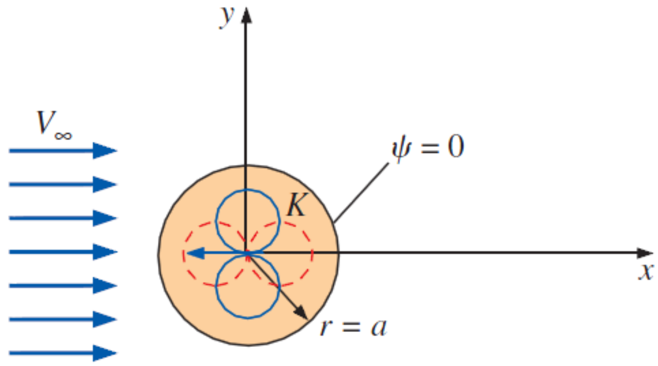
Today, we will:

- Do another example of superposition of irrotational flows – flow over a circular cylinder
- Start discussing the last approximation of Chapter 10: **The Boundary Layer Approx.**

b. Example of superposition: Flow over a circular cylinder

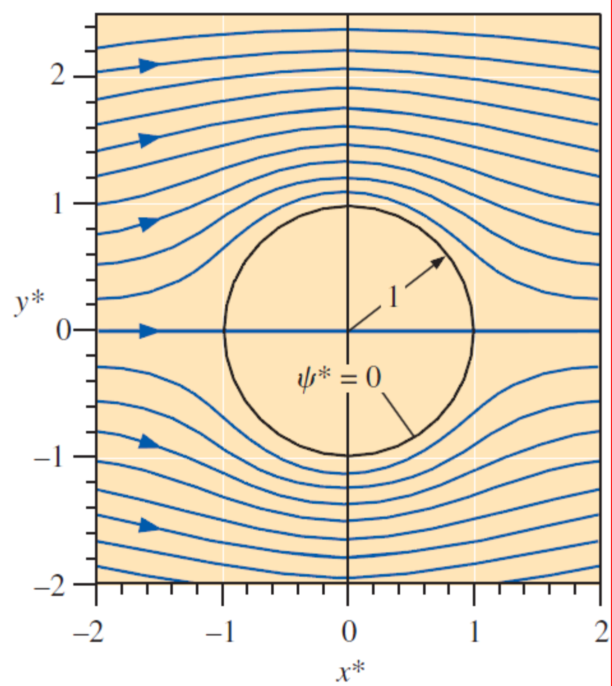
Given: Superpose a uniform stream of velocity V_∞ and a doublet of strength K at the origin.

To do: Plot streamlines, and discuss the flow that results from this superposition.

