Today, we will:

- Begin Chapter 4 – FLUID KINEMATICS
- Discuss the material acceleration and the material derivative, and show examples
- Discuss various kinds of flow patterns and flow visualization techniques
- Begin to discuss other kinematic properties (motion and deformation of fluid particles)

III. FLUID KINEMATICS

A. Descriptions of Fluid Flow – there are two ways to describe fluid flow:

1. Lagrangian description

2. Eulerian description

3. Acceleration field and material derivative

Derivation of Material Acceleration (Section 4-1)

Acceleration of a fluid particle:
$$\vec{a}_{\text{particle}} = \frac{d\vec{V}_{\text{particle}}}{dt} \quad (4\text{–}6)$$

This is a *Lagrangian* description of the acceleration of a fluid particle.

However, at any instant in time t, the velocity of the particle is the same as the local value of the velocity *field* at the location $(x_{\text{particle}}(t), y_{\text{particle}}(t), z_{\text{particle}}(t))$ of the particle, since the fluid particle moves with the fluid by definition. In other words, $\vec{V}_{\text{particle}}(t) \equiv \vec{V}(x_{\text{particle}}(t), y_{\text{particle}}(t), z_{\text{particle}}(t), t)$. To take the time derivative in Eq. 4–6, we must therefore use the *chain rule*, since the dependent variable ($\vec{V}$) is a function of *four* independent variables $(x_{\text{particle}}, y_{\text{particle}}, z_{\text{particle}}, \text{and } t)$,

Recall the ***chain rule***: If f is a function of two variables, t and some variable s which is itself also a function of t, then we take the total derivative of f with respect to t as follows:
$$\frac{df}{dt} = \frac{\partial f}{\partial t}\frac{dt}{dt} + \frac{\partial f}{\partial s}\frac{ds}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial s}\frac{ds}{dt}$$
Now let's apply this chain rule to the time derivative of the fluid particle's velocity:

Note that from the Lagrangian description (following a fluid particle, x_{particle} is a function of time, since the particle's location changes with time. Thus, $x_{\text{particle}} = x_{\text{particle}}(t)$. Similarly, $y_{\text{particle}} = y_{\text{particle}}(t)$ and $z_{\text{particle}} = z_{\text{particle}}(t)$.

Thus, the acceleration of a fluid particle is calculated using the chain rule as follows:
$$\vec{a}_{\text{particle}} = \frac{d\vec{V}_{\text{particle}}}{dt} = \frac{d\vec{V}}{dt} = \frac{d\vec{V}(x_{\text{particle}}, y_{\text{particle}}, z_{\text{particle}}, t)}{dt}$$
$$= \frac{\partial \vec{V}}{\partial t}\frac{dt}{dt} + \frac{\partial \vec{V}}{\partial x_{\text{particle}}}\frac{dx_{\text{particle}}}{dt} + \frac{\partial \vec{V}}{\partial y_{\text{particle}}}\frac{dy_{\text{particle}}}{dt} + \frac{\partial \vec{V}}{\partial z_{\text{particle}}}\frac{dz_{\text{particle}}}{dt} \quad (4\text{–}7)$$

$dt/dt =$	$dx_{\text{particle}}/dt =$	$dy_{\text{particle}}/dt =$	$dz_{\text{particle}}/dt =$

Or, finally,
$$\vec{a}_{\text{particle}}(x, y, z, t) = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} \quad (4\text{–}8)$$

Vorticity and Rotationality (Section 4-5)

The **vorticity vector** is defined as the **curl of the velocity vector**,

Greek letter zeta → $$\vec{\zeta} = \vec{\nabla} \times \vec{V}$$

It turns out that **vorticity is equal to twice the angular velocity of a fluid particle**,

$$\vec{\zeta} = 2\vec{\omega}$$

Thus, ***vorticity is a measure of rotation of a fluid particle***.

if $\vec{\zeta} = 0$, the flow is irrotational

if $\vec{\zeta} \neq 0$, the flow is rotational

Vorticity vector in Cartesian coordinates:

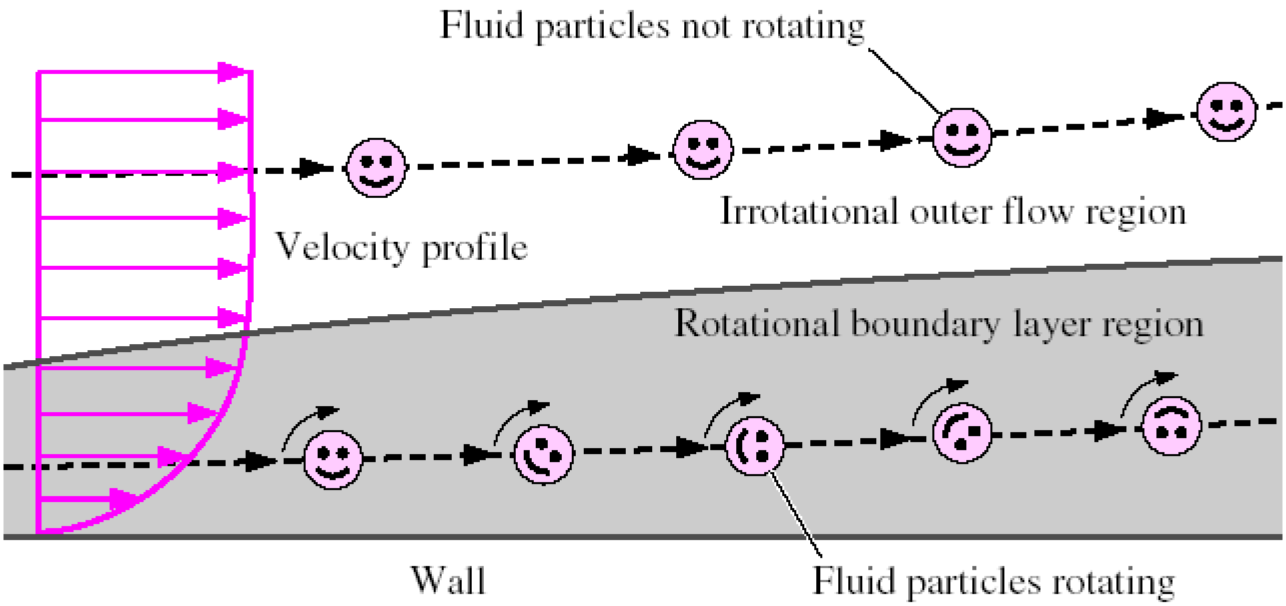
$$\vec{\zeta} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k} \qquad (4\text{–}30)$$

