INTRODUCTION TO COMPUTATIONAL FLUID DYNAMICS

In this lesson, we will:

- Introduce **Computational Fluid Dynamics** (**CFD**) and discuss the general **procedure** for how to apply it to fluid flow problems
- Show some examples of CFD solutions (steady vs. unsteady, two-dimensional vs. three-dimensional, flows with heat transfer, incompressible vs. compressible, inviscid vs. viscous)

Review – Equations of Fluid Flow

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Continuity (1)

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial P}{\partial x}+\rho g_x+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial P}{\partial y}+\rho g_y+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial P}{\partial z}+\rho g_z+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$

Navier - Stokes equation (3)

4 eqs & 4 unknowns (u, v, w, P)

HOW TO SOLVE ?

- Solve analytically & exactly (limited to simple flow geometries)

  Eq's get too hard (impossible) to solve for complicated geometries

- Solve approximately (eliminate some terms in the N-S equation)

  Easier equations, but the approximations may not be valid

- Solve numerically (using computer software) (keep all terms in N-S equation)

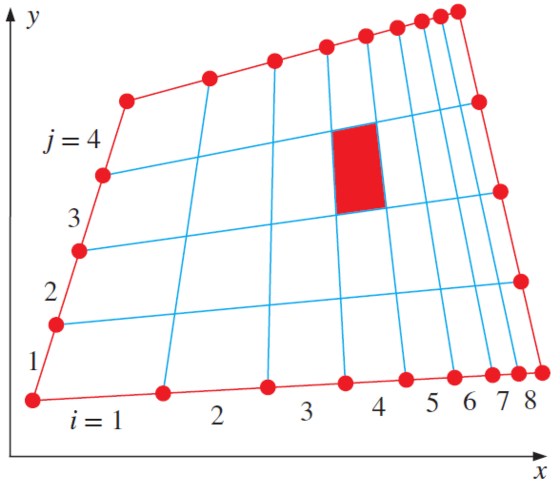
  Can solve for any geometry, but need to do it properly

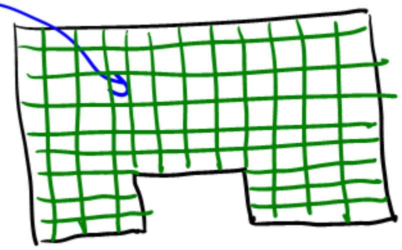
★ CFD

CFD Solution Procedure

All figures from Çengel and Cimbala, Ed. 4.

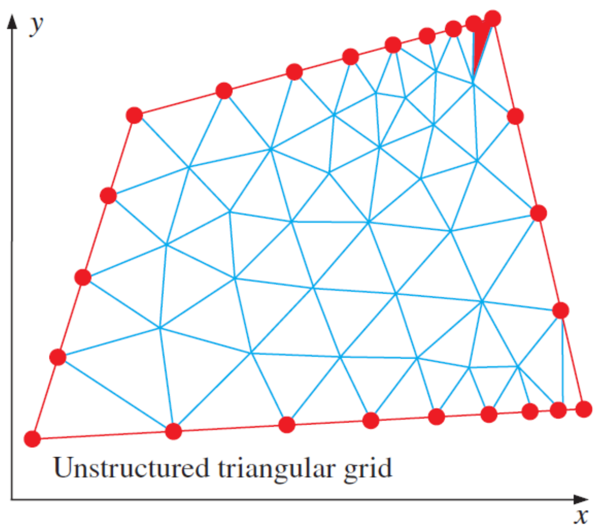
• Step 1: Choose a computational domain & generate a mesh or grid

