PUMP SCALING LAWS

In this lesson, we will:

- Review **Dimensional Analysis** by applying it to **Pump Performance Curves**
- Discuss the **Pump Affinity Laws** and how to apply them to design pumps
- Do an example problem

Dimensional Analysis of Pump Performance Parameters

Start with the functional relationship, and perform the method of repeating variables:

gH occur together

$$gH = f\left(\dot{V}, D, \varepsilon, \omega, \rho, \mu\right)$$

D = pump dia

ω in rad/s [ṅ in RPM] $\{\omega\} = \{\frac{1}{t}\}$ $\{\dot{n}\} = \{\frac{1}{t}\}$

Try it yourself – great review of dimensional analysis and the method of repeating variables:

$$\boxed{\frac{gH}{\omega^2 D^2} = \text{function of}\left(\frac{\dot{V}}{\omega D^3}, \frac{\rho\omega D^2}{\mu}, \frac{\varepsilon}{D}\right)}$$

A similar analysis with input brake horsepower (*bhp*) as a function of the same variables results in

$$\boxed{\frac{bhp}{\rho\omega^3 D^5} = \text{function of}\left(\frac{\dot{V}}{\omega D^3}, \frac{\rho\omega D^2}{\mu}, \frac{\varepsilon}{D}\right)}$$

ωD = a speed $\left[Re = \frac{\rho\, \omega D\, D}{\mu}\right]$

Let's name these Π's:

$$\boxed{C_H = \frac{gH}{\omega^2 D^2} = \text{head coefficient}}$$

$$\boxed{\mathrm{Re} = \frac{\rho\omega D^2}{\mu} = \text{Reynolds number}}$$

$$\boxed{\frac{\varepsilon}{D} = \text{roughness ratio}}$$

Volume flow rate

$$\boxed{C_Q = \frac{\dot{V}}{\omega D^3} = \text{capacity coefficient}}$$

Q is used since Q = same as $\dot{V}$

$$\boxed{C_P = \frac{bhp}{\rho\omega^3 D^5} = \text{power coefficient}}$$

So, we write

$$\boxed{C_H = \text{function}\left(C_Q, \mathrm{Re}, \varepsilon/D\right)} \quad \text{and} \quad \boxed{C_P = \text{function}\left(C_Q, \mathrm{Re}, \varepsilon/D\right)}$$

But for many pumps, effects of Re and roughness are small at high Re, and thus,

$$\boxed{C_H \approx \text{function}\left(C_Q\right)} \quad \text{and} \quad \boxed{C_P \approx \text{function}\left(C_Q\right)}$$

Finally, pump efficiency is already dimensionless, and we write η_{pump} as

$$\boxed{\eta_{\text{pump}} = \frac{\rho\left(\dot{V}\right)\left(gH\right)}{bhp} = \frac{\cancel{\rho}\left(\cancel{\omega}D^{\cancel{3}}C_Q\right)\left(\cancel{\omega^2}\cancel{D^2}C_H\right)}{\cancel{\rho}\cancel{\omega^3}\cancel{D^5}C_P} = \frac{C_Q C_H}{C_P} = \text{function}\left(C_Q\right)}$$

η_{pump} = fnc(CQ) also

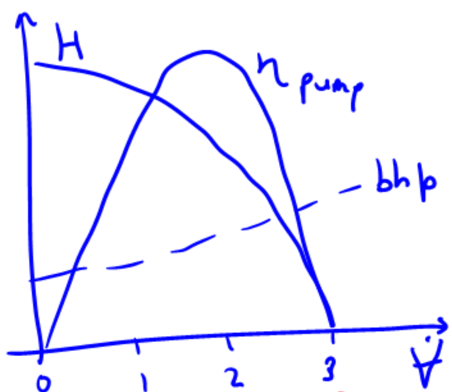
Pump Affinity Laws

Definition of **affinity**: Inherent likeness or agreement.

