Today, we will:

- Derive the "head" form of the energy equation
- Discuss pumps and turbines and their efficiencies
- Do some example problems – energy equation with pumps and turbines
- Discuss grade lines – energy grade line and hydraulic grade line

C. Conservation of Energy (continued)

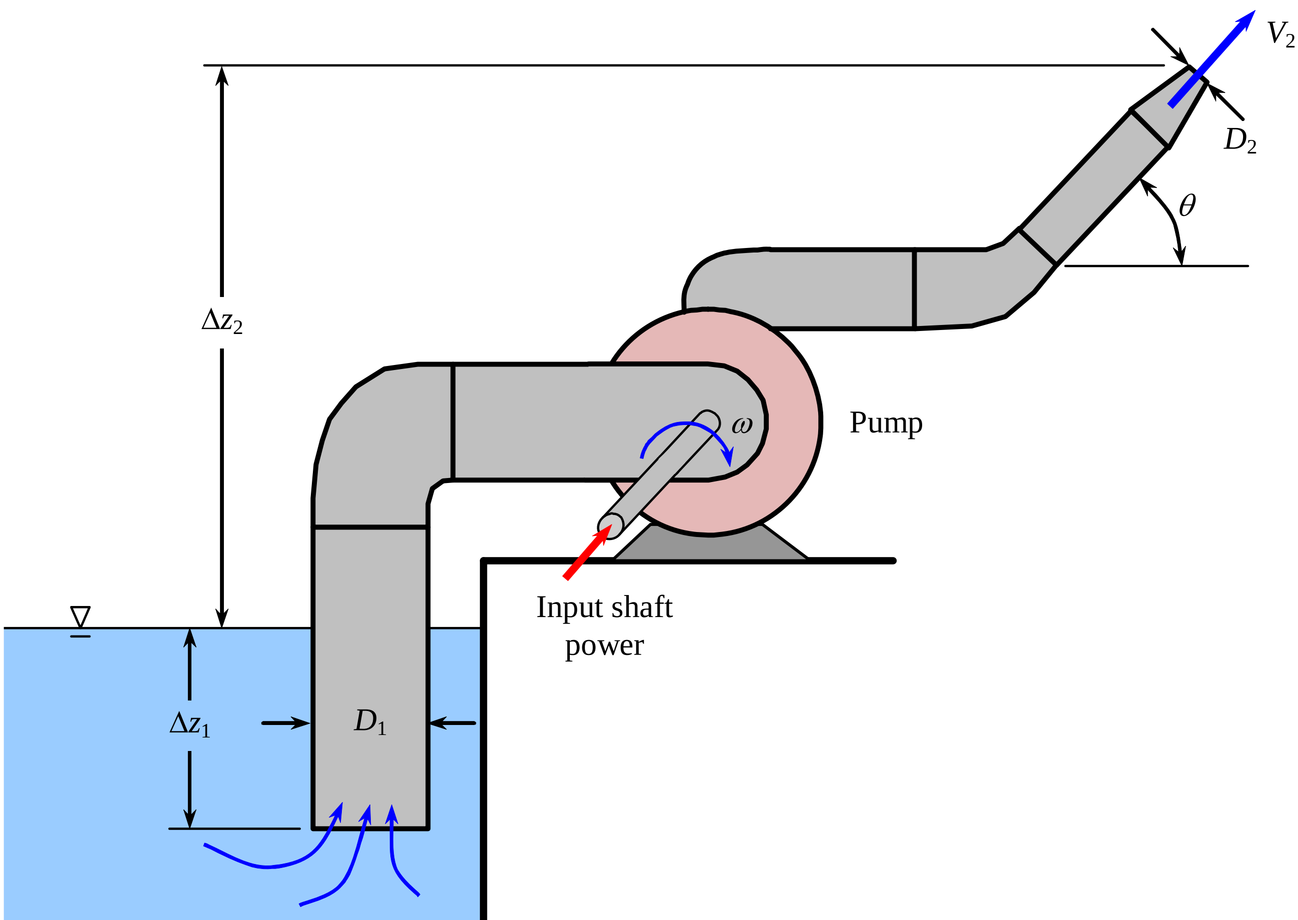
4. The "head" form of the energy equation

From previous lecture…the Steady-State Steady-Flow (SSSF) conservation of energy equation for a fixed control volume:

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{net shaft in}} = \sum_{\text{outlets}} \dot{m}\left(h + \alpha\frac{V^2}{2} + gz\right) - \sum_{\text{inlets}} \dot{m}\left(h + \alpha\frac{V^2}{2} + gz\right) \qquad \textbf{(1)}$$

Example – Fire-fighting water pump

Given: A self-priming pump is used to draw water from a lake and shoot it through a nozzle, as sketched. The diameter of the pump inlet is D_1 = 12.0 cm. The diameter of the nozzle outlet is D_2 = 2.54 cm, and the average velocity at the nozzle outlet is V_2 = 65.8 m/s. The pump efficiency is 80%. The vertical distances are Δz_1 = 1.00 m and Δz_2 = 2.00 m. The irreversible head losses in the piping system (not counting inefficiencies associated with the pump itself) are estimated as h_L = 4.50 m of equivalent water column height. *Note*: Later on, in Chapter 8, you will learn how to calculate the irreversible head losses associated with piping systems on your own. For now, they are given.