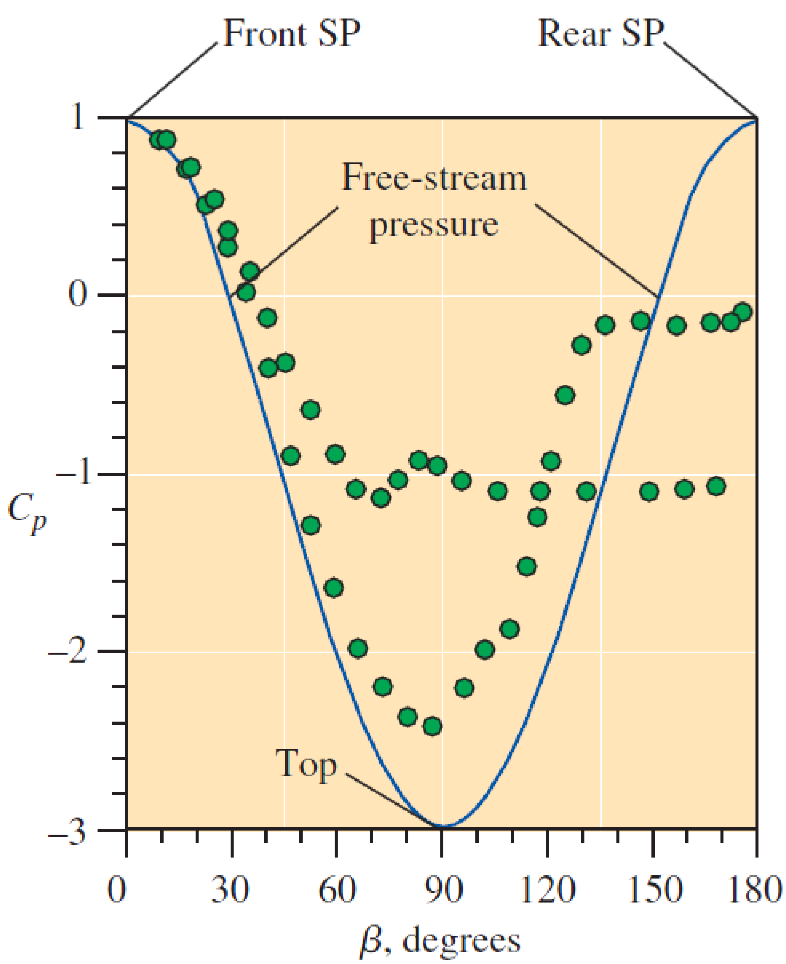


Solution:

- We simply add up the stream functions for the two building block flows: $\psi = \psi_{\text{freestream}} + \psi_{\text{doublet}} = V_\infty y - K\frac{\sin\theta}{r}$.
- But we know that $y = r\sin\theta$, thus, $\psi = V_\infty r\sin\theta - K\frac{\sin\theta}{r}$.
- For "convenience", and with hindsight, we choose to set $\psi = 0$ at $r = a$.
  [It turns out that radius a is a special radius that becomes the radius of the circle.]
- Set $r = a$ in our equation for the stream function:
  $$0 = V_\infty a\sin\theta - K\frac{\sin\theta}{a} \quad \rightarrow \quad K = V_\infty a^2.$$
- Then our final expression for ψ becomes
  $$\psi = V_\infty \sin\theta\left(r - \frac{a^2}{r}\right).$$
- Plot streamlines: [we plot nondimensionally, setting $x^*=x/a$ and $y^*=y/a$]



- From our equation for ψ above, we can calculate the velocity field from the definition of ψ, i.e., $u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta}$ $\quad u_\theta = -\frac{\partial\psi}{\partial r}$. See text for details. On the cylinder ($r = a$),
  $$u_r = 0 \qquad u_\theta = -2V_\infty\sin\theta$$
- We can also define the **pressure coefficient**, $C_p = \frac{P - P_\infty}{\frac{1}{2}\rho V_\infty^{\,2}} = 1 - \frac{V^2}{V_\infty^{\,2}}$.
- *On the cylinder*, it turns out that $C_p = 1 - 4\sin^2\beta$, where β is the angle from the nose.


Front SP
Rear SP
Free-stream
pressure
Top
1
0
–1
–2
–3
C_p
0
30
60
90
120
150
180
β, degrees

