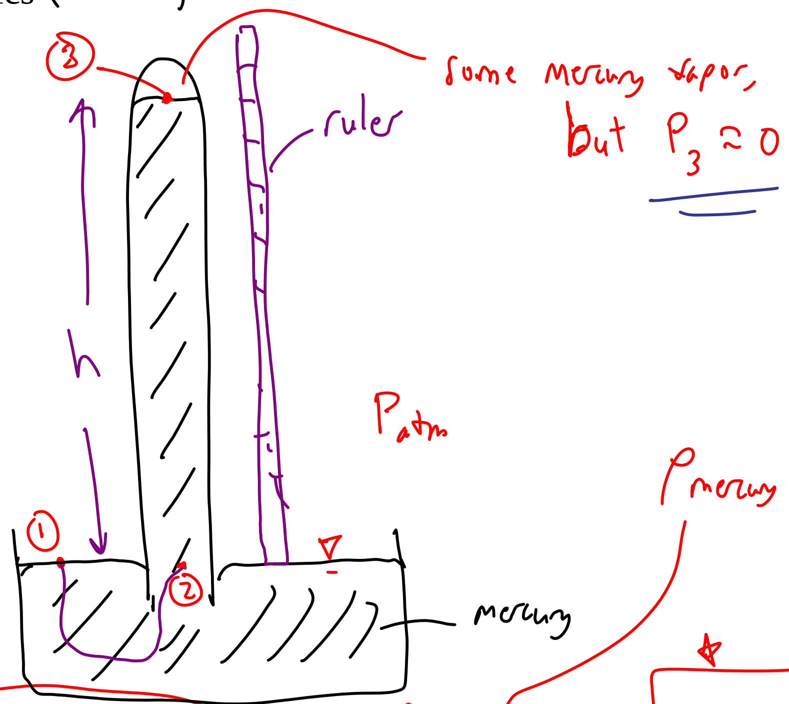
Today, we will:

- Continue Chapter 3 – Pressure and Fluid Statics
- Discuss applications of fluid statics (barometers and U-tube manometers)
- Do some example problems (manometers)

D. Applications of Fluid Statics (continued)

1. Mercury barometer

measures atmospheric pressure



$P_1 = P_2 = P_{atm}$

$\rho_{mercury}$

$$P_{below} = P_{above} + \rho g |\Delta z| \rightarrow P_2 = \cancel{P_3}^{\approx 0} + \rho g h \Rightarrow \boxed{P_{atm} = \rho g h}$$

e.g. "29" of Hg" means

$$P_{atm} = \rho g h = \left(13{,}580 \frac{kg}{m^3}\right)\left(9.807 \frac{m}{s^2}\right)\left(29.0 \text{ in}\right)\left(\frac{0.0254 \text{ m}}{\text{in}}\right)\left(\frac{N}{kg \cdot m/s^2}\right)$$

$$P_{atm} = 98{,}099.7 \frac{N}{m^2} \cong 98{,}100 \text{ Pa} \text{ or } \boxed{98.1 \text{ kPa}}$$

2. "Head" as a pressure measurement

☆ $\boxed{\text{Head = pressure expressed as the equivalent column height of a fluid}}$

[e.g. here, the head associated with 98.1 kPa is 29" of mercury]

3. The U-tube manometer

Purpose: To measure an unknown pressure or pressure difference

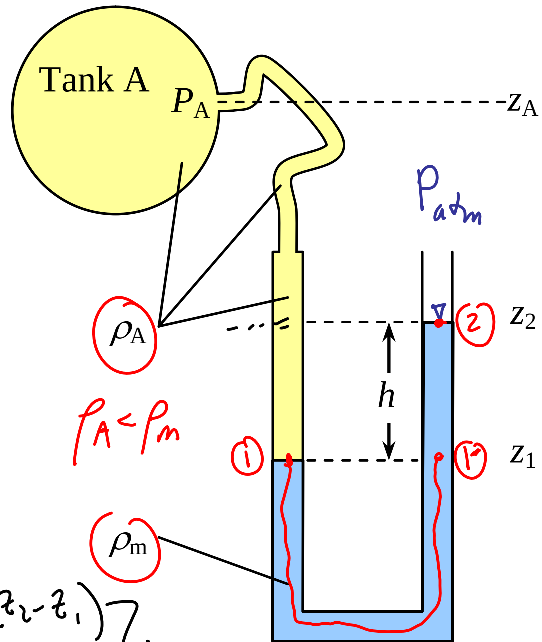
Example: Pressure measurement with a U-tube manometer

Given: A U-tube manometer is used as an instrument to measure the pressure in a tank. The right leg of the manometer is open to atmospheric pressure.

