**Today, we will**:

- Continue our discussion about fluid properties from Chapter 2
- Do some example problems – viscosity and surface tension

3. Other (miscellaneous) properties (continued)
   a. speed of sound
   b. vapor pressure, $P_v$ — see previous lecture
   c. viscosity, $\mu$, and kinematic viscosity, $\nu$

$\mu$ = viscosity or coeff. of viscosity or molecular viscosity

$$\{\mu\} = \left\{\frac{m}{L \cdot t}\right\} \qquad \left(\text{units typically } \frac{kg}{m \cdot s}\right)$$

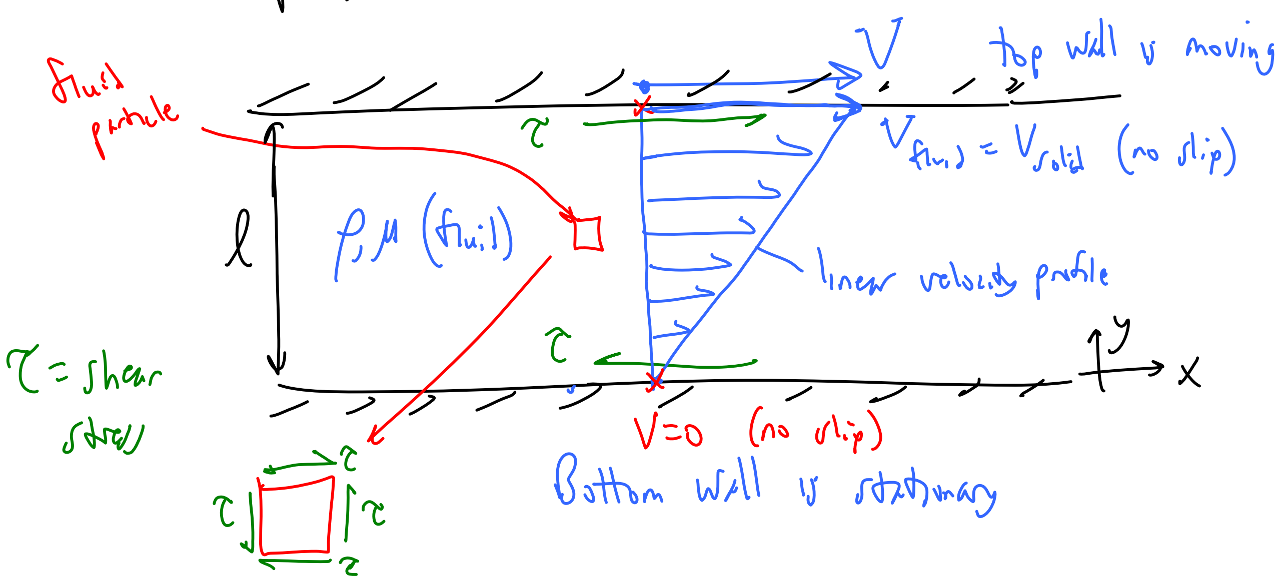
$\nu$ = kinematic viscosity

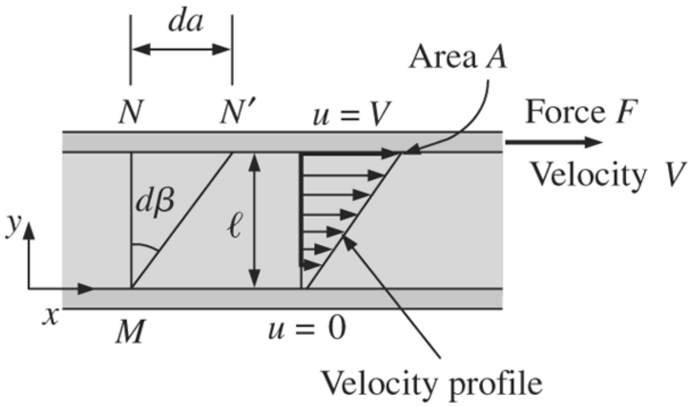
$$\boxed{\nu = \frac{\mu}{\rho}}$$

$$\{\nu\} = \left\{\frac{L^2}{t}\right\} \qquad \left(\text{typical units are } \frac{m^2}{s}\right)$$

