EXAMPLES OF SUPERPOSITION IN POTENTIAL FLOW

In this lesson, we will:

- Show some examples and applications of **Superposition in Potential (Irrotational) Flow**
- Introduce the **Method of Images** and how to apply it to potential flows

The Rankine Half-Body

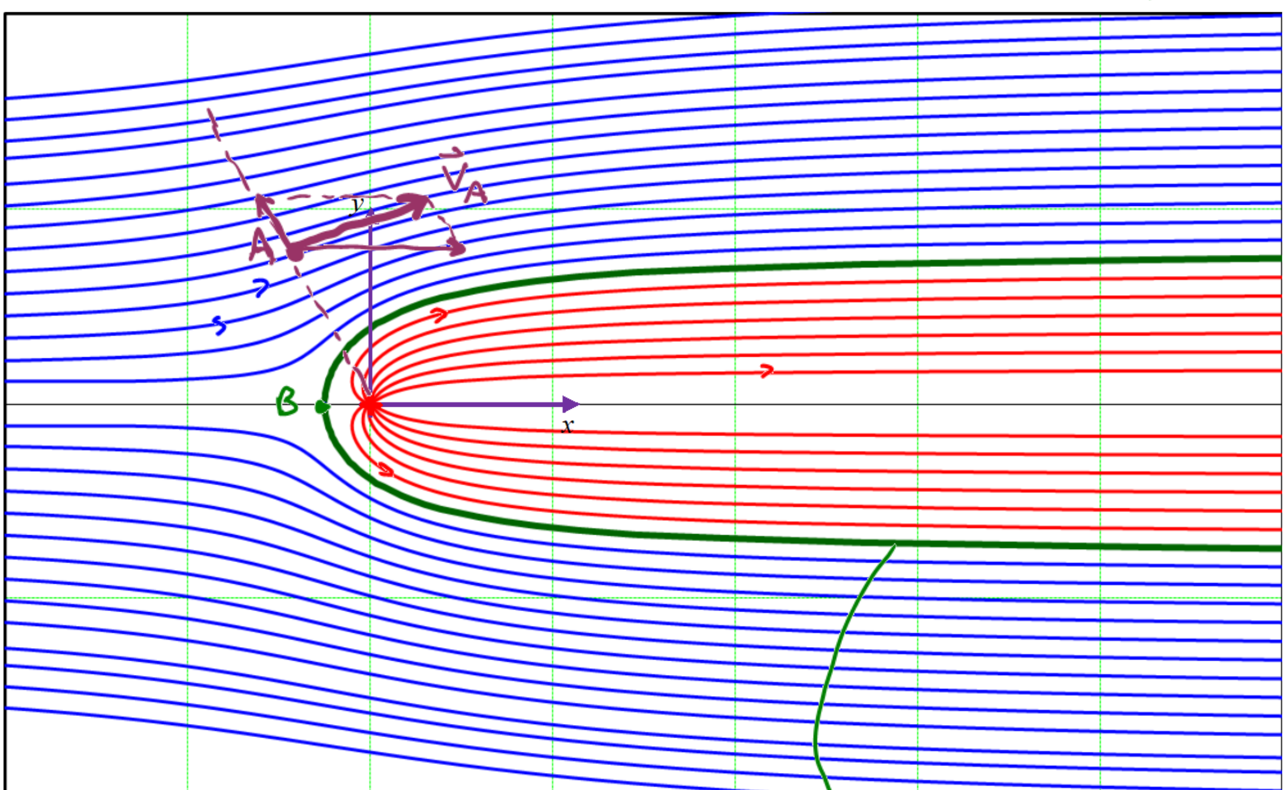
Superpose
- Uniform stream V
- Line source @ origin $\dot{\forall}/L$

