**Today, we will**:

- Discuss dimensional analysis of turbines
- Do an example problem – dimensional analysis with turbines
- Discuss piping networks – how to deal with pipes in series or in parallel

b. Dimensionless parameters in turbine performance

We perform exactly the same dimensional analysis for turbines as we did for pumps. Result:

Dimensionless Parameters: $C_Q = \dfrac{\dot{V}}{\omega D^3}$ $C_H = \dfrac{gH}{\omega^2 D^2}$ $C_P = \dfrac{bhp}{\rho \omega^3 D^5}$

Capacity coefficient Head coefficient Power coefficient

**Example: Scaling up a hydroturbine**

**Given**: An existing hydroturbine (A): Fluid is water at 20°C, $D_A$ = 1.95 m, $\dot{n}_A = 120$ rpm, $bhp_A$ = 220 MW, and $\dot{V}_A = 335\ \text{m}^3/\text{s}$ at $H_A = 72.4$ m. We are designing a new turbine (B) that is geometrically similar, still uses water at 20°C, and $\dot{n}_B = 120$ rpm, but $H_B = 97.4$ m. [Dam B has a higher gross head available than Dam A.]

**To do**: (a) Calculate $D_B$ and $\dot{V}_B$ for operation of turbine B at a homologous point.

(b) Calculate $bhp_B$ and estimate the turbine efficiency of both turbines.

**Solution**:

(a) At homologous points, the two turbines are dynamically similar. Apply the affinity laws:

$$C_{H,A} = \frac{gH_A}{\omega_A^{\ 2} D_A^{\ 2}} = C_{H,B} = \frac{gH_B}{\omega_B^{\ 2} D_B^{\ 2}} \quad \rightarrow \quad \text{solve for } D_B = D_A\left(\frac{\omega_A}{\omega_B}\right)\sqrt{\frac{H_B}{H_A}} = D_A\left(\frac{\dot{n}_A}{\dot{n}_B}\right)\sqrt{\frac{H_B}{H_A}}$$

Plug in numbers:

Similarly, $$C_{Q,A} = \frac{\dot{V}_A}{\omega_A D_A^{\ 3}} = C_{Q,B} = \frac{\dot{V}_B}{\omega_B D_B^{\ 3}} \quad \rightarrow \quad \dot{V}_B = \dot{V}_A\left(\frac{\omega_B}{\omega_A}\right)\left(\frac{D_B}{D_A}\right)^3 = \dot{V}_A\left(\frac{\dot{n}_B}{\dot{n}_A}\right)\left(\frac{D_B}{D_A}\right)^3$$

Plug in numbers:

(b) Similarly, $$C_{P,A} = \frac{bhp_A}{\rho_A \omega_A^{\ 3} D_A^{\ 5}} = C_{P,B} = \frac{bhp_B}{\rho_B \omega_B^{\ 3} D_B^{\ 5}} \quad \rightarrow \quad bhp_B = bhp_A\left(\frac{\rho_B}{\rho_A}\right)\left(\frac{\dot{n}_B}{\dot{n}_A}\right)^3\left(\frac{D_B}{D_A}\right)^5$$

Plug in numbers:

Finally, the efficiency is calculated for each turbine:

$$\eta_{\text{turbine},A} = \frac{bhp_A}{\rho_A g H_A \dot{V}_A} = \frac{220{,}000{,}000\ \text{W}}{\left(1000\,\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)72.4\ \text{m}\left(335\,\frac{\text{m}^3}{\text{s}}\right)}\left(\frac{\text{N}\cdot\text{m}}{\text{W}\cdot\text{s}}\right)\left(\frac{\text{kg}\cdot\text{m}}{\text{s}^2\cdot\text{N}}\right) =$$

$$\eta_{\text{turbine},B} = \frac{bhp_B}{\rho_B g H_B \dot{V}_B} = \frac{461{,}820{,}979\ \text{W}}{\left(1000\,\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)97.4\ \text{m}\left(522.728\,\frac{\text{m}^3}{\text{s}}\right)}\left(\frac{\text{N}\cdot\text{m}}{\text{W}\cdot\text{s}}\right)\left(\frac{\text{kg}\cdot\text{m}}{\text{s}^2\cdot\text{N}}\right) =$$