

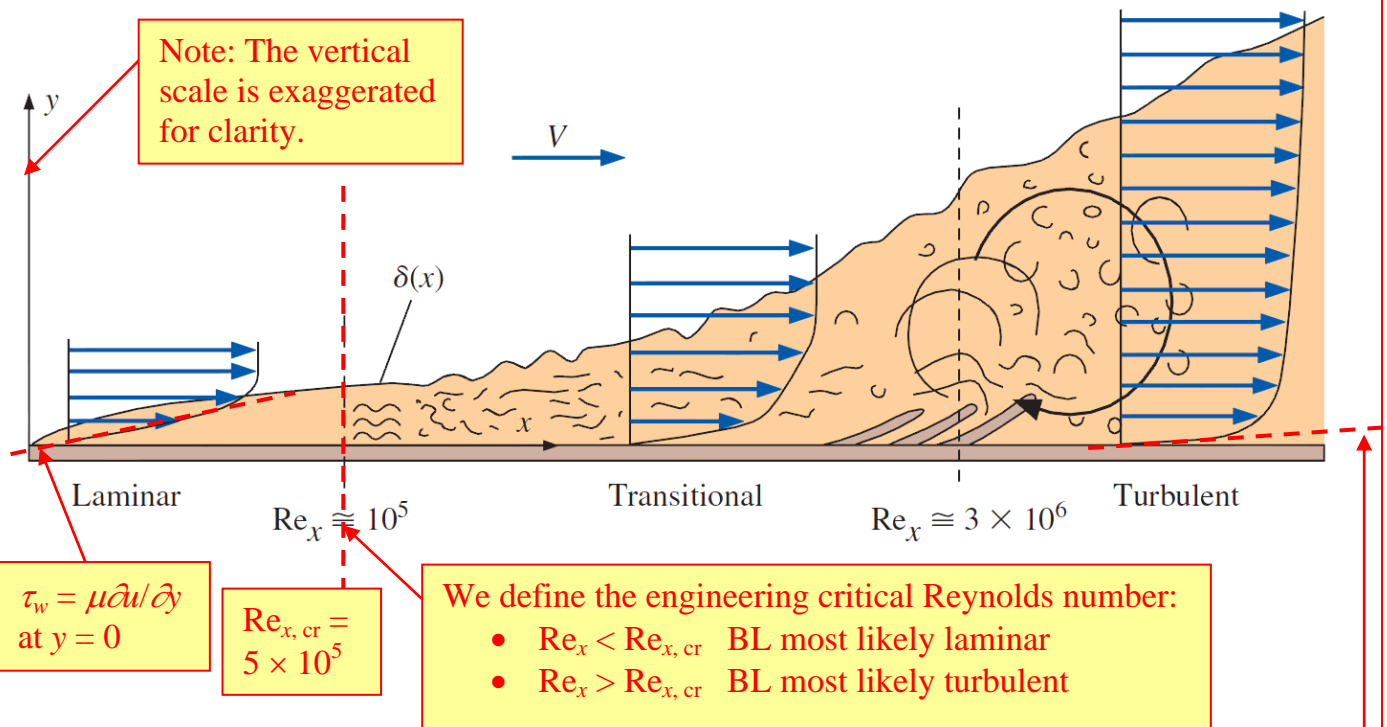
Today, we will:

- Discuss the *turbulent* boundary layer on a flat plate, and compare with laminar flow
- Talk about boundary layers with *pressure gradients*

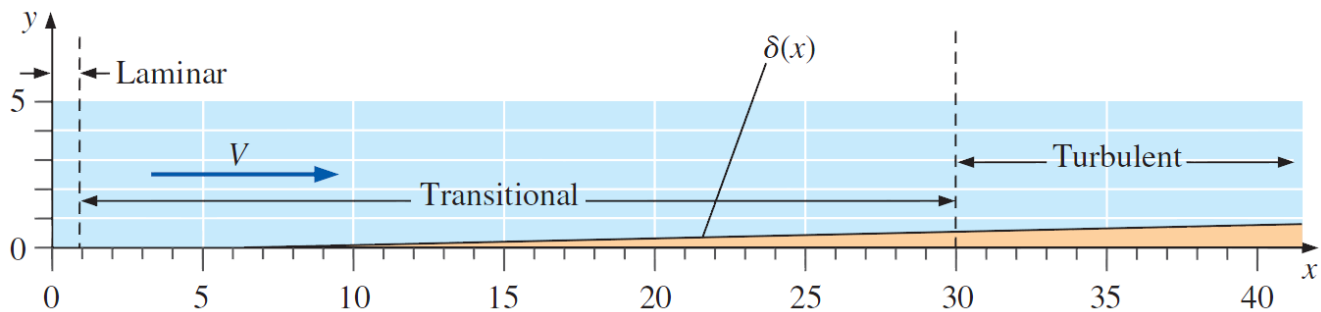
F. The Boundary Layer Approximation (continued)

