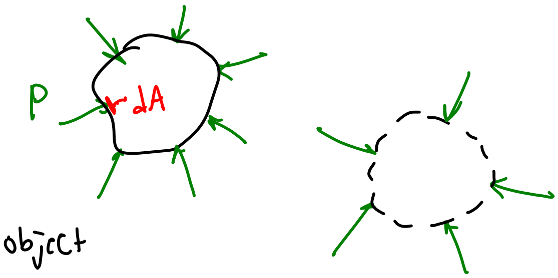
Today, we will:

- Begin Chapter 3 – Pressure and Fluid Statics
- Discuss different kinds of pressure measurement (absolute, gage, vacuum)
- Derive the equation of fluid statics (hydrostatic pressure relation)

II. Pressure and Fluid Statics

A. Pressure, *P*

1. Some basics



- Pressure is a scalar, not a vector
- Pressure acts inward normal on any object
- Pressure causes a surface force not a body force

2. Dimensions and units

P is a stress

$$\{P\} = \left\{\frac{\text{force}}{\text{area}}\right\}$$

Units: English : lbf/in^2 or psi

SI : N/m^2 or Pascals

also "atmospheres"

B. Types of Pressure Measurement

1. Absolute pressure $\equiv$ $\boxed{\text{P relative to a total vacuum}}$

[1 standard atmosphere $P_{atm} = 101{,}325$ Pa $= 14.696$ psi]

P or P_{abs} → Same as the P you use in thermo.

2. Gage pressure (sometimes "gauge")

↳ $\boxed{\text{P relative to the local atmospheric pressure}}$ [P_{gage}]

$$\boxed{P_{gage} = P - P_{atm}}$$ ☆

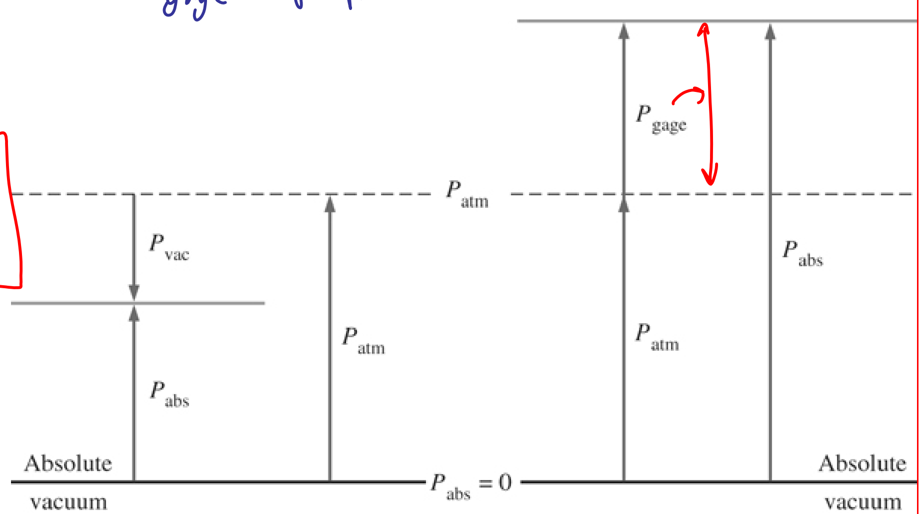
Most gages measure P_{gage}

e.g. car tire gage → 35 psi on the gage

means $P_{gage} = 35$ psi ⟶ $P = 35 + 14.7$ psi $= 49.7$ psi

3. Vacuum pressure

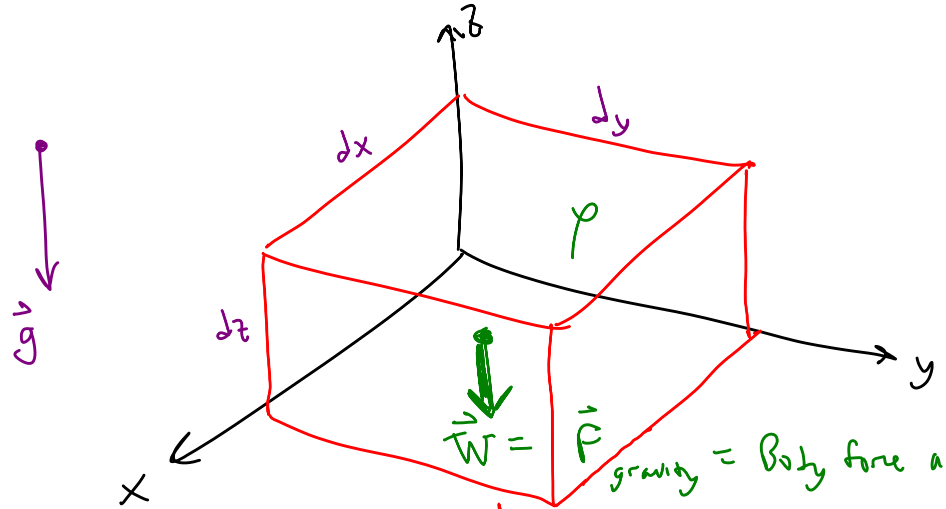
$$\boxed{P_{vac} = P_{atm} - P = -P_{gage}}$$ ☆