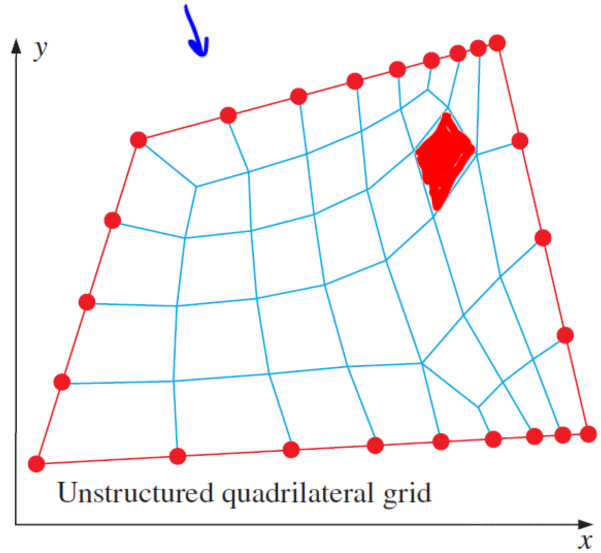


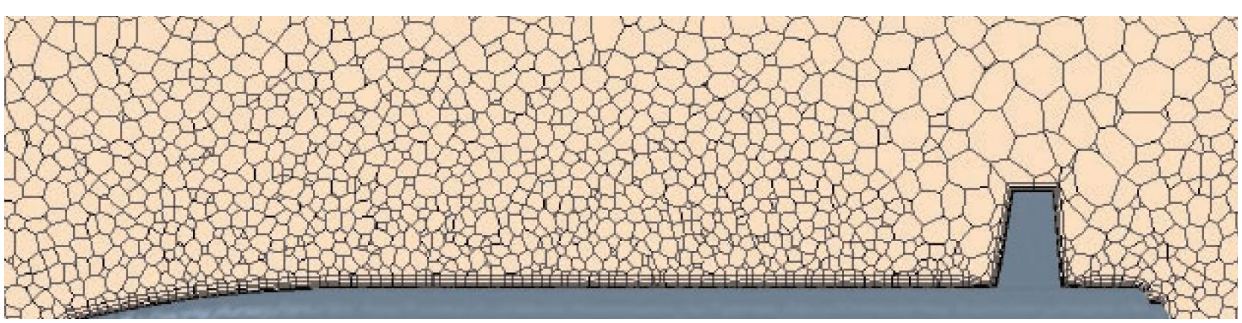


Structured mesh

UNSTRUCTURED mesh

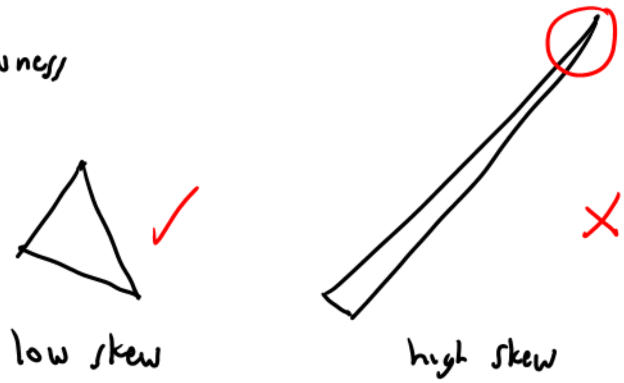




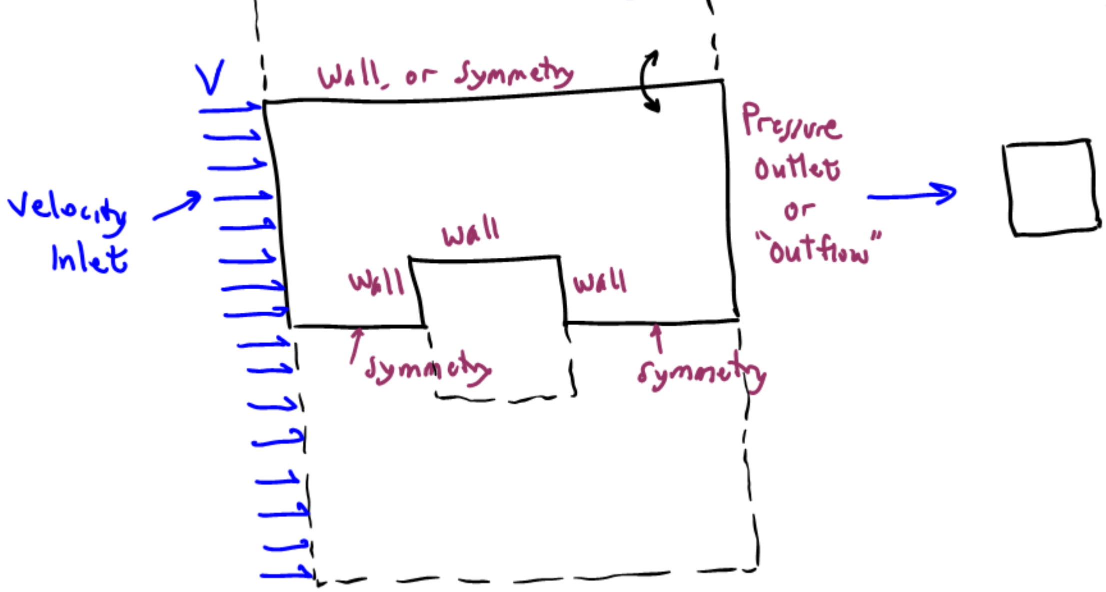


Unstructured polygonal grid, 2-D   3-D → polyhedrals

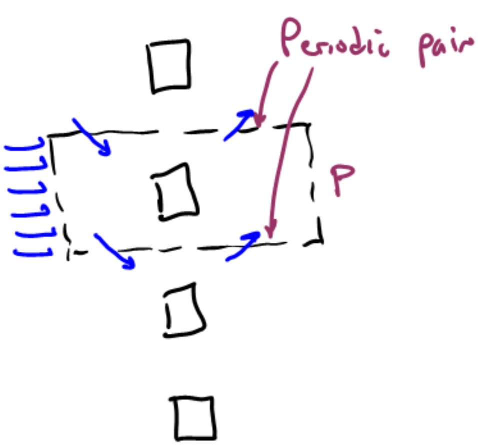
Avoid large skewness



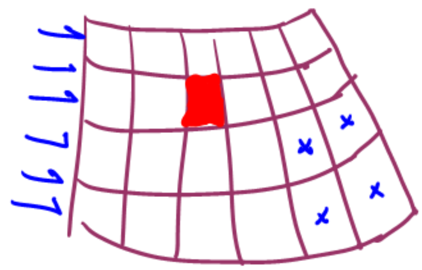
- **Step 2** Specify Boundary Conditions (on each edge (2D) or face (3D) on the boundary of computational domain)



