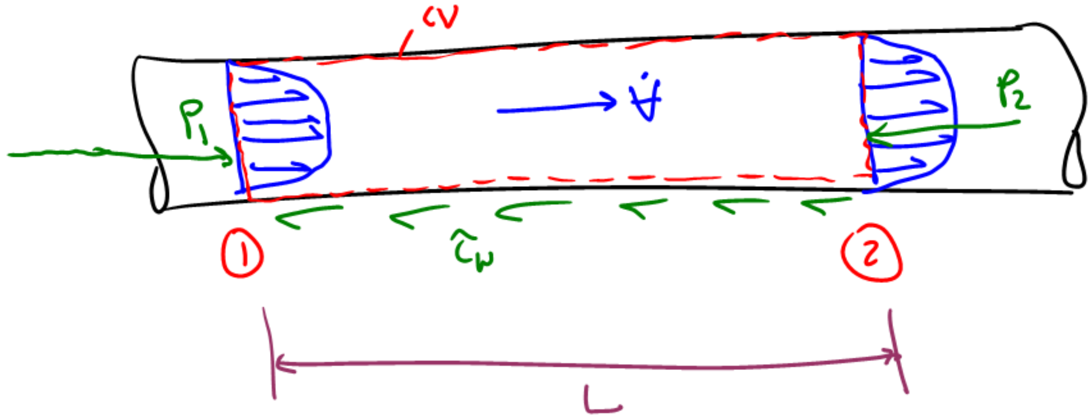
- $\Delta P_{turb} > \Delta P_{lam}$
- $h_{L\,turb} > h_{L\,lam}$

} @ same $\dot{V}$

- Pressure drop in fully developed pipe flow

[For either laminar or turbulent case]

Fully developed, incompressible, & steady (or steady in the mean)



· Cons. of mass

$$\boxed{\dot m_1 = \dot m_2 = \dot m}$$

$$\cancel{\rho}\dot{V}_1 = \cancel{\rho}\dot{V}_2 \rightarrow V_1 \frac{\cancel{\pi D^2}}{\cancel{4}} = V_2 \frac{\cancel{\pi D^2}}{\cancel{4}}$$

$$\boxed{V_1 = V_2}$$

· Cons. of x-mom

$$\sum F_x = \underbrace{\cancel{\sum F_{x_{grav}}}}_{\text{not in x-dir}} + \underbrace{\sum F_{x_{press}}}_{P_1\frac{\pi D^2}{4} - P_2\frac{\pi D^2}{4}} + \underbrace{\sum F_{x_{visc}}}_{-\tau_w \pi D L} + \cancel{\sum F_{x_{other}}}$$

$$= \underbrace{\sum_{out} \cancel{\beta \dot m u} - \sum_{in} \cancel{\beta \dot m u}}_{\text{cancel}}$$

x-mom reduces to $(P_1 - P_2)\frac{\cancel{\pi} D^{\cancel{2}}}{4} = \tau_w \cancel{\pi} \cancel{D} L$

$$\boxed{P_1 - P_2 = 4\tau_w \frac{L}{D}} \quad (1)$$

· Now apply the head form of the energy eq. for the same CV

$$\frac{P_1}{\rho g} + \alpha_1 \cancel{\frac{V_1^2}{2g}} + \cancel{z_1} + \cancel{h_{pump,u}} = \frac{P_2}{\rho g} + \alpha_2 \cancel{\frac{V_2^2}{2g}} + \cancel{z_2} + \cancel{h_{turb,e}} + h_L$$

fully dev.

Energy eq. reduces to $\boxed{P_1 - P_2 = \rho g h_L}$ (2)

EQUATE (1) & (2)

$$4\tau_w \frac{L}{D} = \rho g h_L$$

or $\boxed{h_L = \frac{4\tau_w}{\rho g}\frac{L}{D}}$ (3)

What is τ_w? → LAMINAR – can solve exactly

TURBULENT – need empirical equations

$\boxed{f = \text{Darcy friction factor} = \frac{8\tau_w}{\rho V^2}} = \text{fnc}\left(Re, \frac{\varepsilon}{D}\right)$

$$h_L = \frac{4 f \rho V^2}{8 \rho g}\frac{L}{D} = f\frac{V^2}{2g}\frac{L}{D}$$

$\boxed{h_L = f\frac{L}{D}\frac{V^2}{2g}}$ (4)

Example: Head Loss in a Long, Horizontal, Straight Pipe

Given: Water at 20ºC flows through a long, horizontal, straight section of round pipe. The section of pipe under consideration is fully developed. At a certain operating condition, these are the known values:

$\nu = 1.004 \times 10^{-6}\ \frac{m^2}{s}$

- Pipe diameter = 6.81 cm $= D$
- Pipe length = 35.0 m $= L$
- Reynolds number = 3.21×10^5 (turbulent pipe flow) $= Re$
- Darcy friction factor = 0.0298 $= f$

To do: Calculate the irreversible head loss through the pipe in units of m.

h_L

Solution:

$$Re = \frac{VD}{\nu} \rightarrow \boxed{V = \frac{Re \cdot \nu}{D}}$$

$$h_L = f \frac{L}{D} \frac{V^2}{2g}$$

$$\boxed{h_L = f \frac{L}{D} \left(\frac{Re \cdot \nu}{D}\right)^2 \frac{1}{2g}}$$

ANS. IN VARIABLE FORM

$$h_L = (0.0298) \frac{35.0\ m}{0.0681\ m} \left(\frac{3.21 \times 10^5 \cdot 1.004 \times 10^{-6}\ \frac{m^2}{s}}{0.0681\ m}\right)^2 \frac{1}{2(9.807\ m/s^2)}$$

$$\boxed{h_L = 17.5\ m}$$

irreversible head loss due to losses in the pipe