MORE PIPE FLOW EXAMPLES

In this lesson, we will:

- Do two more example problems – one to calculate **turbine shaft power** and one to calculate **volume flow rate**, which requires ***iteration***

Review and Equations

The main equations we need to solve pipe flow problems with or without pumps or turbines, where we use the **Churchill Equation** for either laminar, transitional, or turbulent flow:

$$\frac{P_1}{\rho_1 g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, }u}=\frac{P_2}{\rho_2 g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, }e}+h_{L\text{, total}}$$

$$h_{L\text{, total}}=\sum h_{L\text{, major}}+\sum h_{L\text{, minor}}$$

$$f=8\left[\left(\frac{8}{\text{Re}}\right)^{12}+\left(A+B\right)^{-1.5}\right]^{\frac{1}{12}}$$

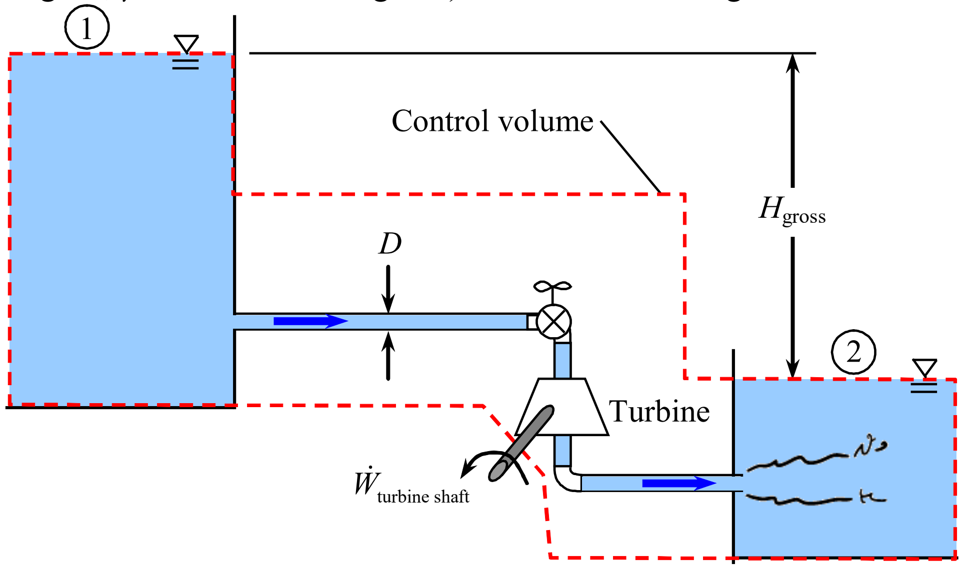
$$A=\left\{2.457\cdot\ln\left[\left(\frac{7}{\text{Re}}\right)^{0.9}+0.27\frac{\varepsilon}{D}\right]\right\}^{16}$$

− (missing a negative sign here)

$$B=\left(\frac{37530}{\text{Re}}\right)^{16}$$

Example: Calculation of Turbine Shaft Power

Given: Water at 20°C (ρ = 998.0 kg/m^3, μ = 1.002 × 10^{-3} kg/m·s) flows from one large reservoir to another, and through a turbine as sketched. The elevation difference between the two reservoir surfaces is H_{gross} = 120.0 m. The pipe is 5.00 cm I.D. cast iron pipe. The total pipe length is 30.8 m. The entrance is slightly rounded; the exit is sharp. There is one regular flanged 90-degree elbow, and one fully open flanged angle valve. The turbine is 81.0% efficient. The volume flow rate through the turbine is 0.00450 m^3/s.



To do: Calculate the shaft power produced by the turbine in units of kilowatts.

Solution:

- First we draw a control volume, as shown by the dashed line. We cut through the surface of both reservoirs (inlet 1 and outlet 2), where we know that the velocity is nearly zero and the pressure is atmospheric. We also slice through the turbine shaft. The rest of the control volume simply surrounds the piping system.