★ Viscosity is a measure of the importance of friction in the fluid

Example: Simple flow of a fluid between two infinite parallel plates






$$u(y) = \frac{y}{\ell} V$$

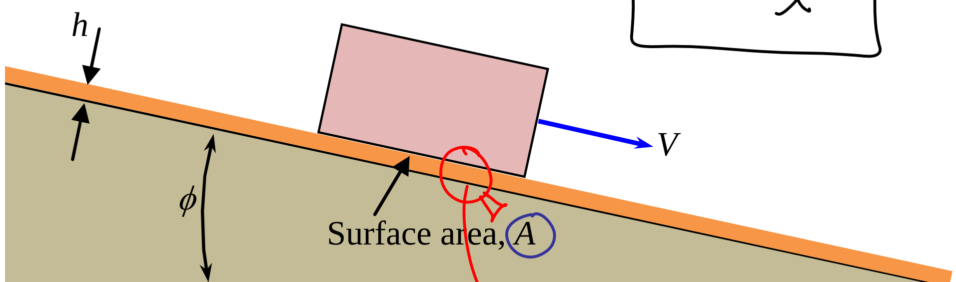
$$\boxed{u(y) = V \frac{y}{\ell}}$$

for this simple flow (linear velocity profile)

$$\boxed{\tau = \mu \frac{du}{dy}} = \text{constant} \; \boxed{= \mu \frac{V}{\ell}}$$

**Example: Viscosity**

**Given**: A block of weight $W$ and bottom surface area $A$ slides steadily down an incline at speed $V$, riding on a thin film of oil of viscosity $\mu$. The oil film thickness is $h$ and the incline angle is $\phi$ as sketched.

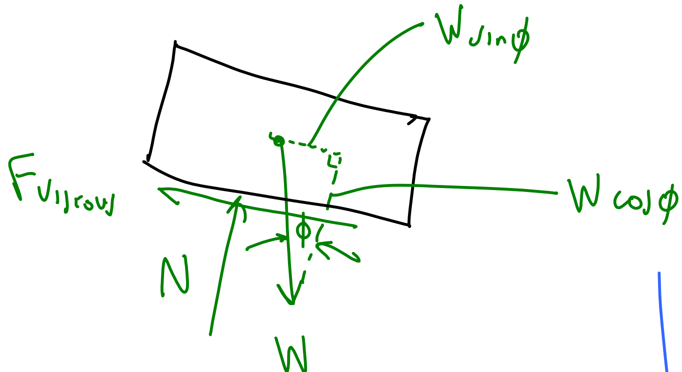


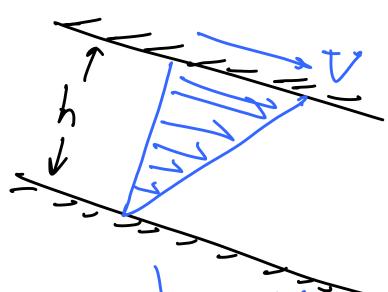

mag. glass

**To do**: Calculate $V$ as a function of the other variables (stay in variable form).

**Solution**:

· FBD




$$\tau = \mu \frac{du}{dy} = \mu \frac{V}{h}$$

$\sum \vec{F} = 0$ since there is no acceleration

- Normal direction $N = W \cos\phi$
- Parallel to the wall $F_{viscous} = W \sin\phi$

$$F_{viscous} = \tau \cdot A$$

$$F_{visc.} = \tau \cdot A = \frac{\mu V A}{h} = W \sin\phi$$

Solve for $V$ $\rightarrow$ $\boxed{V = \frac{W h \sin\phi}{\mu A}}$

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$\mu$ is generally a constant in a given problem