C. Equation of Fluid Statics

Consider a small fluid element of dimensions dx, dy, dz

Do a FBD - sum all forces on this "body"



$\vec{W} = \vec{F}_{gravity}$ = Body force acting down ($-z$ direction)

Eq. $\rightarrow$ $\boxed{\sum \vec{F} = 0}$ ☆ Because there is no acceleration

Recall, a fluid at rest cannot resist a shear stress $\rightarrow$ $\tau = 0$ here

$\therefore$ the only stresses are normal stresses

$$\boxed{\sum \vec{F} = \sum \vec{F}_{\text{body forces}} + \sum \vec{F}_{\text{surface forces}} = 0}$$

$$\boxed{\vec{F}_{grav} = m\vec{g} = \rho \forall \vec{g} = \rho\, dx\, dy\, dz\, \vec{g}}$$

Notation:
$\forall$ = volume
V = velocity

$\vec{g}$ is down in neg. z direction, $\boxed{\vec{g} = -g\vec{k}}$

$$\boxed{\sum \vec{F}_{\text{body forces}} = -\rho g\, dx\, dy\, dz\, \vec{k}}$$

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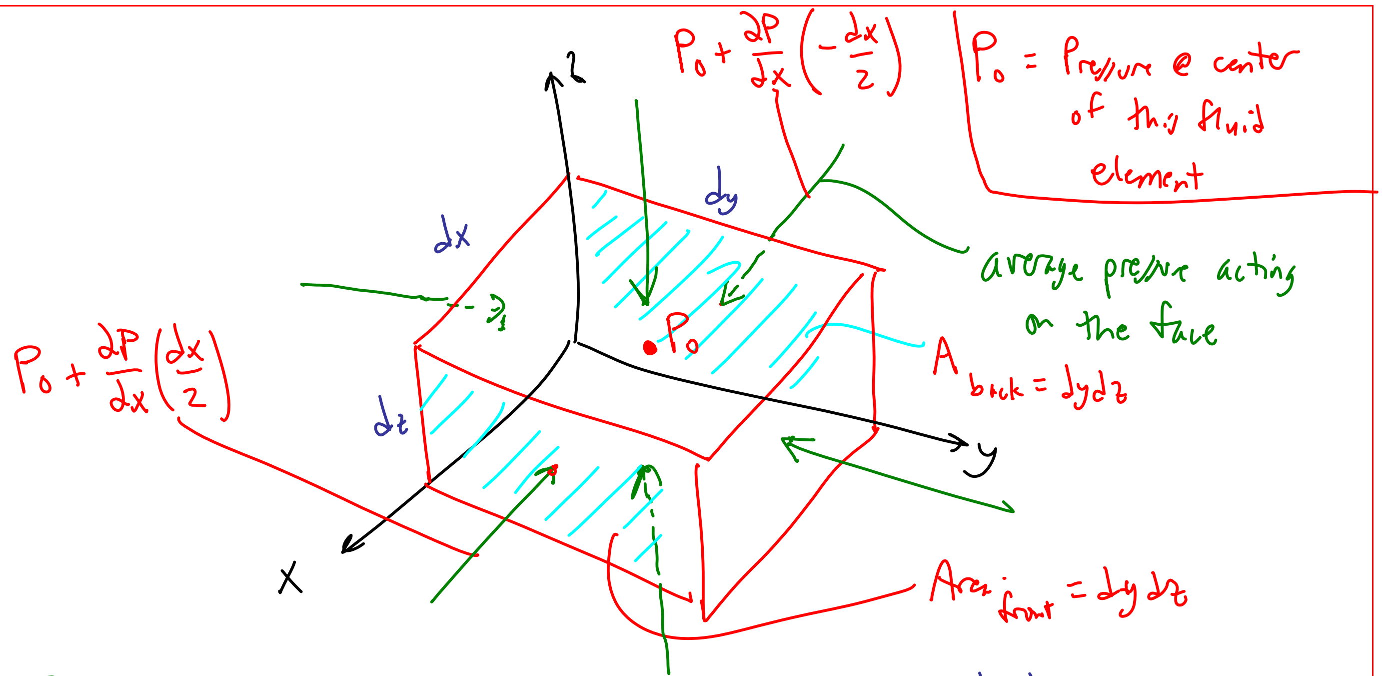
2. Surface Forces (the only surface force is pressure)