***(a)* To do**: Calculate the absolute and gage pressure P_A and $P_{A,gage}$ for the general case in which ρ_A is not small compared to ρ_m.

***(b)* To do**: Simplify for the case in which $\rho_A << \rho_m$ (e.g., A is air and m is mercury).

Solution:



Manometer

$P_2 = P_{atm}$

$\rho_A < \rho_m$

- $\boxed{P_{below} = P_{above} + \rho g |\Delta z|}$
- $P_1 = P_1'$
- From (A) to (1): $P_1 = P_A + \rho_A g (z_A - z_2) + \rho_A g (z_2 - z_1)$
- From (2) to (1'): $P_1' = P_1 = P_{atm} + \rho_m g (z_2 - z_1)$

(a) equate these 2 eq's

$$\boxed{P_A = P_{atm} + (\rho_m - \rho_A) g (z_2 - z_1) - \rho_A g (z_A - z_2)}$$

$$\boxed{P_{A\,gage} = P_A - P_{atm} = (\rho_m - \rho_A) g (z_2 - z_1) - \rho_A g (z_A - z_2)}$$

(b)

$\rho_m >> \rho_A$, $\rho_m - \rho_A \cong \rho_m$

$$\boxed{P_A \cong P_{atm} + \rho_m g (z_2 - z_1) - \underbrace{\rho_A g (z_A - z_2)}_{\text{this term may or may not be negligible}}}$$

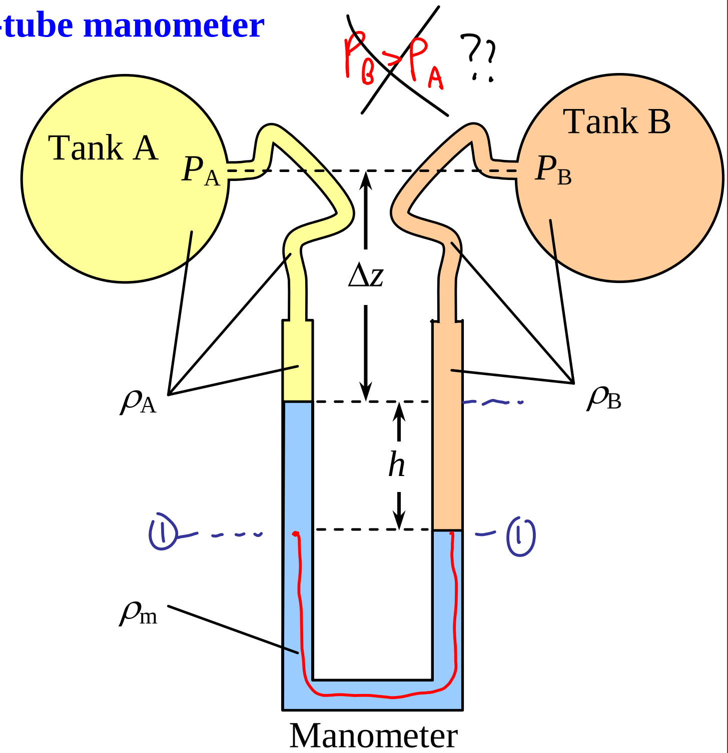
Example: Pressure measurement with a U-tube manometer

Given: A U-tube manometer is used as a differential pressure measurement instrument to measure the pressure difference between two tanks. The two tanks are at the same elevation.

(*a*) To do: Calculate the pressure difference $P_B - P_A$ for the general case in which ρ_A is not the same as ρ_B (they are different fluids.

(*b*) To do: Simplify for the case in which $\rho_A = \rho_B$ (they are the same fluid).

