

Today, we will:

- Continue Chapter 5 – Conservation of mass for control volumes
- Do some example problems, conservation of mass
- Begin to discuss conservation of energy

B. Conservation of Mass (continued)**1. Equations and definitions (continued)**

Last lecture, we derived the **control volume equation for conservation of mass**:

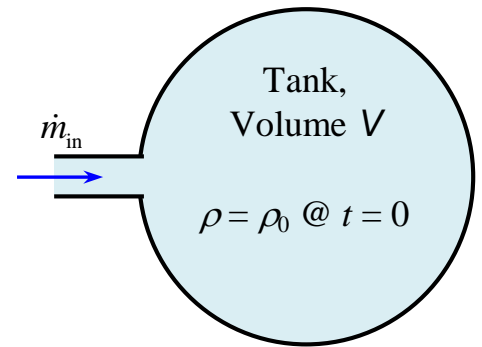
$$\frac{d}{dt} \int_{CV} \rho dV + \int_{CS} \rho (\vec{V} \cdot \vec{n}) dA = 0 \quad (1)$$

Example: Unsteady conservation of mass (flow into a tank)

Given: Air is pumped into a rigid tank of volume V . The mass flow rate of the air entering the tank is constant, \dot{m}_{in} . We assume that the process is slow enough that the air in the tank remains at the same temperature (isothermal conditions).

To do: Generate an equation for density ρ in the tank as a function of time.

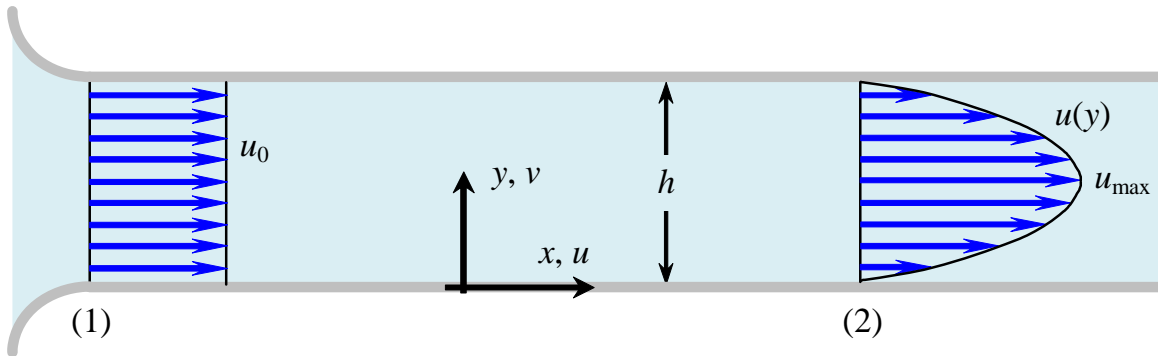
Solution:



Example: Velocity profiles in 2-D channel flow

Given: Consider steady, incompressible, two-dimensional flow of a liquid between two very long parallel plates as sketched. At the inlet (1) there is a nice bell mouth, and the velocity is nearly uniform (except for a very thin boundary layer, not shown).

- At (1), $u = u_0 = \text{constant}$, $v = 0$, and $w = 0$.
- At (2), the flow is fully developed, and $u = ay(h - y)$, $v = 0$, and $w = 0$, where a is a constant.

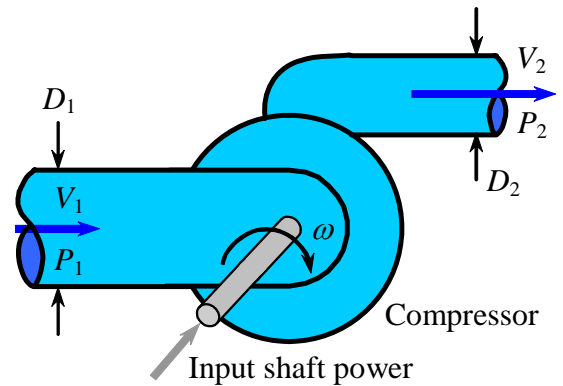


To do: Generate expressions for constant a and speed u_{\max} in terms of the given variables.

Solution:

Example: Control volume energy equation applied to an air compressor

Given: A large air compressor takes in air at absolute pressure $P_1 = 14.0$ psia, at temperature $T_1 = 80^\circ\text{F}$ (539.67 R), and with mass flow rate $\dot{m} = 20.0$ lbm/s. The diameter of the compressor inlet is $D_1 = 24.5$ inches. At the outlet, $P_2 = 70.0$ psia and $T_2 = 500^\circ\text{F}$ (959.67 R). The diameter of the compressor outlet is $D_2 = 7.50$ inches. The shaft driving the compressor supplies 3100 horsepower to the compressor.



(a) **To do:** Calculate the average velocity of the air entering the compressor.

Solution: At the inlet, $\dot{m} \approx \rho_{1, \text{avg}} V_{1, \text{avg}} A_1 = \rho_1 V_1 A_1$ where the subscripts "avg" have been dropped for convenience. Thus, $V_1 = \frac{\dot{m}}{\rho_1 A_1} = \frac{4\dot{m}}{\pi \rho_1 D_1^2} = \frac{4RT_1 \dot{m}}{\pi P_1 D_1^2}$, where we have used the ideal gas law

$P = \rho RT$ to calculate the density of the air. Substitution of the values yields

$$V_1 = \frac{4RT_1 \dot{m}}{\pi P_1 D_1^2} = \frac{4 \left(53.34 \frac{\text{ft} \cdot \text{lbf}}{\text{lbm} \cdot \text{R}} \right) (539.67 \text{ R}) \left(20.0 \frac{\text{lbm}}{\text{s}} \right)}{\pi \left(14.0 \frac{\text{lbf}}{\text{in}^2} \right) (24.5 \text{ in})^2} = 87.229 \frac{\text{ft}}{\text{s}} \approx \mathbf{87.2 \frac{\text{ft}}{\text{s}}}$$

(b) **To do:** Calculate the average velocity of the air leaving the compressor.

Solution: Similarly, using the pressure, temperature, and diameter at the compressor outlet, we get $V_2 = 331.051$ ft/s, or $V_2 = \mathbf{331. \text{ ft/s}}$ (to three significant digits of precision)

(c) **To do:** Calculate the net rate of heat transfer from the air compressor into the room in units of Btu/hr.

Solution: First we choose a control volume. We draw the control volume around the entire compressor, cutting through the shaft, and cutting through the inlet and outlet, as sketched. Note that we draw the net rate of heat transfer $\dot{Q}_{\text{net in}}$ into the control volume to keep the signs straight. We expect a negative value since the compressor will actually give off heat into the room.

Next, we apply the approximate form of the control volume energy equation,

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft net in}} = \underbrace{\frac{d}{dt} \int_{\text{CV}} \rho dV}_{\text{steady}} + \sum_{\text{out}} \dot{m} \left(h + \frac{V^2}{2} + gz \right) - \sum_{\text{in}} \dot{m} \left(h + \frac{V^2}{2} + gz \right)$$

and we solve for $\dot{Q}_{\text{net in}}$.

Solution to be completed in class.