add component flows to generate a new, more involved flow

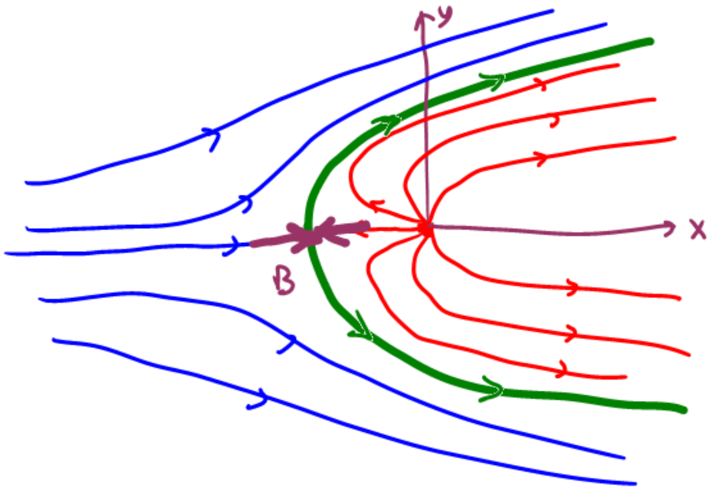
- Let's add stream functions $\psi = Vy + \frac{\dot{\forall}/L}{2\pi}\theta$ $(y = r\sin\theta)$

$$\boxed{\psi = Vr\sin\theta + \frac{\dot{\forall}/L}{2\pi}\theta}$$

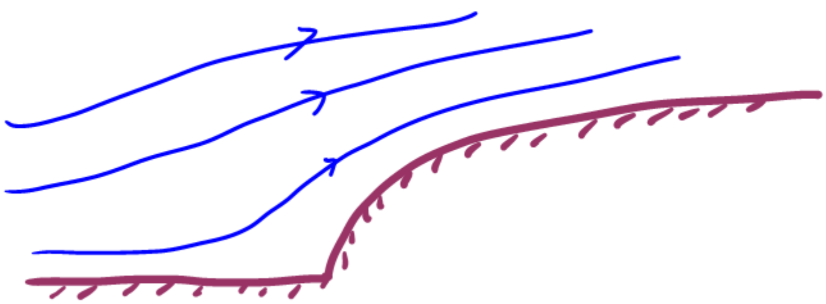
From this eq. for ψ, we can
- Plot streamlines (ψ = const)
- Calculate velocity field
- Then calculate pressure field

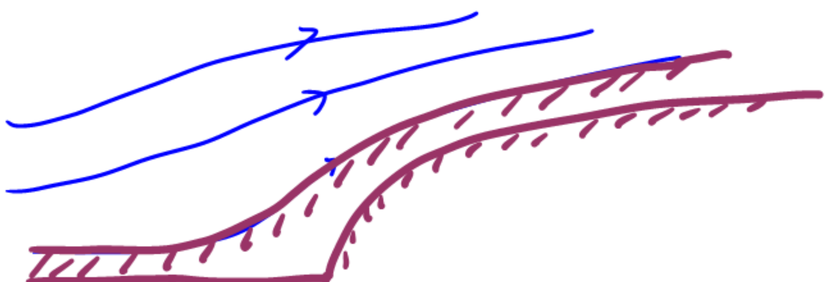


At B, $\vec{V} = 0$ B is a stagnation point



ANY STREAMLINE IN A POTENTIAL FLOW CAN BE THOUGHT OF AS A WALL





ANALYSIS:

$$\boxed{\psi = V r \sin\theta + \frac{\dot{\forall}/L}{2\pi}\theta}$$

$$u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{1}{\cancel{r}} V \cancel{r} \cos\theta + \frac{1}{r}\frac{\dot{\forall}/L}{2\pi} \Rightarrow \boxed{u_r = V\cos\theta + \frac{1}{2\pi r}\frac{\dot{\forall}}{L}}$$

$$u_\theta = -\frac{\partial \psi}{\partial r} = -V\sin\theta \Rightarrow \boxed{u_\theta = -V\sin\theta}$$