1. Turbulent BL on a Flat Plate

The Turbulent Flat Plate Boundary Layer (Section 10-6, Çengel and Cimbala)



Here is what the actual BL looks like to scale:



The turbulent flat plate boundary layer velocity profile:

The time-averaged turbulent flat plate (zero pressure gradient) boundary layer velocity profile is much *fuller* than the laminar flat plate boundary layer profile, and therefore has a larger slope $\partial u / \partial y$ at the wall, leading to greater skin friction drag along the wall.

There are three common empirical relationships for the turbulent flat plate boundary layer velocity profile:

- **The log law:**

$$\text{The log law:} \quad \frac{u}{u_*} = \frac{1}{\kappa} \ln \frac{yu_*}{\nu} + B \quad (10-83)$$

where

$$\text{Friction velocity:} \quad u_* = \sqrt{\frac{\tau_w}{\rho}} \quad (10-84)$$

• **Spalding's law of the wall:**

$$\frac{yu_*}{\nu} = \frac{u}{u_*} + e^{-\kappa B} \left[e^{\kappa(u/u_*)} - 1 - \kappa(u/u_*) - \frac{[\kappa(u/u_*)]^2}{2} - \frac{[\kappa(u/u_*)]^3}{6} \right] \quad (10-85)$$

• **The one-seventh-power law:**

$$\frac{u}{U} \cong \left(\frac{y}{\delta} \right)^{1/7} \quad \text{for } y \leq \delta, \quad \rightarrow \quad \frac{u}{U} \cong 1 \quad \text{for } y > \delta \quad (10-82)$$

Quantities of interest for the turbulent flat plate boundary layer:

Just as we did for the laminar (Blasius) flat plate boundary layer, we can use these expressions for the velocity profile to estimate quantities of interest, such as the 99% boundary layer thickness δ , the displacement thickness δ^* , the local skin friction coefficient $C_{f,x}$, etc. These are summarized in Table 10-4 in the text.

Column (b) expressions are generally preferred for engineering analysis.

TABLE 10-4

Summary of expressions for laminar and turbulent boundary layers on a smooth flat plate aligned parallel to a uniform stream*

Property	(a)		(b)
	Laminar	Turbulent ^(†)	Turbulent ^(‡)
Boundary layer thickness	$\frac{\delta}{x} = \frac{4.91}{\sqrt{\text{Re}_x}}$	$\frac{\delta}{x} \cong \frac{0.16}{(\text{Re}_x)^{1/7}}$	$\frac{\delta}{x} \cong \frac{0.38}{(\text{Re}_x)^{1/5}}$
Displacement thickness	$\frac{\delta^*}{x} = \frac{1.72}{\sqrt{\text{Re}_x}}$	$\frac{\delta^*}{x} \cong \frac{0.020}{(\text{Re}_x)^{1/7}}$	$\frac{\delta^*}{x} \cong \frac{0.048}{(\text{Re}_x)^{1/5}}$
Momentum thickness	$\frac{\theta}{x} = \frac{0.664}{\sqrt{\text{Re}_x}}$	$\frac{\theta}{x} \cong \frac{0.016}{(\text{Re}_x)^{1/7}}$	$\frac{\theta}{x} \cong \frac{0.037}{(\text{Re}_x)^{1/5}}$
Local skin friction coefficient	$C_{f,x} = \frac{0.664}{\sqrt{\text{Re}_x}}$	$C_{f,x} \cong \frac{0.027}{(\text{Re}_x)^{1/7}}$	$C_{f,x} \cong \frac{0.059}{(\text{Re}_x)^{1/5}}$

Note that $C_{f,x}$ is the local skin friction coefficient, applied at only one value of x .

To these we add the integrated **average skin friction coefficients** for one side of a flat plate of length L , noting that C_f applies to the entire plate from $x = 0$ to $x = L$ (see Chapter 11):

Laminar: $C_f = \frac{1.33}{\text{Re}_L^{1/2}} \quad \text{Re}_L \leq 5 \times 10^5 \quad (11-19)$

Turbulent: $C_f = \frac{0.074}{\text{Re}_L^{1/5}} \quad 5 \times 10^5 \leq \text{Re}_L \leq 10^7 \quad (11-20)$

For cases in which the laminar portion of the plate is taken into consideration, we use:

$$C_f = \frac{0.074}{Re_L^{1/5}} - \frac{1742}{Re_L} \quad 5 \times 10^5 \leq Re_L \leq 10^7 \quad (11-22)$$

Turbulent flat plate boundary layers with wall roughness:

Finally, all of the above are for *smooth* flat plates. However, if the plate is *rough*, the average skin friction coefficient C_f increases with roughness ϵ . This is similar to the situation in pipe flows, in which Darcy friction factor f increases with pipe wall roughness.

