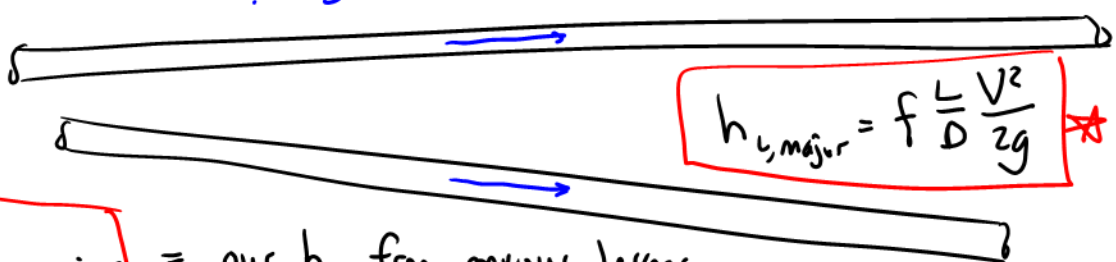
MAJOR HEAD LOSSES, LAMINAR

In this lesson, we will:

- Discuss the difference between so-called **Major Losses** and **Minor Losses** in pipe flows
- Derive an equation for **Darcy Friction Factor** for **Fully Developed Laminar Pipe Flow**
- Do an example problem to show how to apply this equation

Major and Minor Losses

MAJOR LOSSES – associated with long, straight sections of pipe & fully developed



$$\boxed{h_{L,\text{major}} = f\frac{L}{D}\frac{V^2}{2g}}$$

$\boxed{h_{L,\text{major}}}$ = our h_L from previous lessons

MINOR LOSSES – associated with everything else in the pipe system

$\boxed{h_{L,\text{minor}}}$ fittings, valves, elbows, tees, inlet regions, outflows

NOTE: Major losses are not necessarily larger than minor losses

In the head form of the energy eq., use $h_{L,\text{total}} = h_{L,\text{major}} + h_{L,\text{minor}}$

Notation: From now on, we replace h_L from previous lessons by $h_{L,\text{ total}}$.

Review

Why do we need to generate an equation for Darcy friction factor?

Recall from previous lessons: For ***fully developed pipe flow*** (using our new notation):

$$\Delta P_{\text{irreversible loss}} = \rho g h_{L,\text{ major}} \qquad h_{L,\text{ major}} = \frac{4\tau_w}{\rho g}\frac{L}{D} \qquad h_L = h_{L,\text{ major}} = f\frac{L}{D}\frac{V^2}{2g}$$

We also know from dimensional analysis that

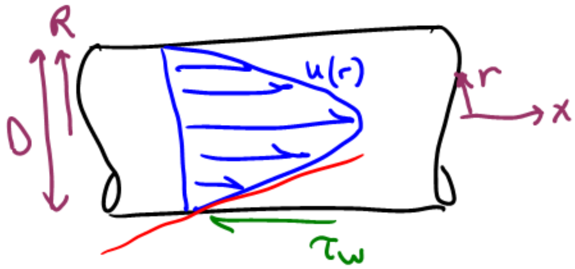
$$f = \text{fnc}\left(\text{Re}, \frac{\varepsilon}{D}\right) \quad \text{where} \quad f = \frac{8\tau_w}{\rho V^2} \qquad \text{Re} = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$$

For laminar flow, $\frac{\varepsilon}{D}$ is not important $\quad \boxed{f = \text{fnc}(\text{Re})}$

Darcy Friction Factor for Fully Developed Laminar Pipe Flow

We start with the velocity profile equation for fully developed laminar pipe flow,

$$\boxed{u(r) = 2V\left(1 - \frac{r^2}{R^2}\right)}$$



$$\tau_w = \mu\left|\frac{du}{dr}\right| = \mu\left|\frac{2V(-2r)}{R^2}\right|$$

$$= \frac{4\mu V}{R} \quad \text{@ wall } (r=R)$$

$$\tau_w = \frac{4\mu V}{D/2} = \frac{8\mu V}{D} \longrightarrow f = \frac{8\tau_w}{\rho V^2}$$

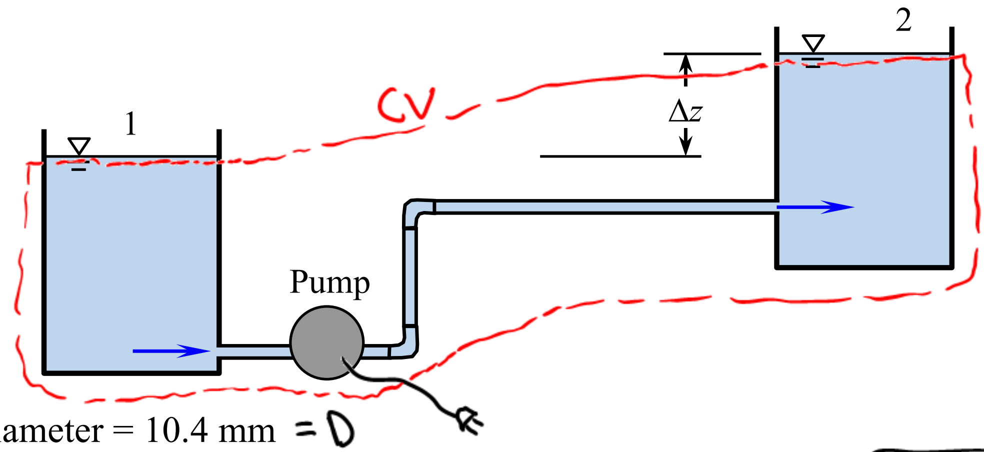
$$f = \frac{64\mu}{\rho V D} \rightarrow \frac{1}{Re}$$