$u_\theta = 0$ when $\theta = 0, \pi, 2\pi, \ldots$

★ STAGNATION PT → set $\vec{V} = 0 \Rightarrow u_r = 0 \quad u_\theta = 0$

Here $u_\theta = 0$ when $\boxed{\theta = \pi}$

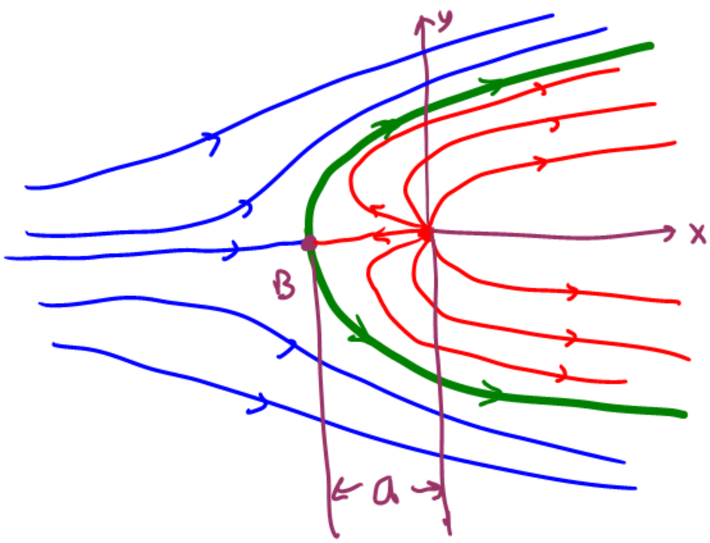
$u_r = 0$ when $-V\cos\theta = \dfrac{1}{2\pi r}\dfrac{\dot{\forall}}{L}$

$(\cos\pi = -1)$

$$V = \frac{1}{2\pi r}\frac{\dot{\forall}}{L}$$

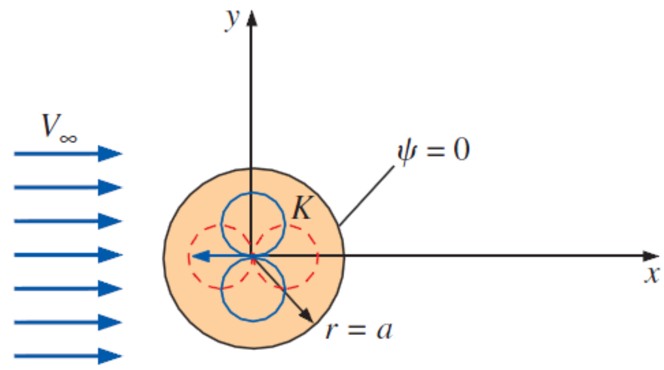
let this $\underline{r = a}$ @ stagnation pt

$$\Downarrow$$



$$\boxed{a = \frac{\dot{\forall}/L}{2\pi V}}$$ ☆

Flow Over a Circular Cylinder



Figures from Çengel and Cimbala, Ed. 4.

Given: Superpose a uniform stream of velocity V_∞ and a doublet of strength K at the origin.

To do: Plot streamlines, and discuss the flow that results from this superposition.

Solution:

- We simply add up the stream functions for the two building block flows:

$$\boxed{\psi = \psi_{\text{freestream}} + \psi_{\text{doublet}} = V_\infty y - K\frac{\sin\theta}{r}}$$

- But we know that $\boxed{y = r\sin\theta}$, thus, $\boxed{\psi = V_\infty r\sin\theta - K\frac{\sin\theta}{r}}$ ☆

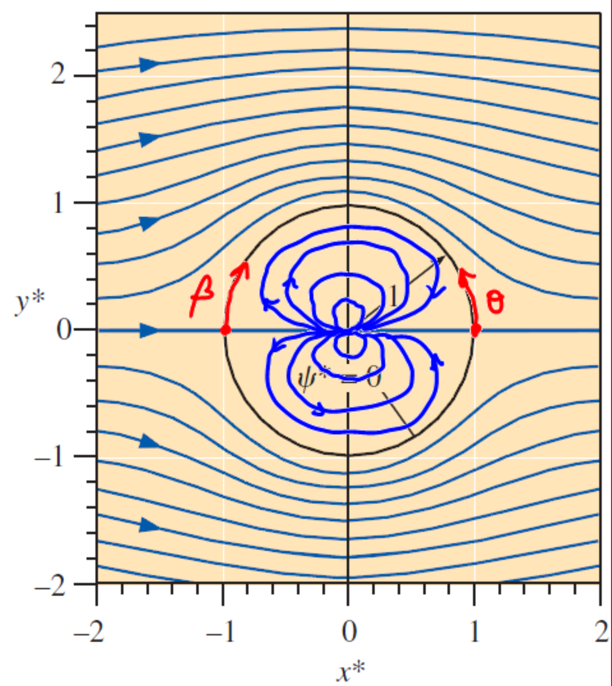
- For "convenience", and with hindsight, we choose to set $\psi = 0$ at $r = a$
  [It turns out that radius a is a special radius that becomes the radius of the circle.]