- We apply the head form of the energy equation from the inlet (1) to the outlet (2):

$$\boxed{P_1 = P_2 = P_{atm}}$$

$$\frac{\cancel{P_1}}{\cancel{\rho} g} + \alpha_1 \frac{\cancel{V_1^2}}{\cancel{2}g} + z_1 + \cancel{h_{\text{pump,u}}} = \frac{\cancel{P_2}}{\cancel{\rho} g} + \alpha_2 \frac{\cancel{V_2^2}}{\cancel{2}g} + z_2 + h_{\text{turbine,e}} + h_{L,\text{ total}}$$

$$\boxed{V_1 = V_2 \approx 0}$$

- But by definition of turbine efficiency, $\boxed{h_{\text{turbine, }e} = \frac{\dot{W}_{\text{turbine shaft}}}{\eta_{\text{turbine}} \dot{m} g}}$ where $\boxed{\dot{m} = \rho \dot{V}}$. Also, since the reference velocity is the same for all the major and minor losses (the pipe diameter is constant throughout), we may use the simplified version of the equation for $h_{L,\text{total}}$: $\boxed{h_{L,\text{total}} = \frac{V^2}{2g}\left(f\frac{L}{D} + \sum K_L\right)}$. Therefore, we solve the energy equation for the desired unknown, namely, turbine shaft power, $\boxed{\dot{W}_{\text{turbine shaft}} = \eta_{\text{turbine}} \rho \dot{V} g \left[H_{\text{gross}} - \frac{V^2}{2g}\left(f\frac{L}{D} + \sum K_L\right)\right]}$. This is our answer in variable form, but we still need to calculate the values of some of the variables.

- $V = \frac{\dot{V}}{A} = \frac{4\dot{V}}{\pi D^2} = \frac{4(0.00450 \text{ m}^3/\text{s})}{\pi (0.0500 \text{ m})^2} = \underline{2.29183 \frac{\text{m}}{\text{s}} = V}$

- $Re = \frac{\rho V D}{\mu} = \frac{(998.0 \text{ kg/m}^3)(2.29183 \frac{\text{m}}{\text{s}})(0.0500 \text{ m})}{0.001002 \frac{\text{kg}}{\text{m}\cdot\text{s}}} = \underline{\underline{114134 = Re}}$

TURBULENT

- Tables $\rightarrow$ $\underline{\underline{\varepsilon = 0.26 \text{ mm}}}$

$\varepsilon / D = \frac{0.26 \text{ mm}}{50.0 \text{ mm}} = \underline{\underline{0.0052 = \frac{\varepsilon}{D}}}$

- Churchill $\rightarrow$ $\underline{\underline{f = 0.03176}}$

- $\sum K_L = 0.12 + 5.0 + 0.30 + \underbrace{\alpha_{\text{outlet}}}_{\boxed{1.05}} = \underline{\underline{6.47 = \sum K_L}}$

- NOW WE PLUG IN ALL THE #s & SOLVE

$$\dot{W}_{\text{turbine shaft}} = (0.810)\left(998.0\ \frac{kg}{m^3}\right)\left(0.00450\ \frac{m^3}{s}\right)\left(9.807\ \frac{m}{s^2}\right)\cdot$$

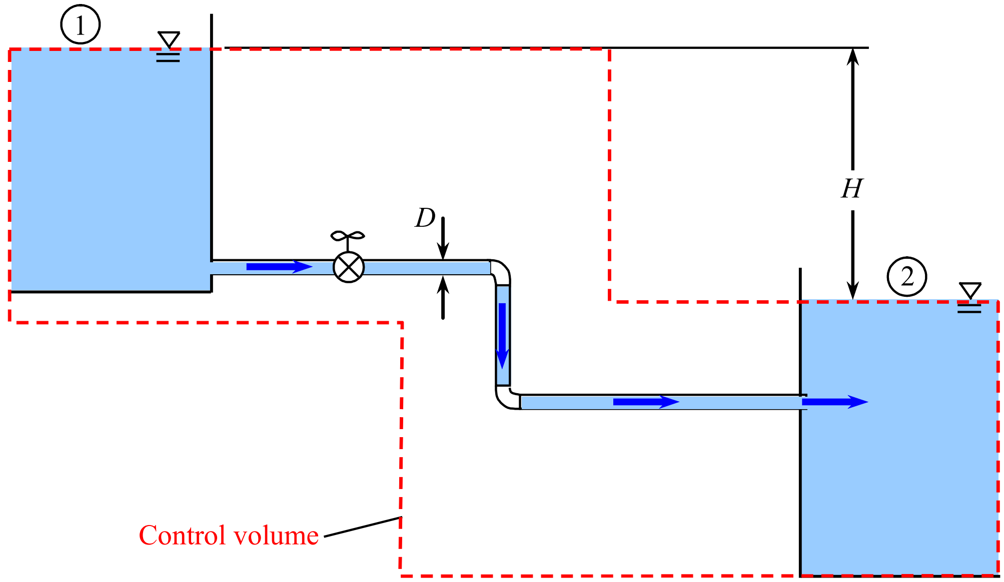
$$\left[120.0\ m - \frac{\left(2.29183\ \frac{m}{s}\right)^2}{2\left(9.807\ \frac{m}{s^2}\right)}\left(\underbrace{0.03176}_{f \text{ from Churchill}}\ \frac{30.8\ m}{0.0500\ m} + 6.47\right)\right]\left(\frac{N\cdot s^2}{kg\cdot m}\right)\left(\frac{kW\cdot s}{1000\ N\cdot m}\right)$$