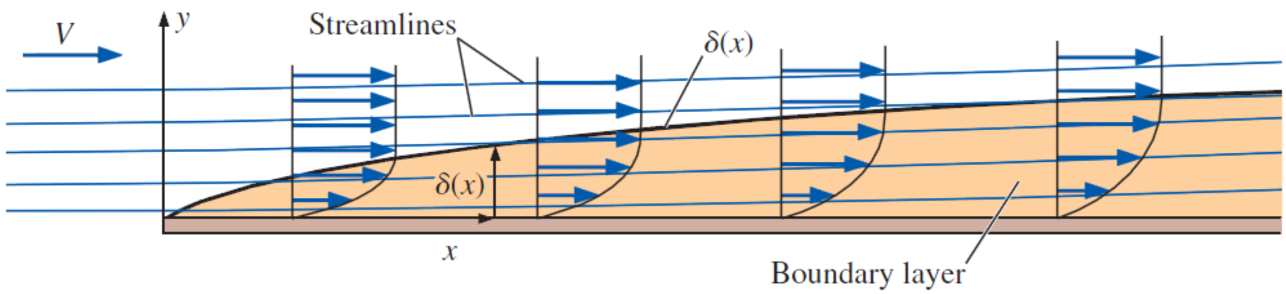
F. The Boundary Layer Approximation

1. Introduction

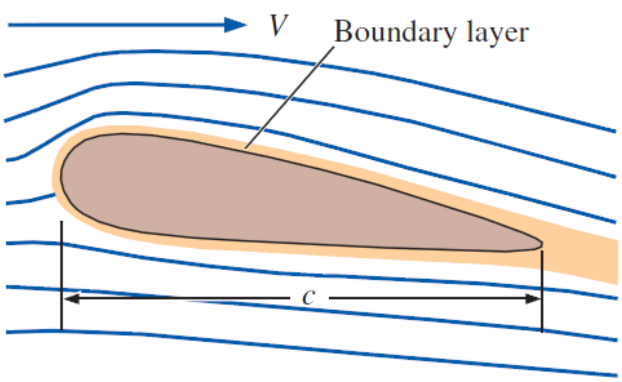
Definition: A ***boundary layer*** is a thin layer in which viscous effects and vorticity are significant, and cannot be ignored.

Examples

- BL on a flat plate aligned with the freestream flow (we show top side only):

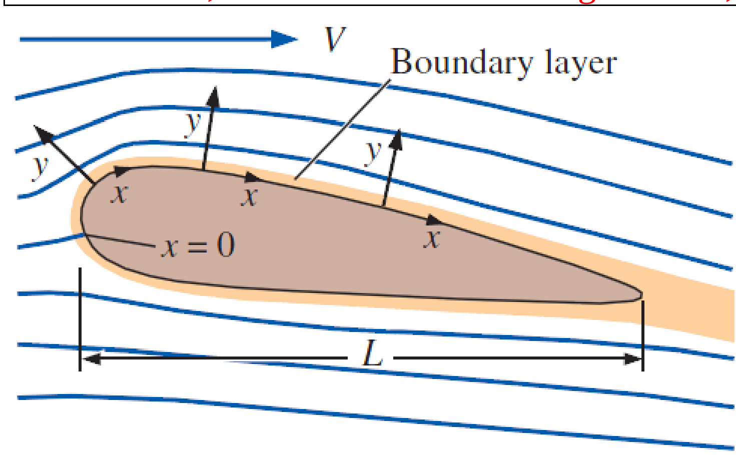


- BL on an airfoil:

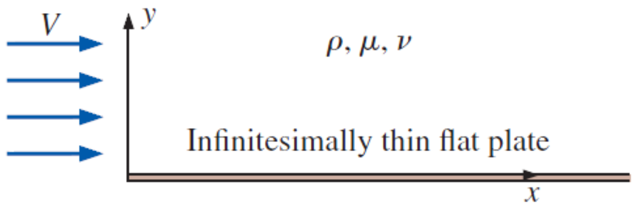


2. The Boundary Layer Coordinate System

In a 2-D flow, we let x = distance along the wall, and y = distance normal to the wall.

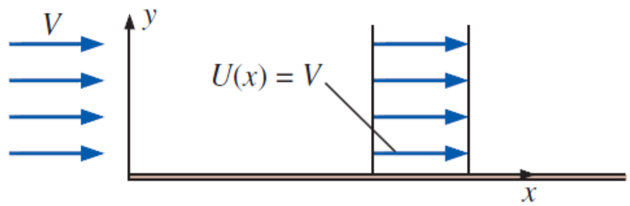


The Laminar Flat Plate Boundary Layer Solution of Blasius
(Example 10-10, Çengel and Cimbala)



We go through the steps of the boundary layer procedure:

- **Step 1**: The outer flow is ***U*(*x*) = *U* = *V* = constant**. In other words, the outer flow is simply a uniform stream of constant velocity.
- **Step 2**: A very thin boundary layer is assumed (so thin that it does not affect the outer flow). In other words, the outer flow does not even know that the boundary layer is there.



- **Step 3**: The boundary layer equations must be solved; they reduce to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

There are four required boundary conditions,

$$u = 0 \quad \text{at } y = 0 \qquad u = U \quad \text{as } y \to \infty$$

$$v = 0 \quad \text{at } y = 0 \qquad u = U \quad \text{for all } y \text{ at } x = 0$$

This equation set was first solved by **P. R. H. Blasius** in 1908 – numerically, but *by hand*!

Blasius introduced a **similarity variable** η that combines independent variables *x* and *y* into one nondimensional independent variable,

Similarity variable

$$\eta = y\sqrt{\frac{U}{\nu x}}$$

and he solved for a nondimensionalized form of the *x*-component of velocity,

$$f' = \frac{u}{U} = \text{function of } \eta$$

The similarity solution is f' as a function of η.