**But** $\rightarrow$ $\mu$ = fnc. of $T$ (only a very weak fnc. of $P$)

$\mu \uparrow$ as $T \uparrow$ in a gas

$\mu \downarrow$ as $T \uparrow$ in a liquid

**Newtonian fluids** $\equiv$ shear stress varies linearly with rate of deformation

e.g. $\boxed{\tau = \mu \frac{du}{dy}}$ (common fluids like air, water, oil, etc. are Newtonian)

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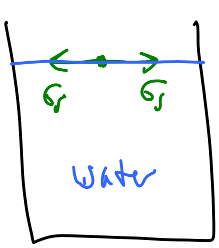
d. Surface Tension _ $\sigma_s$

$$\{\sigma_s\} = \left\{\frac{\text{Force}}{\text{length}}\right\} \quad \left(\text{typ. units are } \frac{N}{m}\right)$$

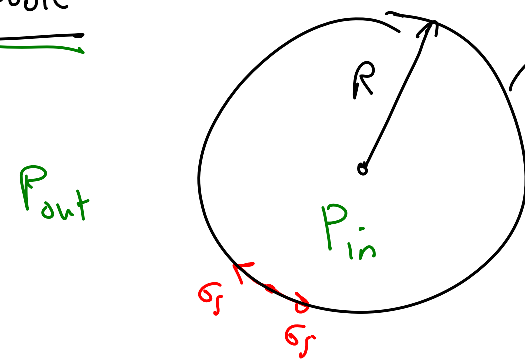
$$= \left\{\frac{m}{t^2}\right\}$$




like a membrane on the surface under tension

$\sigma_s$ always acts parallel to the surface of the liquid

Ex. Soap Bubble

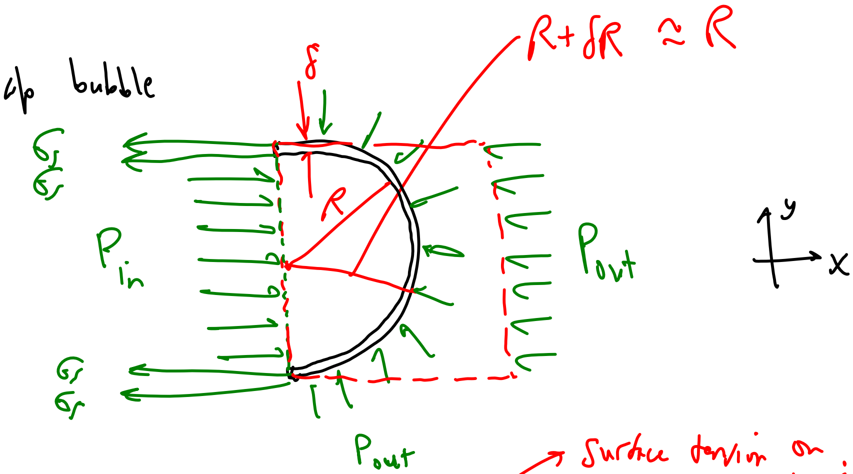



skin is a very thin membrane pulled by surface tension

$$\boxed{P_{in} > P_{out}}$$

To analyze:

FBD of half the soap bubble




$\sum F_x = 0$

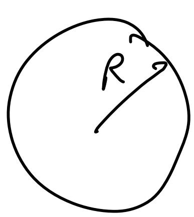
$$P_{in}\,\pi R^2 = P_{out}\,\pi R^2 + 2\,\sigma_s\,2\pi R$$

