Today, we will:

- Discuss dimensional homogeneity
- Discuss dimensional analysis and similarity, and the method of repeating variables

IV. DIMENSIONAL ANALYSIS AND MODELING (Chapter 7)

A. Primary Dimensions (see previous lecture): {m}, {L}, {t}, {T}, {I}, {C}, {N}.
All other dimensions can be formed by combination of these 7 primary dimensions.

B. Dimensional Homogeneity

Steps in the Method of Repeating Variables

There are 6 steps that comprise the method of repeating variables. These are listed concisely in Fig. 7-22 in the text, as repeated below:

The Method of Repeating Variables

Step 1: List the parameters in the problem and count their total number n.

Step 2: List the primary dimensions of each of the n parameters.

Step 3: Set the *reduction* j as the number of primary dimensions. Calculate k, the expected number of Π's,

$$k = n - j$$

Step 4: Choose j *repeating parameters*.

Step 5: Construct the k Π's, and manipulate as necessary.

Step 6: Write the final functional relationship and check your algebra.

Step 4 is often the most difficult or mysterious step. There are guidelines provided in Table 7-3, but it takes practice to know which repeating variables to choose wisely.

FIGURE 7–22
A concise summary of the six steps that comprise the *method of repeating variables*.

The final functional relationship is given as the *dependent* Π, Π_1, as a function of the *independent* Π's, Π_2, Π_3, … , Π_k, i.e., $\Pi_1 = f\left(\Pi_2, \Pi_3, \ldots, \Pi_k\right)$

Guidelines for choosing the repeating variables in Step 4 of the method of repeating variables: (See Table 7-3 in the text for more details):

1. Never pick the *dependent* variable. Otherwise, it may appear in all the Π's, which is undesirable.
2. The chosen repeating parameters must not *by themselves* be able to form a dimensionless group. Otherwise, it would be impossible to generate the rest of the Π's.
3. The chosen repeating parameters must represent *all* the primary dimensions in the problem.
4. Never pick parameters that are already dimensionless. These are Π's already, all by themselves.
5. Never pick two parameters with the *same* dimensions or with dimensions that differ by only an exponent.
6. Whenever possible, choose dimensional constants over dimensional variables so that only *one* Π contains the dimensional variable.
7. Pick common parameters since they may appear in each of the Π's.

Guidelines for Manipulating the Π Parameters

There are several guidelines for manipulating the Π parameters. These guidelines are listed concisely in Table 7-4 in the text, as summarized below: See Table 7-4 for more details.

1. We may impose a constant (dimensionless) exponent on a Π or perform a functional operation on a Π.
2. We may multiply a Π by a pure (dimensionless) constant.
3. We may form a product (or quotient) of any Π with any other Π in the problem to replace one of the Π's.
4. We may use any of guidelines 1 to 3 in combination.
5. We may substitute a dimensional parameter in the Π with other parameter(s) of the same dimensions.

The goal is to get each Π into a form that looks like one of the common *established* nondimensional parameters that are listed in Table 7-5 in the text. Some of the most popular and often-used ones are listed below. A more exhaustive list is given in the text.

TABLE 7–5

Some common established nondimensional parameters or Π's encountered in fluid mechanics and heat transfer*

Name	Definition	Ratio of Significance
Darcy friction factor	$f = \frac{8\tau_w}{\rho V^2}$	$\frac{\text{Wall friction force}}{\text{Inertial force}}$
Drag coefficient	$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$	$\frac{\text{Drag force}}{\text{Dynamic force}}$
Froude number	$\text{Fr} = \frac{V}{\sqrt{gL}} \left(\text{sometimes } \frac{V^2}{gL}\right)$	$\frac{\text{Inertial force}}{\text{Gravitational force}}$
Lift coefficient	$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A}$	$\frac{\text{Lift force}}{\text{Dynamic force}}$
Mach number	$\text{Ma (sometimes } M) = \frac{V}{c}$	$\frac{\text{Flow speed}}{\text{Speed of sound}}$
Reynolds number	$\text{Re} = \frac{\rho V L}{\mu} = \frac{VL}{\nu}$	$\frac{\text{Inertial force}}{\text{Viscous force}}$
Strouhal number	$\text{St (sometimes S or Sr)} = \frac{fL}{V}$	$\frac{\text{Characteristic flow time}}{\text{Period of oscillation}}$

Reynolds number is the most important nondimensional parameter in fluid mechanics.