Periodic BC



- **Step 3** Specify fluid properties ρ, μ, etc
- **Step 4** Specify numerical options & solution options (e.g., how to discretize the equations)



- **Step 5** Specify initial conditions or initial guesses for each variable in each cell
- **Step 6** Solve the discretized equations

Typically involves iteration till convergence

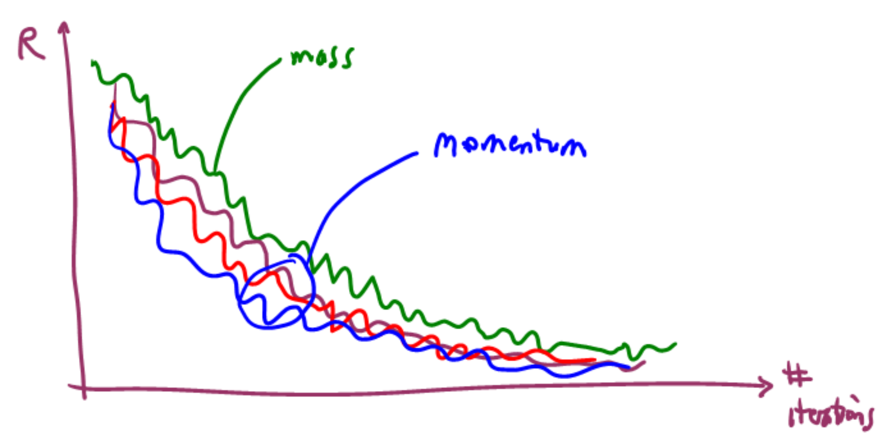
Residual → The error associated with an equation
→ There is a residual for each cell & each equation

e.g., continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

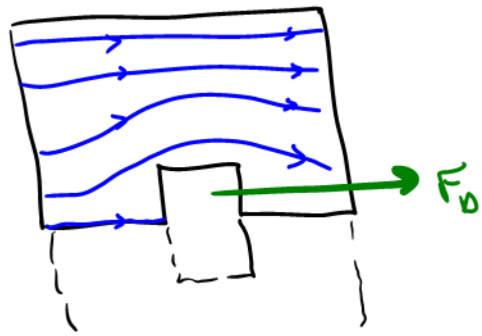
$$\frac{\Delta u}{\Delta x} + \frac{\Delta v}{\Delta y} + \frac{\Delta w}{\Delta z} = R = \text{residual}$$