Surface tension on outside & inside surfaces

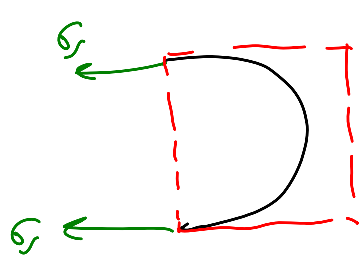
$\div \pi R^2 \quad \rightarrow$

$$\boxed{\Delta P = P_{in} - P_{out} = \frac{4\sigma_s}{R}} \quad \star$$

Similar analysis for a gas bubble in a liquid

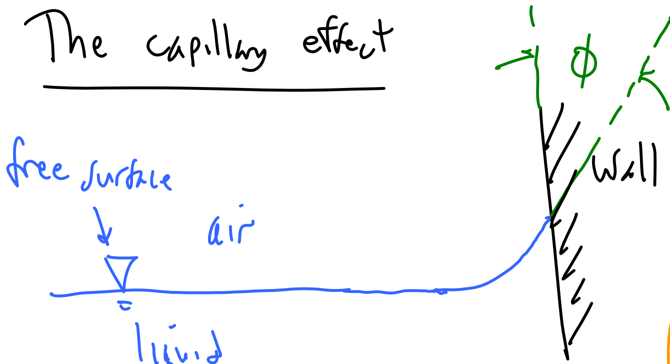



FBD




$$\boxed{\Delta P = \frac{2\sigma_s}{R}} \quad \star$$

# The capillary effect



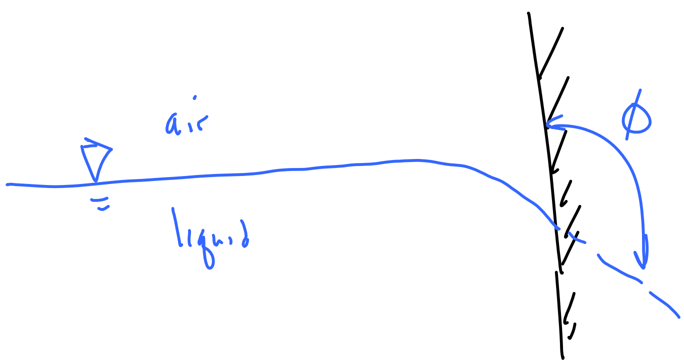

$\phi$ = contact angle

fnc. of liquid & the solid

e.g. water & a glass wall

$\phi \approx 0$

"Wetting" fluid $(\phi < 90^\circ)$

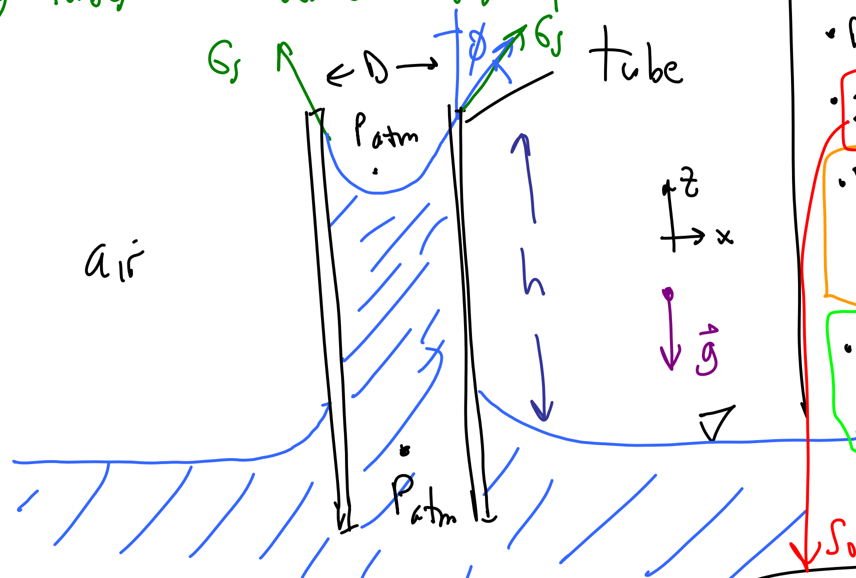



e.g. mercury & glass

non-wetting fluid

$(\phi > 90^\circ)$

Capillary tubes → water rises up!!




ANALYSIS:

- Draw FBD (see below)
- $\Sigma F_z = 0$

DOWNWARD FORCE

- $W = mg = \rho g \forall$
$$= \rho g \frac{\pi D^2}{4} h$$

- Surf. ten. force
$$= \sigma_s \pi D \cos\phi$$

UPWARD FORCE

Solve for h:

$$\boxed{h = \frac{4 \sigma_s \cos\phi}{\rho g D}}$$

FBD of the liquid in the tube

(pressure forces all cancel)