The key here is that ***one single similarity velocity profile holds for any x-location along the flat plate***. In other words, the velocity profile shape is the same ("similar") at any location, but it is merely *stretched vertically* as the boundary layer grows down the plate. This is illustrated in Fig. 10-98 in the text.

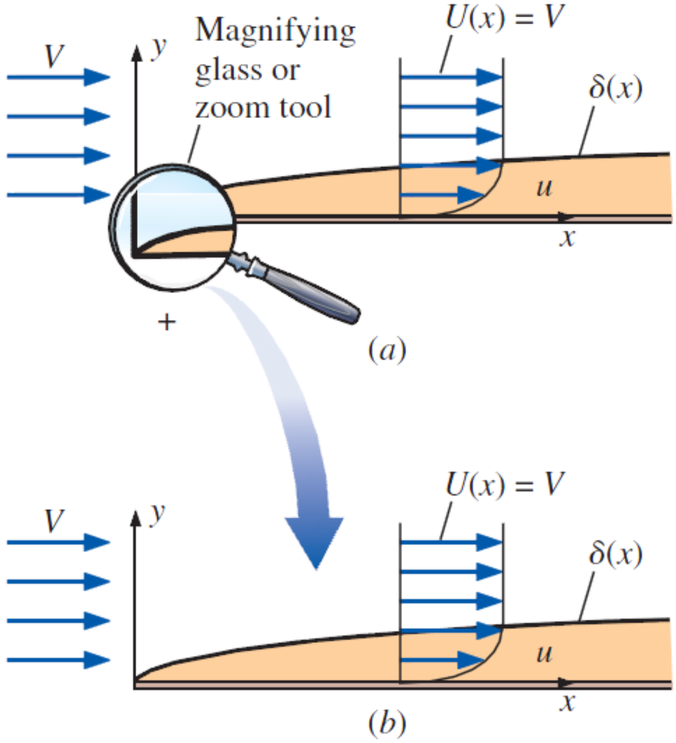
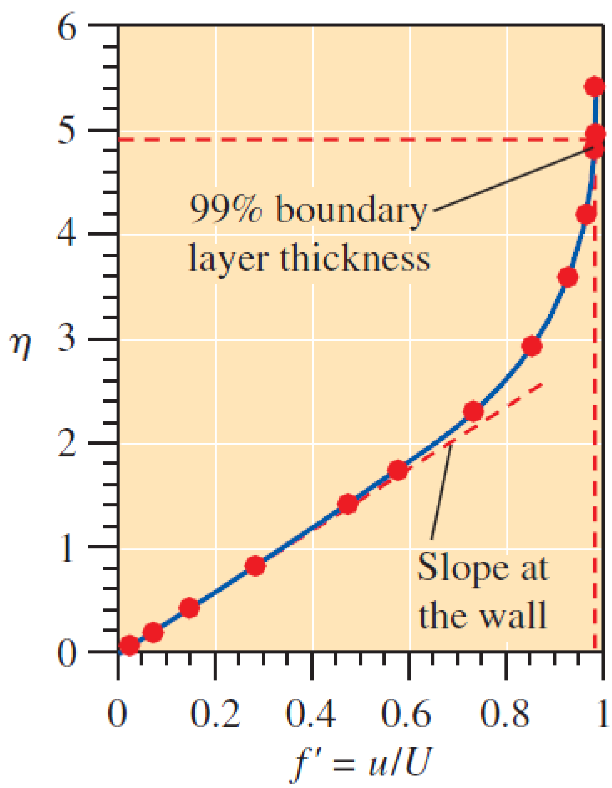


FIGURE 10–98
A useful result of the similarity assumption is that the flow looks the same (is *similar*) regardless of how far we zoom in or out; (*a*) view from a distance, as a person might see, (*b*) close-up view, as an ant might see.

The similarity solution itself is tabulated in Table 10-3, and is plotted in Fig. 10-99.

FIGURE 10–99
The Blasius profile in similarity variables for the boundary layer growing on a semi-infinite flat plate. Experimental data (circles) are at $Re_x = 3.64 \times 10^5$.
From Panton (1996).



This one velocity profile, plotted in nondimensional form as above, applies at *any x*- location in the boundary layer.