- Set $r = a$ in our equation for the stream function:

$$0 = V_\infty a\sin\theta - K\frac{\sin\theta}{a} \quad \rightarrow \quad \boxed{K = V_\infty a^2}$$

- Then our final expression for ψ becomes

$$\boxed{\psi = V_\infty \sin\theta\left(r - \frac{a^2}{r}\right)}$$



- Plot streamlines: [we plot nondimensionally, setting $x^*=x/a$ and $y^*=y/a$]

- From our equation for ψ above, we can calculate the velocity field from the definition of ψ, i.e., $u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta}$ $\quad u_\theta = -\frac{\partial\psi}{\partial r}$. See text for details. On the cylinder ($r = a$), $(r^* = 1)$

$\boxed{u_r = 0 \quad u_\theta = -2V_\infty \sin\theta}$ VELOCITY COMPONENTS ON THE "WALL" OF THE CYLINDER

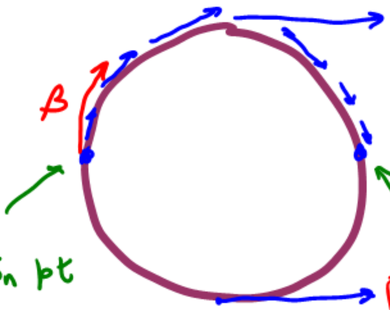
- We then apply the Most Beloved Bernoulli Equation to generate an expression for the pressure field $P + \frac{1}{2}\rho V^2 = P_\infty + \frac{1}{2}\rho V_\infty^2$

- We define the **pressure coefficient**, $C_p = \frac{P - P_\infty}{\frac{1}{2}\rho V_\infty^2} = 1 - \frac{V^2}{V_\infty^2}$

- *On the cylinder*, it turns out that $\boxed{C_p = 1 - 4\sin^2\beta}$ where β is the angle from the nose.

$\boxed{V^2 = 4V_\infty^2 \sin^2\theta}$

$V_\infty \rightarrow$



Front stagnation pt

rear stagnation point

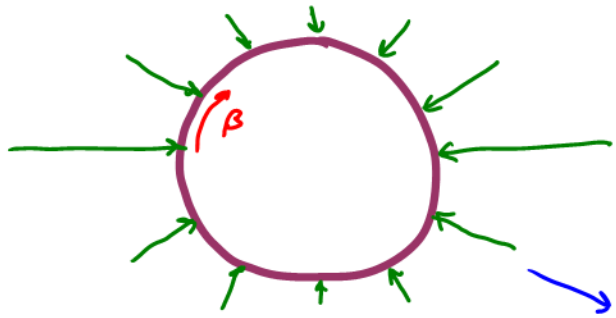
$\boxed{V_{top} = 2V_\infty}$ (@ $\beta = 90°$)

MAX SPEED

[since $u_\theta = -2V_\infty$]