$$\boxed{\dot{W}_{\text{turbine shaft}} = 4.03\ kW}$$

(Also verified in software)

Example: Calculate Unknown Volume Flow Rate

Given: Water at 20°C ($\rho = 998.$ kg/m^3, $\mu = 1.002 \times 10^{-3}$ kg/m·s) flows *by gravity alone* from one large tank to another, as sketched. The elevation difference between the two surfaces is $H = 35.0$ m. The pipe is 2.50 cm I.D. with an average roughness of 0.010 cm. The total pipe length is 20.0 m. The entrance and exit are sharp. There are two regular threaded 90-degree elbows, and one fully open threaded globe valve.



To do: Calculate the volume flow rate through this piping system.

Solution:

- First we draw a control volume, as shown by the dashed line. We cut through the surface of both reservoirs (inlet 1 and outlet 2), where we know that the velocity is nearly zero and the pressure is atmospheric. The rest of the control volume simply surrounds the piping system.
- We apply the head form of the energy equation from the inlet (1) to the outlet (2):

$P_1 = P_2 = P_{atm}$

$$\cancel{\frac{P_1}{\rho g}} + \alpha_1 \cancel{\frac{V_1^2}{2g}} + z_1 + \cancel{h_{\text{pump,u}}} = \cancel{\frac{P_2}{\rho g}} + \alpha_2 \cancel{\frac{V_2^2}{2g}} + z_2 + \cancel{h_{\text{turbine,e}}} + h_{L,\text{ total}}$$

$V_1 = V_2 \approx 0$

Therefore, the energy equation reduces to $\boxed{h_{L,\text{ total}} = z_1 - z_2 = H}$

- Next, we add up all the irreversible head losses, both major and minor. Since the reference velocity is the same for all the major and minor losses (the pipe diameter is constant throughout), we may use the simplified version of the equation for h_L,

$$\boxed{h_{L,\text{ total}} = \left(f\frac{L}{D} + \sum K_L \right)\frac{V^2}{2g}}, \ \& \ \boxed{\text{Re} = \frac{\rho D V}{\mu}} \ \boxed{\dot{V} = V\frac{\pi D^2}{4}} \ \boxed{\frac{\varepsilon}{D} = \frac{0.010 \text{ cm}}{2.50 \text{ cm}} = 0.0040}$$

- We also need the Churchill equation to solve this problem.

$$\sum K_L = 0.50 + 2(0.90) + 10.0 + 1.05 = \underline{13.35 = \sum K_L}$$

$$f = \text{fnc}\left(\text{Re}, \frac{\varepsilon}{D}\right) \leftarrow \underline{\text{Churchill}}$$

NOT KNOWN!

ITERATION SCHEME:

- Solve for $\boxed{V = \sqrt{\dfrac{2gH}{f\frac{L}{D} + \Sigma K_L}}}$ ☆

Guess f	Calc V (m/s)	Calc Re	Churchill → new f
0.04 →	3.8907 →	96880.1 →	0.02978
0.02978 →	4.2973 →	107005 →	0.029667
0.029667 →	4.3026 →	107135 →	0.029666
0.029666 →	4.302599 →	107136.4 →	$\boxed{0.0296659}$
0.0296659 →	4.302603 →	107136.5 →	$\boxed{0.0296659}$

CONVERGED

☆ VERY RAPID CONVERGENCE!

$$\dot{V} = V\frac{\pi D^2}{4} = \left(4.302603\ \frac{m}{s}\right)\frac{\pi(0.0250\ m)^2}{4} = 0.0021120\ \frac{m^3}{s}$$

$$\boxed{\dot{V} = 0.00211\ \frac{m^3}{s}}$$ ☆

(I verified with software)