Vorticity vector in cylindrical coordinates:

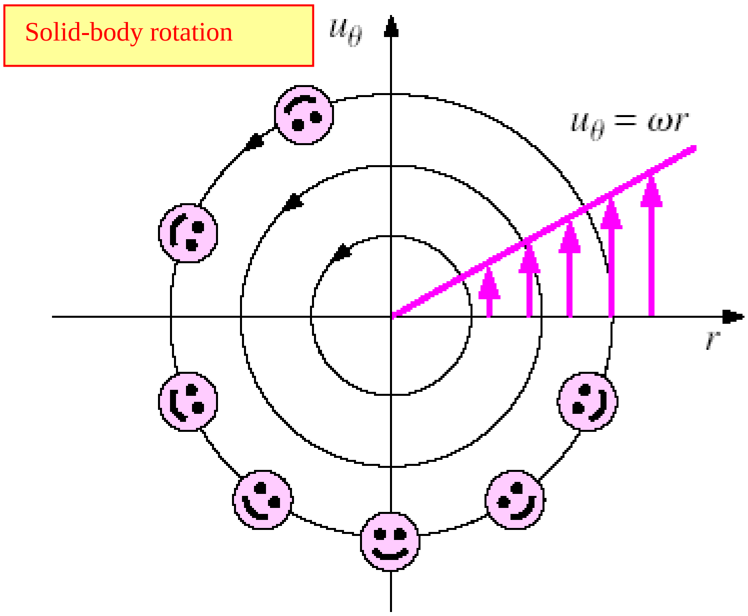
$$\vec{\zeta} = \left(\frac{1}{r}\frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z}\right)\vec{e}_r + \left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right)\vec{e}_\theta + \frac{1}{r}\left(\frac{\partial (r u_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta}\right)\vec{e}_z \qquad (4\text{–}32)$$

Examples:

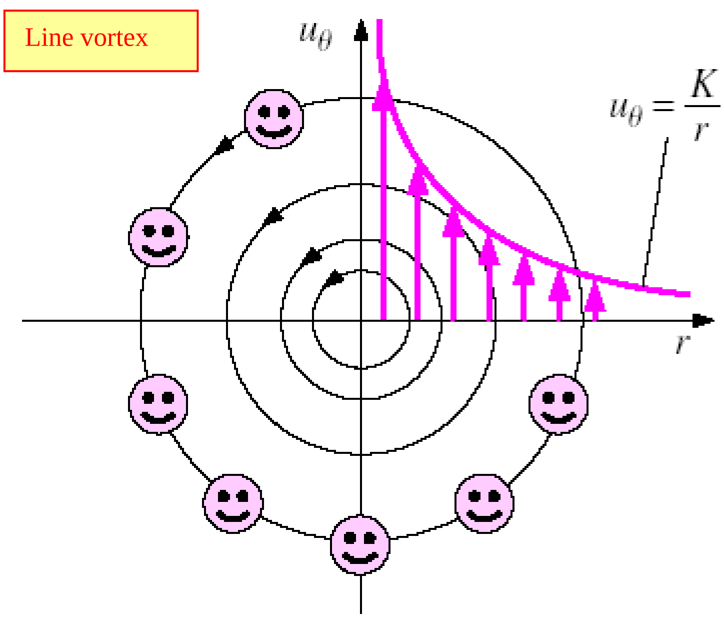
1. Inside a **boundary layer**, where viscous forces are important, the flow in this region is *rotational* ($\vec{\zeta} \neq 0$). However, outside the boundary layer, where viscous forces are not important, the flow in this region is *irrotational* ($\vec{\zeta} = 0$).



2. A **solid-body rotation** (rigid-body rotation) flow is *rotational* ($\vec{\zeta} \neq 0$). In fact, since vorticity is equal to twice the angular velocity, $\vec{\zeta} = 2\vec{\omega}$ *everywhere* in the flow field. Fluid particles rotate as they revolve around the center of the flow. This is analogous to a merry-go-round or a roundabout.



3. A **line vortex** flow, however, is *irrotational* ($\vec{\zeta} = 0$), and fluid particles do not rotate, even though they revolve around the center of the flow. This is analogous to a Ferris wheel.



See text for details and calculations.