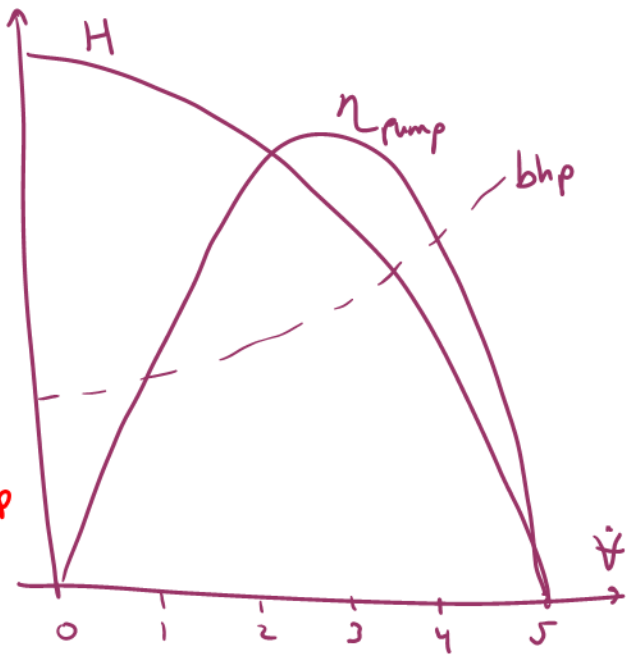
If two pumps are geometrically similar & dynamically similar, then their pump performance curves fall on top of each other when plotted nondimensionally

E.g.,

PUMP A



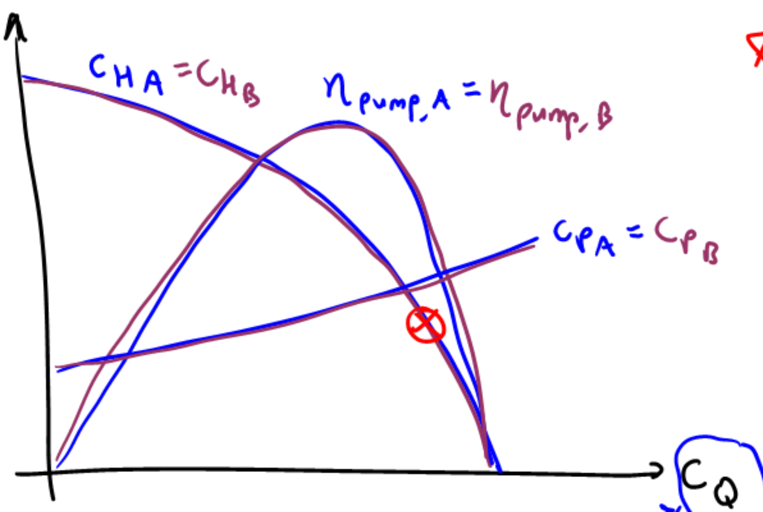
PUMP B



DIMENSIONAL PLOTS OF H & bhp

(η_{pump} is nondimensional)

Re-draw these plots in nondimensional form



ONE SET OF PUMP PERFORMANCE CURVES APPLIES TO ALL SIMILAR PUMPS

- geometrically similar
- dynamically similar

+ Re & $\frac{\varepsilon}{D}$ at small Re

The Pump Affinity Laws:

Set $C_{Q_A} = C_{Q_B}$

THEN $C_{H_A} = C_{H_B}$

$C_{P_A} = C_{P_B}$

& $\eta_{pump,A} = \eta_{pump,B}$

$$\frac{\dot{V}_B}{\dot{V}_A} = \left(\frac{\omega_B}{\omega_A}\right)^{1}\left(\frac{D_B}{D_A}\right)^3$$

$$\frac{H_B}{H_A} = \left(\frac{\omega_B}{\omega_A}\right)^{2}\left(\frac{D_B}{D_A}\right)^2$$

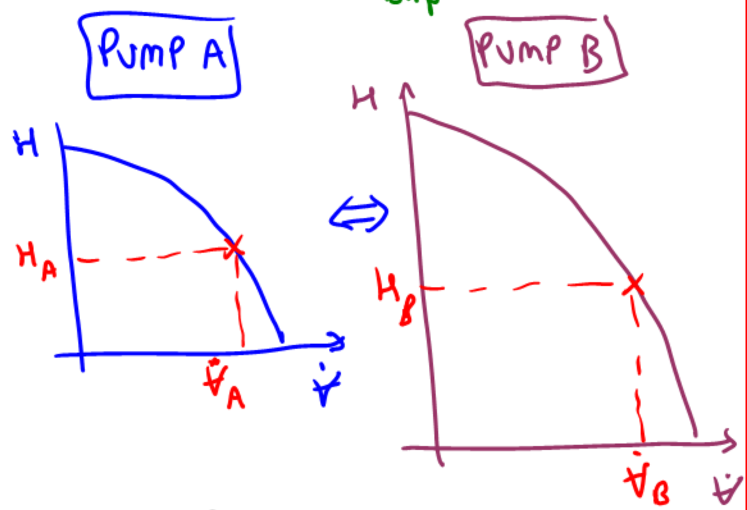
$$\frac{bhp_B}{bhp_A} = \frac{\rho_B}{\rho_A}\left(\frac{\omega_B}{\omega_A}\right)^{3}\left(\frac{D_B}{D_A}\right)^5$$

"VERY HARD PROBLEMS ARE AS EASY AS 1-2-3!"