Solution:



- $P_{below} = P_{above} + \rho g |\Delta z|$
- Go from A to B in one swoop.

$$P_A + \underbrace{\rho_A g \Delta z + \rho_m g h}_{\text{going down } +} \underbrace{- \rho_B g h - \rho_B g \Delta z}_{\text{going up } -} = P_B$$

So,

$$\boxed{P_B - P_A = \underset{+}{(\rho_m - \rho_B) g h} + \underset{+ \text{ or } -}{(\rho_A - \rho_B) g \Delta z}}$$

(b)

$$\boxed{P_B - P_A = (\rho_m - \rho_B) g h}$$

4. Some Notes about Manometry [from pdf file on website]

Author: John M. Cimbala, Penn State University
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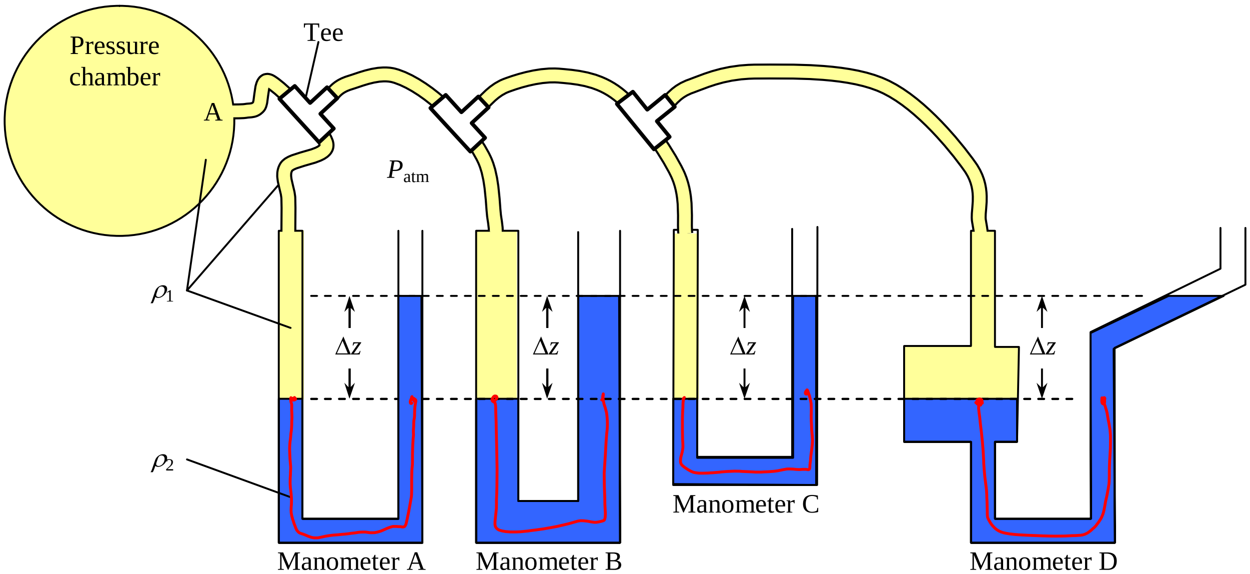
The elevation difference Δ*z* in a U-tube manometer does *not* depend on the following:

1. **U-Tube diameter** (provided that the tube diameter is large enough that capillary effects are negligible). In the sketch below, for a given pressure in the tank, Δ*z* is the same in manometers A and B, even though the tube diameter of manometer B is larger than that of manometer A. Note that the amount of manometer liquid in each of the U-tube manometers has been adjusted such that the level of the interface between fluids 1 and 2 on the left side of each manometer is at the same elevation, for direct horizontal comparison.



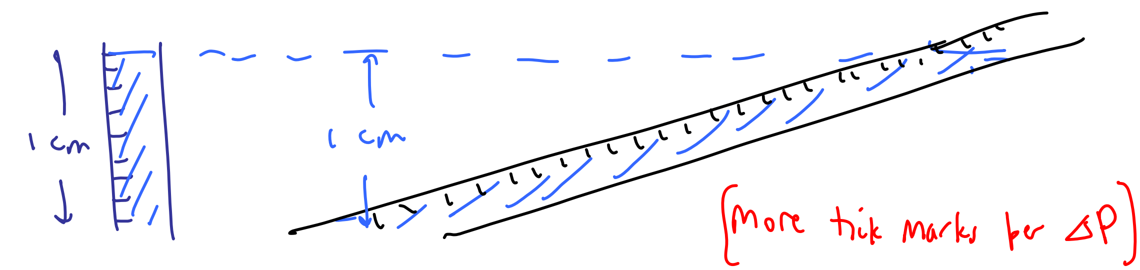
Why? $P_{below} = P_{above} + \rho g |\Delta z|$ → does not depend on diameter!

2. **U-Tube length** (provided that the tubes are long enough to include elevation difference Δ*z*). In the sketch, Δ*z* is the same in manometers A and C, even though manometer C is shorter than manometer A.