As we iterate, R (hopefully) decreases



• **Step 7** Perform Post-Processing

e.g., plots

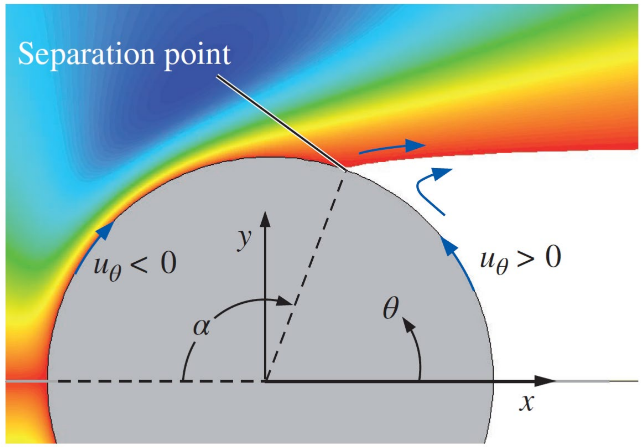


• **Step 8** Calculate Global Properties

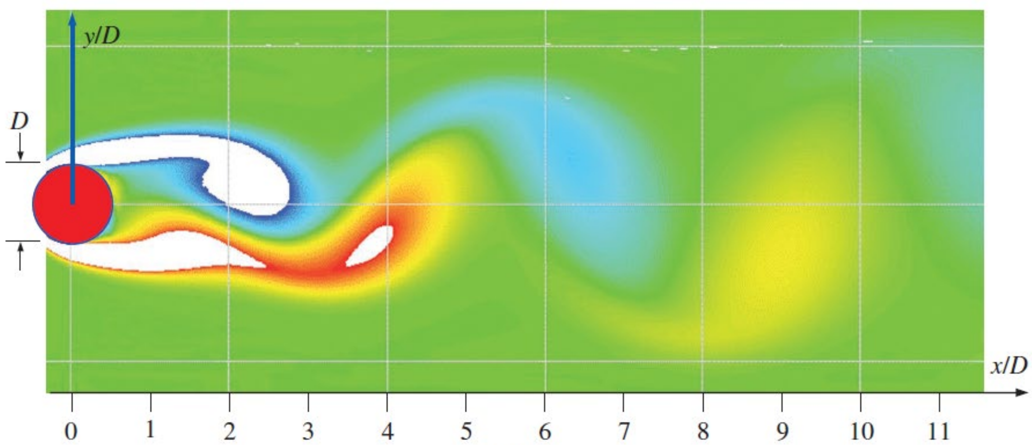
e.g., drag force

Examples of CFD Solutions (All figures from Çengel and Cimbala, Ed. 4.)

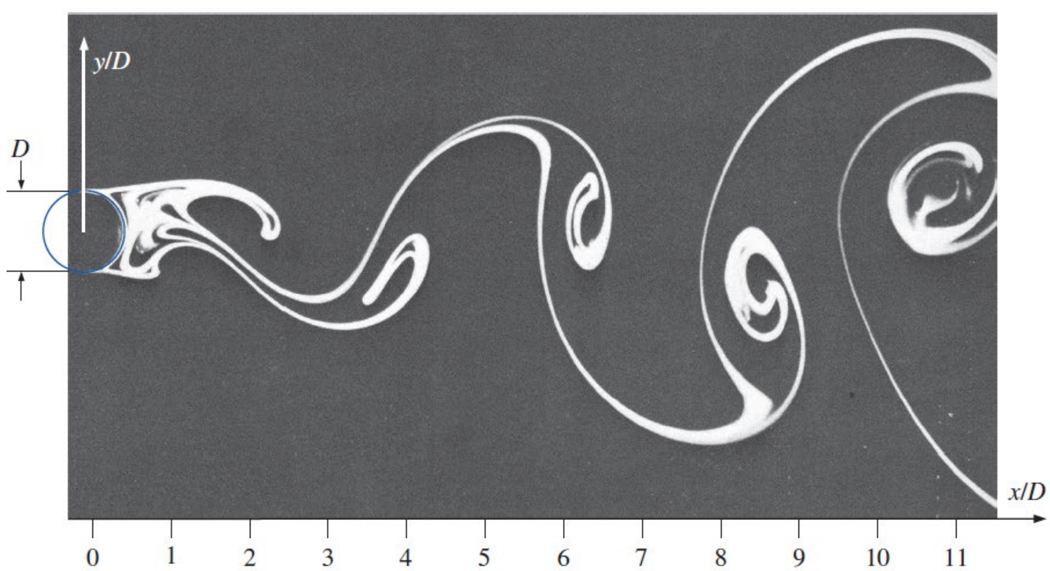
Steady Two-Dimensional Flow: This is laminar flow over a circular cylinder. Shown are contours of the of tangential velocity component.



Unsteady Two-Dimensional Flow: This is laminar flow over a circular cylinder. Image (a) shows vorticity contours from CFD. Image (b) shows dye streaklines from experiment.

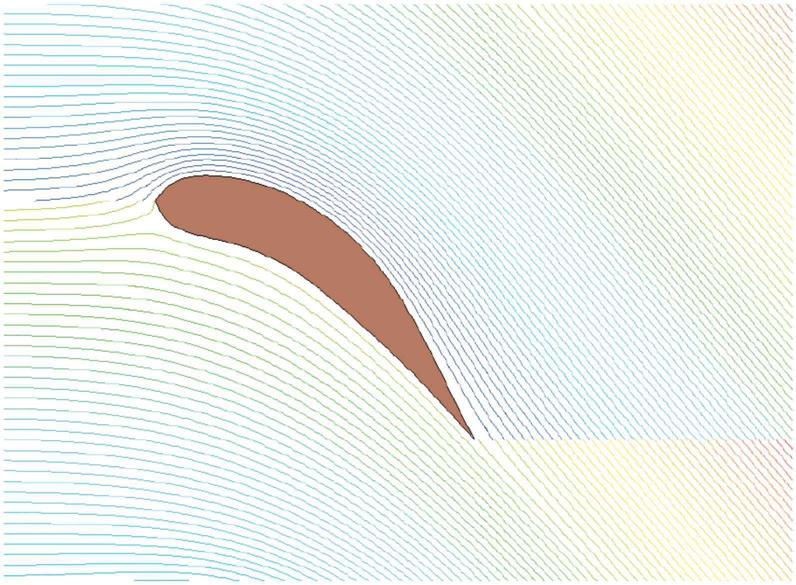


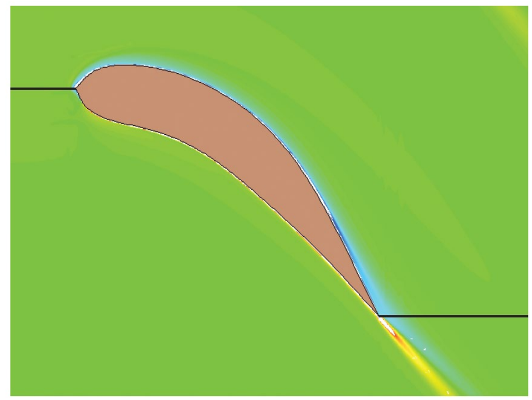
(a)