THUS, $\boxed{f = \dfrac{64}{Re}}$ for fully developed laminar pipe flow (round pipe)

Use this with $\boxed{h_{L,\,major} = f\,\dfrac{L}{D}\,\dfrac{V^2}{2g}}$

Example: Fully Developed Laminar Pipe Flow

Given: Water at 20ºC is pumped by a small aquarium pump from a lower tank to an upper tank as sketched (not to scale).



- Pipe diameter = 10.4 mm $= D$
- Pipe length = 15.8 m $= L$
- The surface elevation difference is 4.13 m $= \Delta z$
- Volume flow rate = 2.06 L/min $= \dot{V}$
- Irreversible head losses from the inlet, outlet, and elbows is estimated to be 0.112 m
- The efficiency of the pump/motor assembly is 76.7% $= \eta_{pump\text{-}motor}$

$h_{L,minor}$

To do: Calculate the electrical power (in W) that must be delivered to the pump motor in order to pump the water at the given flow conditions.

Solution:

$$\dot{V} = 2.06\,\frac{L}{min}\left(\frac{1\ m^3}{1000\,L}\right)\left(\frac{1\ min}{60\ s}\right) = 3.43333\times10^{-5}\,\frac{m^3}{s}$$

$$V = \frac{\dot{V}}{A} = \frac{4\dot{V}}{\pi D^2} = \frac{4(3.43333\times10^{-5}\ m^3/s)}{\pi(0.0104\ m)^2} = \boxed{0.404166\,\frac{m}{s} = V}$$

- Pick a wise CV

- Energy eq in head form:

$$\cancel{\frac{P_1}{\rho g}} + \alpha_1 \cancel{\frac{V_1^2}{2g}} + z_1 + h_{pump,u} = \cancel{\frac{P_2}{\rho g}} + \alpha_2\cancel{\frac{V_2^2}{2g}} + z_2 + \cancel{h_{turbine,e}} + \boxed{h_{L,total}}$$

$P_1 = P_{atm}$ $\quad P_2 = P_{atm}$

$$\boxed{h_{L,total} = h_{L,major} + h_{L,minor}}$$

$$\boxed{h_{pump,u} = (z_2 - z_1) + h_{L,minor} + h_{L,major}} \quad (1)$$

$$h_{L,major} = f\frac{L}{D}\frac{V^2}{2g}$$

Laminar flow $\rightarrow$ $f = \frac{64}{Re}$, $h_{L,major} = \frac{64}{Re}\frac{L}{D}\frac{V^2}{2g}$

$Re = \frac{\rho V D}{\mu}$ $\rightarrow$ $\boxed{h_{L,major} = \frac{32\mu L V}{\rho g D^2}}$ (2)

$\dot{W}_{elec} = \frac{\dot{m} g h_{pump,u}}{\eta_{pump\text{-}motor}} = \boxed{\frac{\rho \dot{\forall} g h_{pump,u}}{\eta_{pump\text{-}motor}} = \dot{W}_{elec}}$ (3)

Plugging (1) & (2) into (3)

$$\boxed{\dot{W}_{elec} = \frac{\rho \dot{\forall} g}{\eta_{pump\text{-}motor}}\left[(z_2 - z_1) + h_{L,minor} + \frac{32\mu L V}{\rho g D^2}\right]}$$

ANS. IN VARIABLES

$$\dot{W}_{elec} = \frac{\left(998.0\ \frac{kg}{m^3}\right)\left(3.43333\times10^{-5}\ \frac{m^3}{s}\right)\left(9.807\ \frac{m}{s^2}\right)}{0.767}$$

(convert from %)

$$\left[4.13\ m + 0.112\ m + \frac{32\left(1.002\times10^{-3}\ \frac{kg}{m\cdot s}\right)\left(15.8\ m\right)\left(0.404166\ \frac{m}{s}\right)}{\left(998.0\ \frac{kg}{m^3}\right)\left(9.807\ \frac{m}{s^2}\right)\left(0.0104\ m\right)^2}\right]\left(\frac{N\cdot s^2}{kg\cdot m}\right)\left(\frac{W\cdot s}{N\cdot m}\right)$$

$$\boxed{\dot{W}_{elec} = 1.94\ W}$$

WRONG !!

We assumed laminar flow $\rightarrow$ is it laminar?

Critical step → check Re to see if flow is laminar

$$Re = \frac{\rho V D}{\mu} = \frac{\left(998.0 \frac{kg}{m^3}\right)\left(0.404166 \frac{m}{s}\right)\left(0.0104\ m\right)}{1.002 \times 10^{-3}\ \frac{kg}{m \cdot s}} = \underline{\underline{4187}}$$

Our $Re > 4000$ → ∴ THIS FLOW IS TURBULENT

OUR CALCULATIONS ARE WRONG

We must repeat the analysis, but for turbulent flow

(see next lesson)