PUMP AFFINITY LAWS

E.g., we double the RPM

- $\dot{V}\uparrow$ by factor of $2^1 = 2$
- $H\uparrow$ ... $2^2 = 4$
- $bhp\uparrow$ ... $2^3 = 8$



At some operating point for Pump A, there is a corresponding operating point for Pump B

THESE ARE CALLED HOMOLOGOUS POINTS

If $C_{Q_A} = C_{Q_B} \longrightarrow C_{H_A} = C_{H_B}$ & $C_{P_A} = C_{P_B}$ & $\eta_{pump,A} = \eta_{pump,B}$

@ Homologous points

$$C_{Q_A} = \frac{\dot{V}_A}{\omega_A D_A^3} = C_{Q_B} = \frac{\dot{V}_B}{\omega_B D_B^3} \Rightarrow \boxed{\frac{\dot{V}_B}{\dot{V}_A} = \frac{\omega_B}{\omega_A}\left(\frac{D_B}{D_A}\right)^3}$$

OUR FIRST AFFINITY LAW

Example: Scaling up a pump using the affinity laws

Given: An existing pump (A): Fluid is water at 20°C, D_A = 6.50 cm, and $\dot{n}_A = 1500$ rpm. At BEP, $\dot{V}_A = 455$ cm³/s at $H_A = 1.44$ m. We are designing a new larger pump (B) that is geometrically similar with D_B = 8.20 cm. It still uses water at 20°C, but rotates at a higher rpm, $\dot{n}_B = 1750$ rpm.

To do: (a) Predict $\dot{V}_B$ and H_B for operation of pump B at its BEP.
(b) Estimate the % increase in required brake horsepower from pump A to pump B.

Solution:
(a) At homologous points, the two turbines are dynamically similar. Apply the affinity laws:

$$\frac{\dot{V}_B}{\dot{V}_A} = \frac{\omega_B}{\omega_A}\left(\frac{D_B}{D_A}\right)^3 \qquad \frac{H_B}{H_A} = \left(\frac{\omega_B}{\omega_A}\right)^2\left(\frac{D_B}{D_A}\right)^2 \qquad \frac{bhp_B}{bhp_A} = \frac{\rho_B}{\rho_A}\left(\frac{\omega_B}{\omega_A}\right)^3\left(\frac{D_B}{D_A}\right)^5$$

$$\dot{V}_B = \dot{V}_A\left(\frac{\omega_B}{\omega_A}\right)\left(\frac{D_B}{D_A}\right)^3 = \dot{V}_A\left(\frac{\dot{n}_B}{\dot{n}_A}\right)\left(\frac{D_B}{D_A}\right)^3$$

$\dot{n}$? $\left(\frac{2\pi \text{ rad}}{\text{rot}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)$

$$\dot{V}_B = \left(455\ \frac{\text{cm}^3}{\text{s}}\right)\left(\frac{1750 \text{ rpm}}{1500 \text{ rpm}}\right)\left(\frac{8.20 \text{ cm}}{6.50 \text{ cm}}\right)^3 = \underline{1065.76\ \frac{\text{cm}^3}{\text{s}}}$$

$$\boxed{\dot{V}_B = 1070\ \frac{\text{cm}^3}{\text{s}}}$$

$$H_B = H_A\left(\frac{\dot{n}_B}{\dot{n}_A}\right)^2\left(\frac{D_B}{D_A}\right)^2 \rightsquigarrow \boxed{H_B = 3.12 \text{ m}}$$

(b) $$\frac{bhp_B}{bhp_A} = \frac{\rho_B}{\rho_A}\left(\frac{\dot{n}_B}{\dot{n}_A}\right)^3\left(\frac{D_B}{D_A}\right)^5 = (1)\left(\frac{1750}{1500}\right)^3\left(\frac{8.20}{6.50}\right)^5 = 5.0739$$

Here $\rho_B = \rho_A$

≈ 5 times as much power !!

$$\underline{\%\text{ increase}} = \frac{bhp_B - bhp_A}{bhp_A} \times 100\% = \left(\frac{bhp_B}{bhp_A} - 1\right) \times 100\% = \boxed{407\% \text{ increase in bhp}}$$