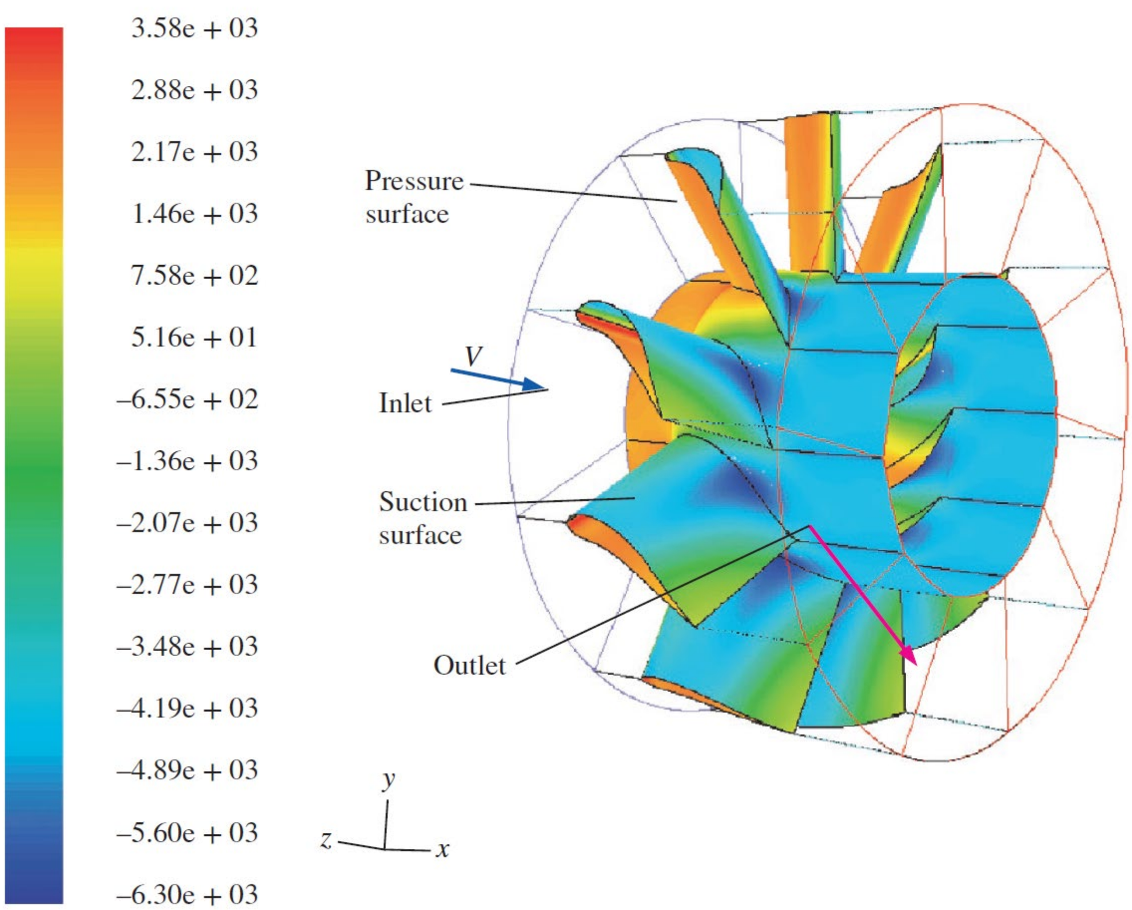
(b)

Steady Two-Dimensional Flow: This is flow over a stator blade in a periodic array of stator blades of an axial flow fan. Plotted are streamlines and vorticity contours.