- In general, $P = P(x, y, z)$ → So, we use partial derivatives in our eqn's (∂)

- Use Truncated Taylor series expansions

At some distance Δs from a given point,

$$\underline{P(\Delta s) = P_0 + \frac{\partial P}{\partial s}\Delta s} + \cancel{\frac{1}{2!}\frac{\partial^2 P}{\partial s^2}\Delta s^2} + \cdots$$

Ignore this & higher-order terms as Δs shrinks to very small

- In x-direction,

$$\sum F_{surface,x} = -\underbrace{\left(\cancel{P_0} + \frac{\partial P}{\partial x}\frac{dx}{2}\right)dy\,dz}_{\text{Front face}} + \underbrace{\left(\cancel{P_0} - \frac{\partial P}{\partial x}\frac{dx}{2}\right)dy\,dz}_{\text{Back face}}$$

cancel

$$\boxed{\sum F_{surface,x} = -\frac{\partial P}{\partial x}\,dx\,dy\,dz}$$

similarly,

$$\boxed{\sum F_{surface,y} = -\frac{\partial P}{\partial y}\,dx\,dy\,dz}$$

$$\boxed{\sum F_{surface,z} = -\frac{\partial P}{\partial z}\,dx\,dy\,dz}$$

3. Hydrostatic Pressure Relation:

$$\sum \vec{F} = 0 \rightarrow \sum \vec{F}_{body} + \sum \vec{F}_{surface} = 0$$

x-direction: $0 - \frac{\partial P}{\partial x}\, dx\, dy\, dz = 0 \rightarrow \boxed{\frac{\partial P}{\partial x} = 0}$ ✩

✩ In hydrostatics, P does not vary in the x-direction in a fluid

y-direction: $0 - \frac{\partial P}{\partial y}\, dx\, dy\, dz = 0 \rightarrow \boxed{\frac{\partial P}{\partial y} = 0}$ ✩

" P " " " y " " "

z-direction: $-\rho g\, dx\, \cancel{dy}\, dz - \frac{\partial P}{\partial z}\, dx\, \cancel{dy}\, dz = 0$

$$\boxed{\frac{\partial P}{\partial z} = -\rho g} \quad ✩ \qquad (1)$$

✩ ✩ ✩ In hydrostatics, P does not vary horizontally, but P varies vertically

$\uparrow z$ $\downarrow \vec{g}$

Eq. (1) means that P increases as you go down

Since $P \neq$ fnc. of x or y, $P =$ fnc(z) only, $\therefore$ (1) becomes

E.g.

$$✩ \boxed{\frac{dP}{dz} = -\rho g} \quad (2)$$

z_2 P_2 $\uparrow z$

z_1 P_1

$(2) \rightarrow dP = -\rho g\, dz$

$$\int_{P_1}^{P_2} dP = -\rho g \int_{z_1}^{z_2} dz \quad \text{[assuming } \rho \text{ \& } g \text{ are constant]}$$

$$\boxed{P_2 - P_1 = -\rho g (z_2 - z_1)}$$ For an incompressible fluid at rest

Hydrostatic Pressure Relation:

$$\boxed{\frac{\partial P}{\partial x} = 0 \qquad \frac{\partial P}{\partial y} = 0 \qquad \frac{dP}{dz} = -\rho g}$$

OR, easier to remember

☆ $$\boxed{P_{below} = P_{above} + \rho g |\Delta z|}$$

OR, $$\boxed{P_{below} = P_{above} + \gamma_s |\Delta z|}$$

[γ_s = specific weight $= \rho g$]

☆ With this simple equation, we can solve any hydrostatics problem!

[See pdf file on website — Some Rules about Hydrostatics]