$\boxed{V_{bottom} = 2V_\infty}$

$\boxed{P = P_\infty}$

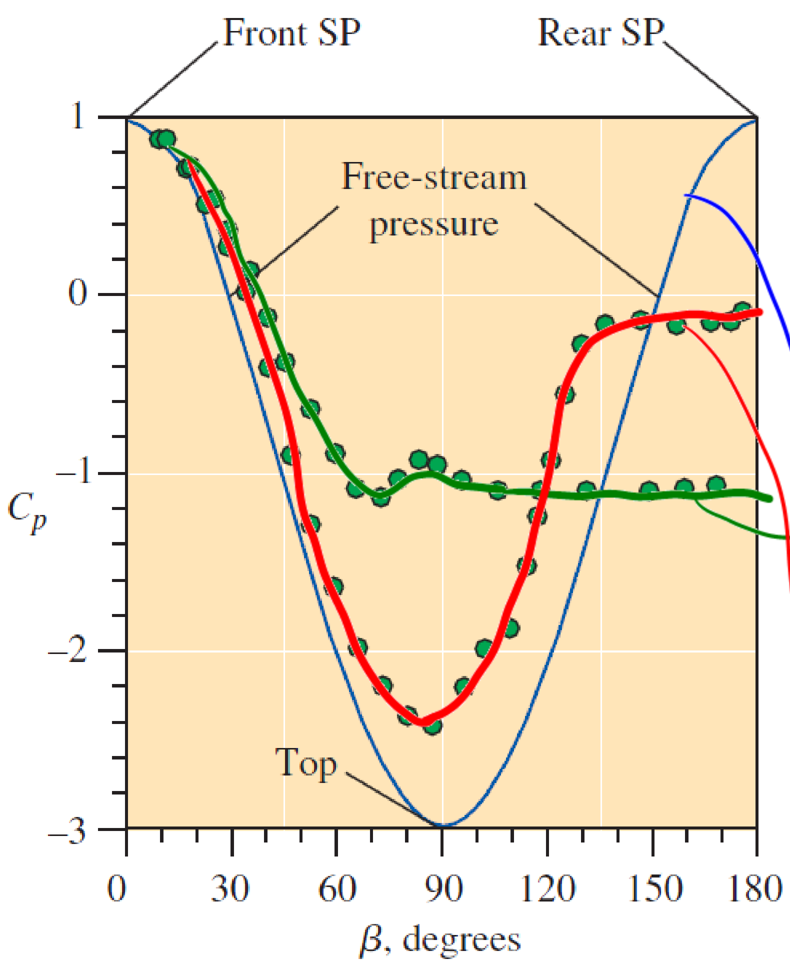


MAX P @ the two stagnation points

NOTICE: P = symmetric both front & back & top & bottom

★ $\boxed{\text{DRAG = 0 ON } \underline{\underline{\text{ANY}}} \text{ NON-LIFTING BODY IN IRROTATIONAL FLOW}}$