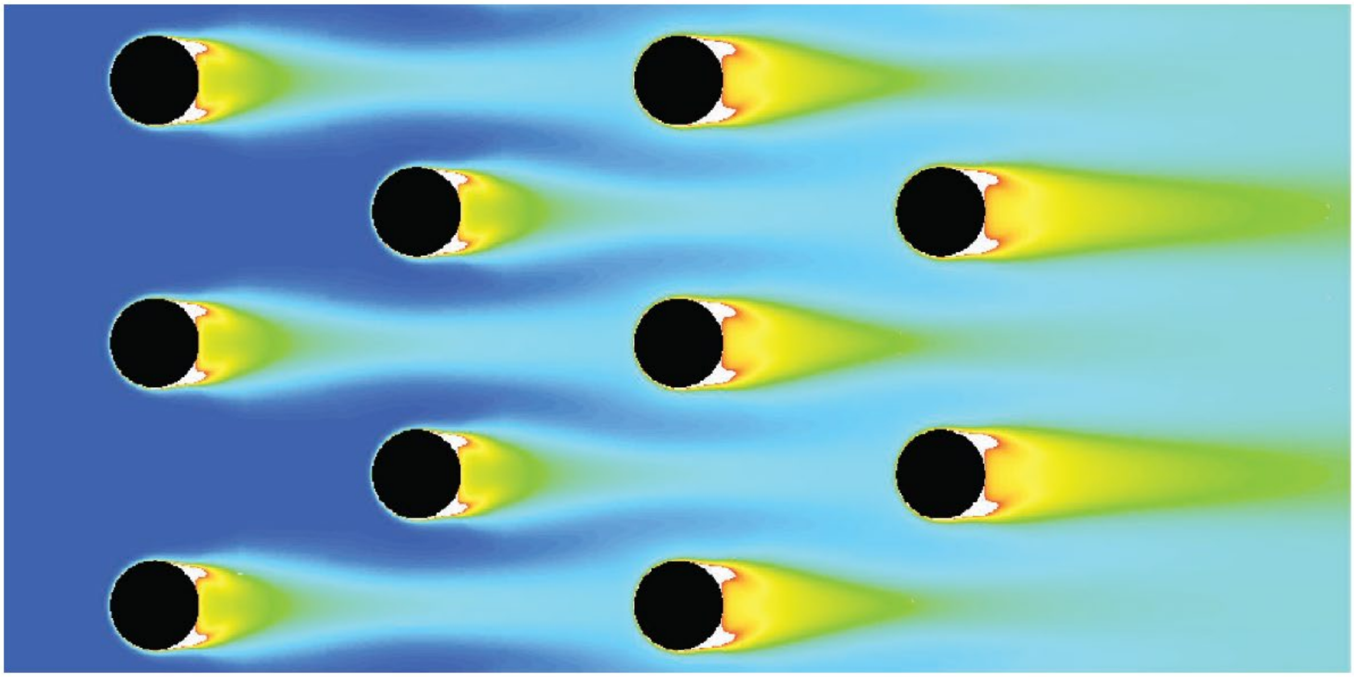




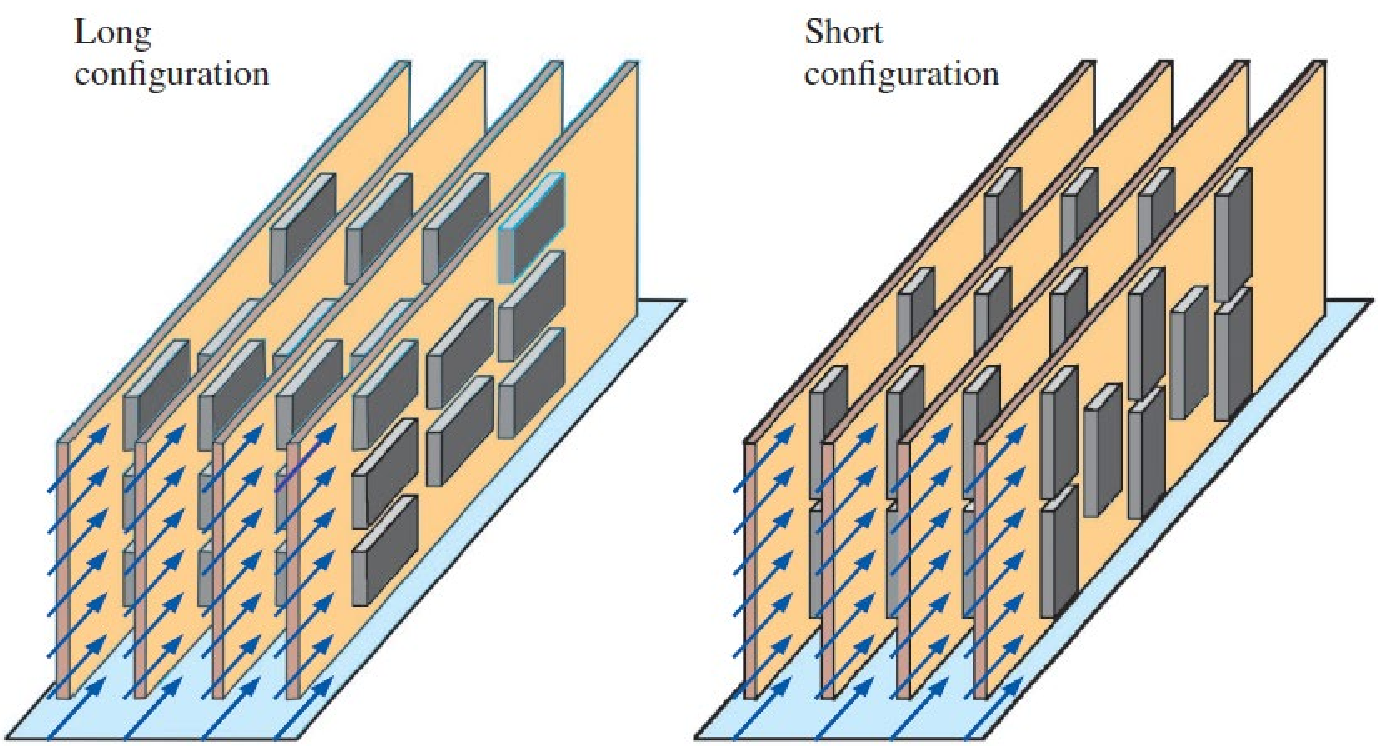
Steady Three-Dimensional Flow: This is flow through the stator blades of an axial flow fan (rotor blaces are downstream, not shown). Plotted are pressure contours.