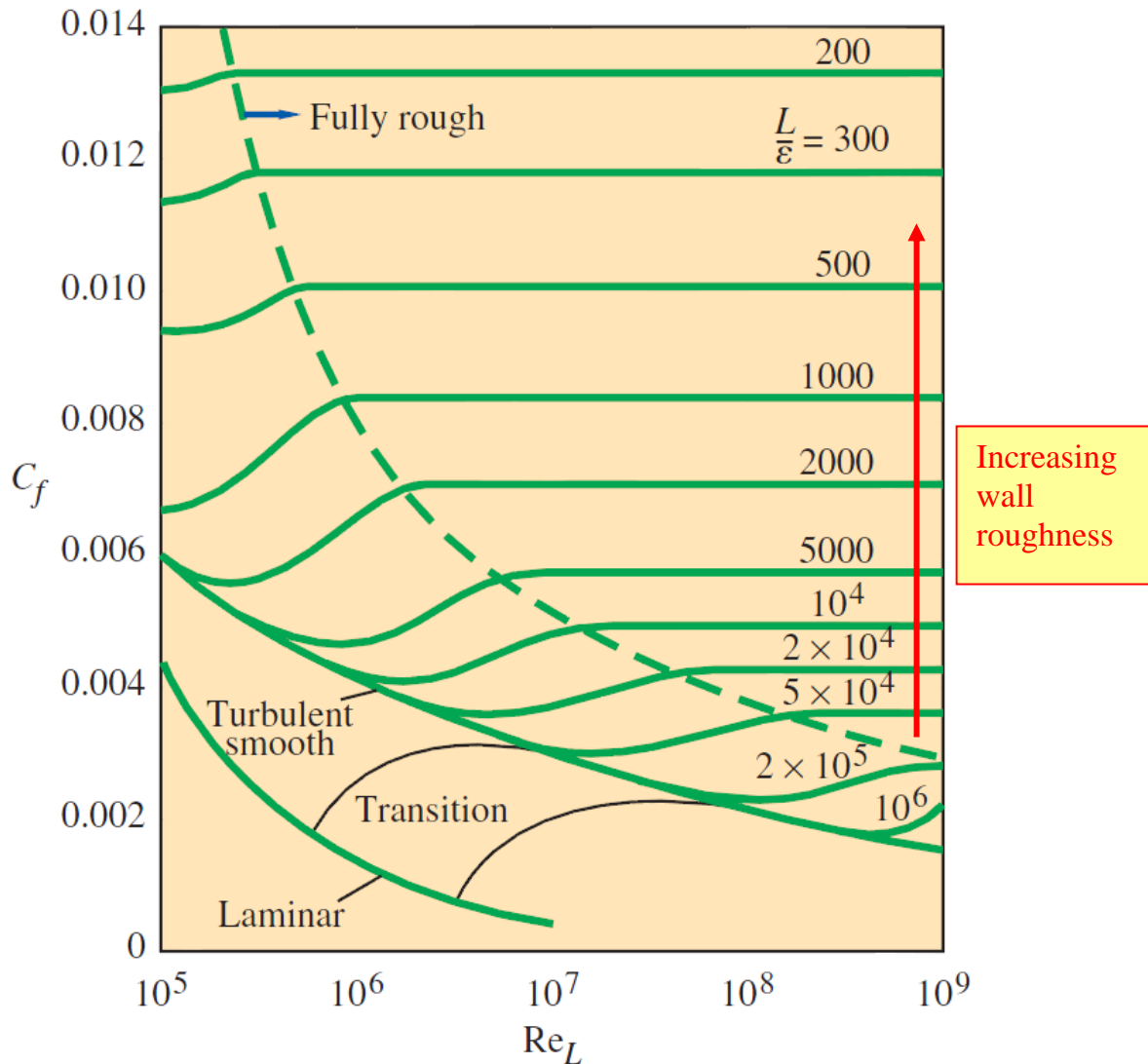


FIGURE 11-31

Friction coefficient for parallel flow over smooth and rough flat plates.

Just as with pipe flows, at high enough Reynolds numbers, the boundary layer becomes “fully rough”. For a **fully rough flat plate turbulent boundary layer** with average wall roughness height ϵ ,

Fully rough turbulent regime:
$$C_f = \left(1.89 - 1.62 \log \frac{\epsilon}{L} \right)^{-2.5} \quad (11-23)$$

This equation represents the flat portions of Fig. 11-31 that are labeled “Fully rough”.