(**a**) **To do**: Calculate the volume flow rate of the water in m^3/hr and gallons per minute (gpm).

Solution: At the outlet, $\dot{V} = V_{2,\text{ avg}} A_2 = V_2 \frac{\pi D_2^{\ 2}}{4} = \left(65.8 \frac{\text{m}}{\text{s}}\right) \frac{\pi (0.0254 \text{ m})^2}{4} = 0.033341 \frac{\text{m}^3}{\text{s}}$, where we have dropped the subscript "avg" for convenience. We convert to the required units as follows:

$\dot{V} = 0.033341 \frac{\text{m}^3}{\text{s}} \left(\frac{3600 \text{ s}}{\text{hr}}\right) = \mathbf{120.\frac{m^3}{hr}}$ and $\dot{V} = 0.033341 \frac{\text{m}^3}{\text{s}} \left(\frac{15{,}850 \text{ gpm}}{\text{m}^3\text{/hr}}\right) = \mathbf{528.\ gpm}$, where both answers are given to three significant digits of precision.

(**b**) **To do**: Calculate the power delivered by the pump to the water, i.e. calculate the ***water horsepower*** $\dot{W}_{\text{water horsepower}}$ in units of kW.

(**c**) **To do**: Calculate the required shaft power to the pump, i.e. calculate the ***brake horsepower*** bhp in units of kW.

Solutions for parts (b) and (c) to be completed in class.

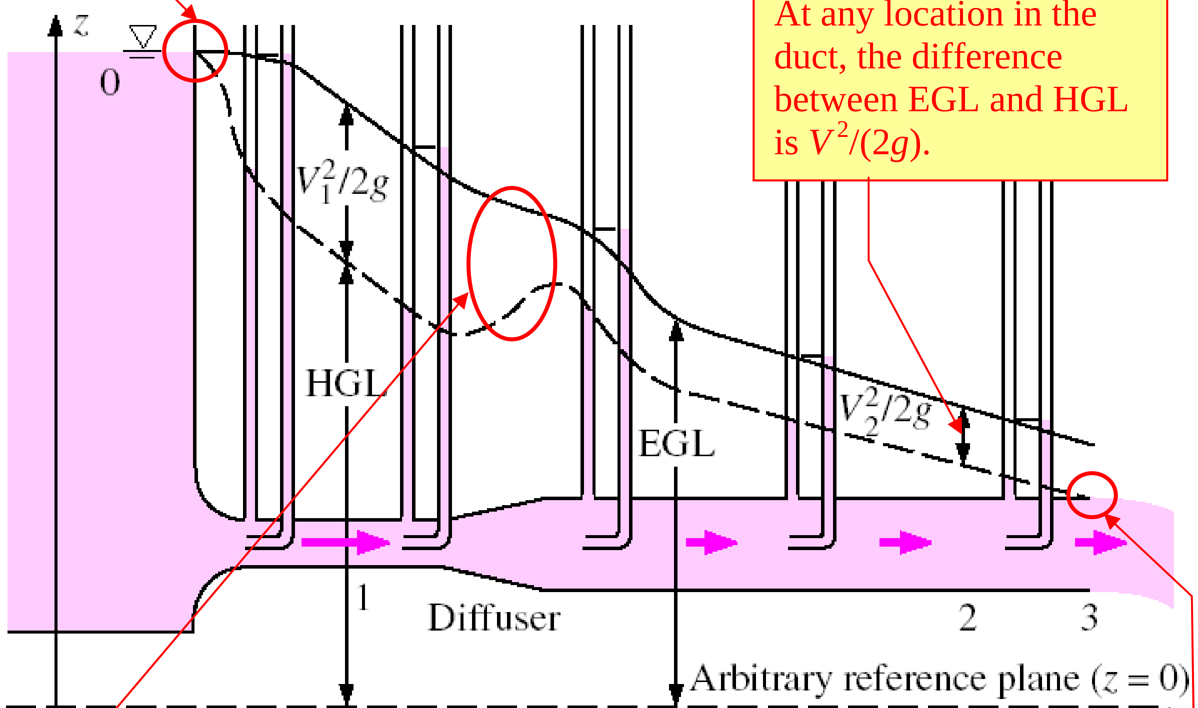
Example of Grade Lines in a Fluid Flow

At point 0, HGL = EGL inside the tank, since the fluid is at rest ($V = 0$). Neither EGL or HGL can rise above this value unless work is added to the flow (e.g., with a pump).

FIGURE 5-35

The *hydraulic grade line* (HGL) and the *energy grade line* (EGL) for free discharge from a reservoir through a horizontal pipe with a diffuser.

At any location in the duct, the difference between EGL and HGL is $V^2/(2g)$.



EGL continually falls due to irreversible losses, but HGL can rise or fall. Overall, however, HGL also must fall. In fact, HGL can *never* rise above EGL.

Since the jet exits at atmospheric pressure at the outlet of the pipe, $P_3 = P_{atm}$, and HGL is equal to the height of the free surface of the liquid.