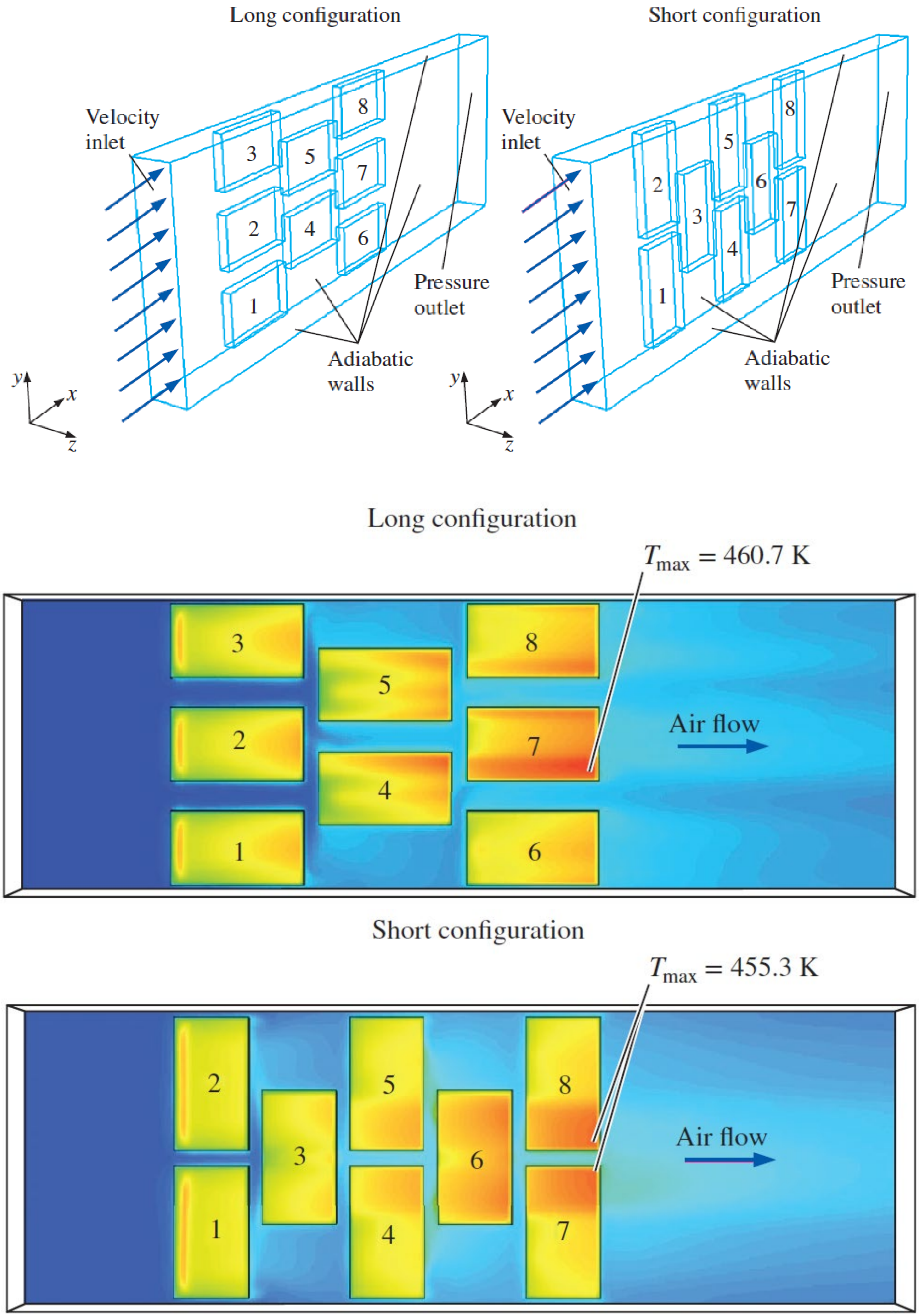


Steady Two-Dimensional Flow with Heat Transfer: This is flow over an array of cylinders in a heat exchanger. Plotted are temperature contours.