D'Alembert's Paradox



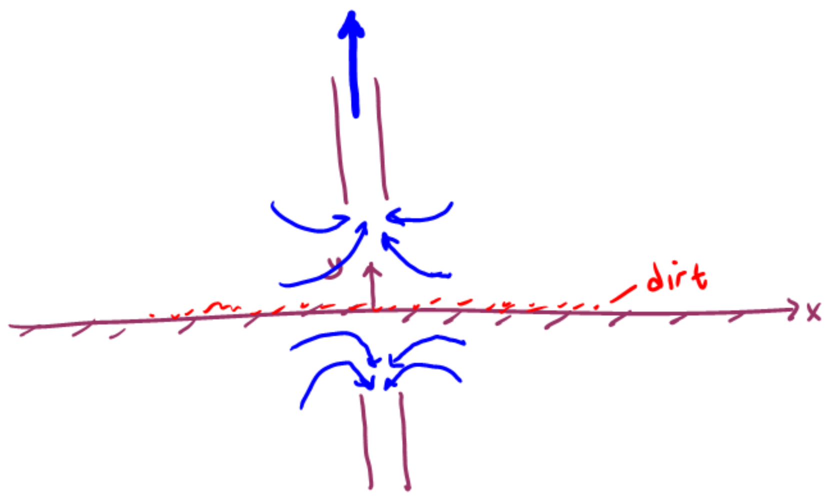
OUR POTENTIAL FLOW ANALYSIS

LAMINAR BL ON THE CYLINDER WALLS

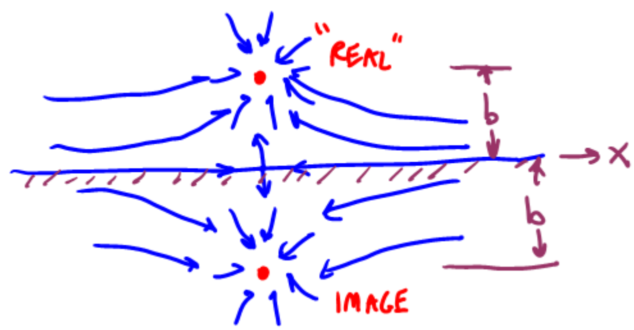
TURBULENT BL ON THE CYLINDER WALLS

The Method of Images

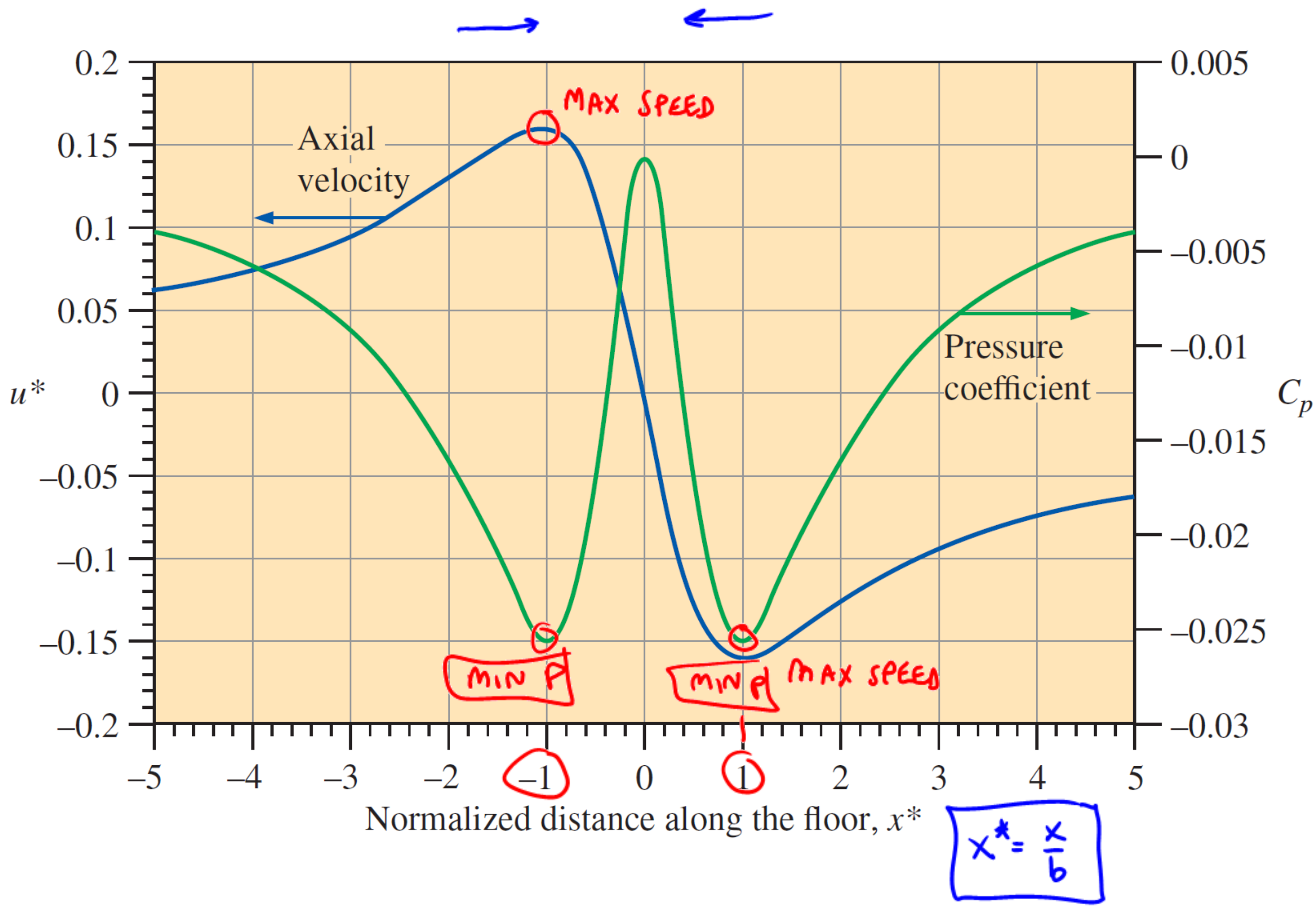
EXAMPLE: FLOW INTO THE 2-D ATTACHMENT OF A VACUUM CLEANER



WE MODEL THIS FLOW BY TWO SINKS



Results:



VIDEO BY CHRIS SMITH