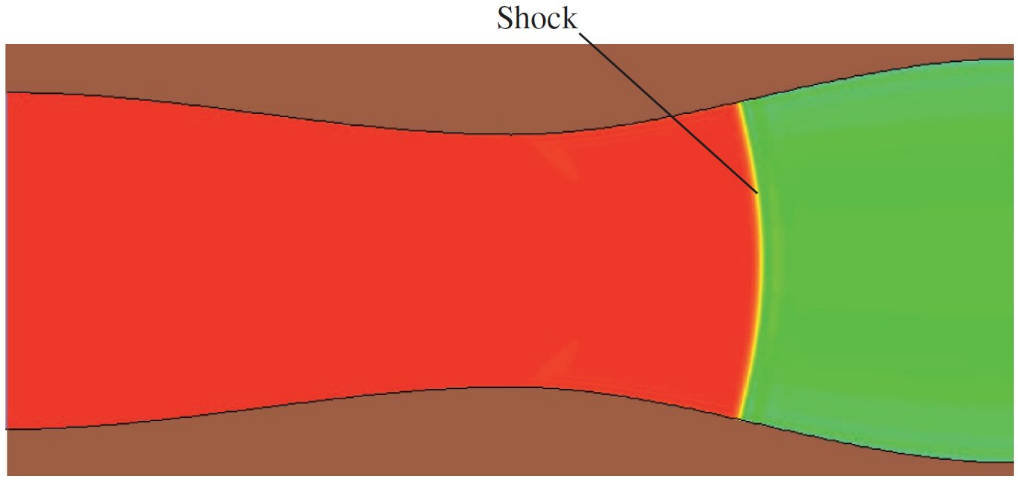
Steady Three-Dimensional Flow with Heat Transfer: This is flow over a printed circuit board, comparing chip surface temperature as a function of chip orientation. Shown below are the two configurations. Shown on the next page are boundary conditions and temperature contours.

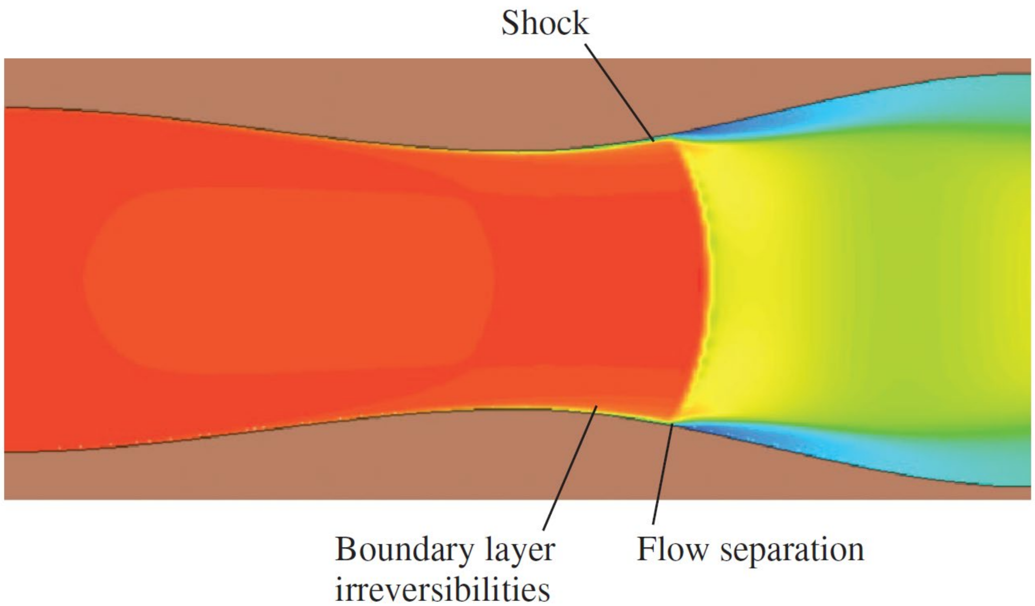




Long configuration
Short configuration
Velocity inlet
Pressure outlet
Adiabatic walls
y
x
z
1
2
3
4
5
6
7
8
Long configuration
T_{max} = 460.7 K
Air flow
Short configuration
T_{max} = 455.3 K
Air flow

Steady 2-D Compressible Flow: This is flow through a converging-diverging nozzle, generating supersonic flow and a shock wave. Plotted are contours of stagnation pressure. In the top image, friction on the walls is not included. In the bottom image, friction is included.





Steady 2-D Compressible Flow: This is supersonic flow over three two-dimensional wedges with different wedge angles: θ = (a) 10°, (b) 20°, and (c) 30°.