Why? Below the surface, nothing impacts our calculations or interface

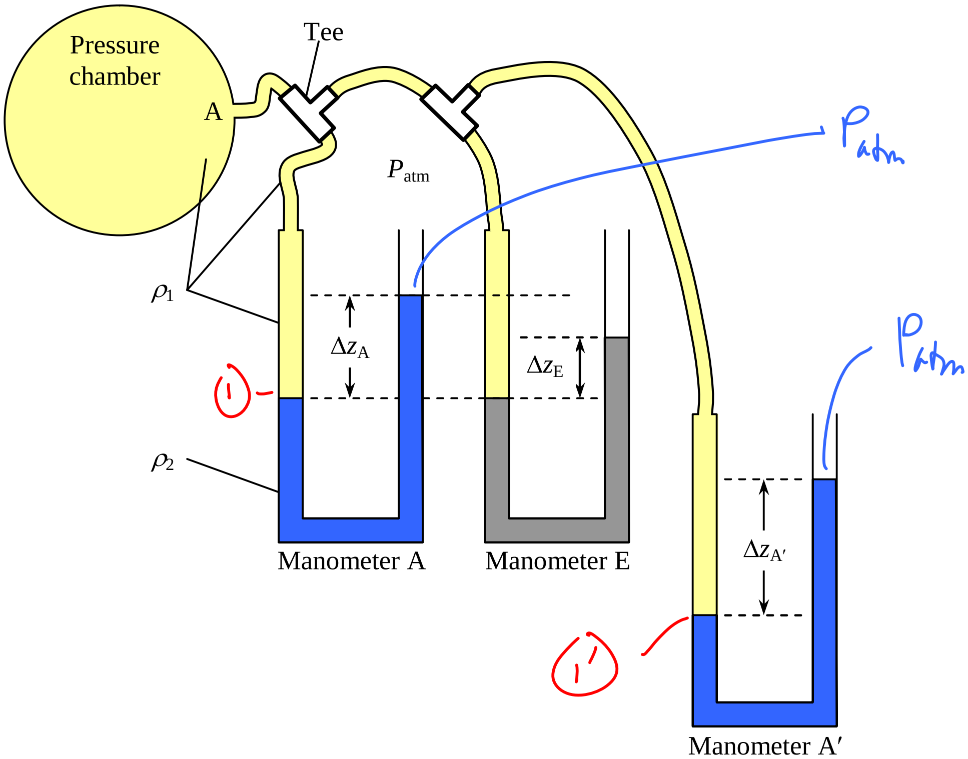
3. **U-Tube shape** (again provided that capillary effects are not important and the relative elevation is the same). In the sketch, Δ*z* is the same in manometers A and D, even though manometer D is oddly shaped. Can you think of an advantage of the "inclined manometer" configuration of manometer D?

Inclined manometer can read to finer resolution



However, the elevation difference Δ*z* in a U-tube manometer *does* depend on the following:

1. **Manometer fluid**. For example, if we replace the blue manometer fluid in the above sketch with a *higher density* (gray colored) fluid, as in the sketch below, Δ*z* would *decrease*. In other words, $\Delta z_E < \Delta z_A$.



Which manometer (A or E) would have better *resolution*?

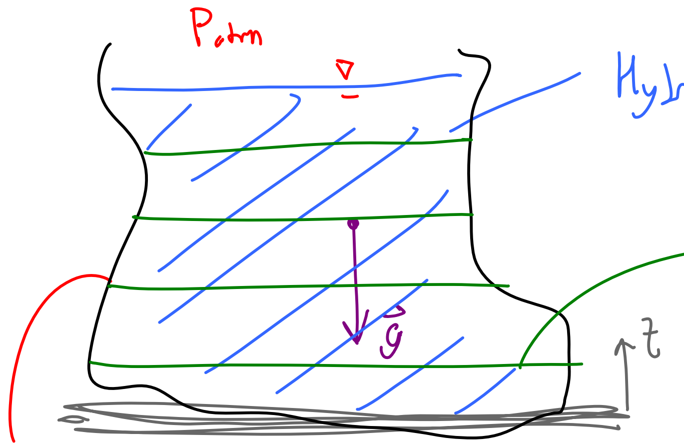
A → more tick marks when h is higher ∴ better resolution

2. **Vertical location of the manometer**. For example, if we move manometer A to a lower elevation, all else being the same, and ignoring changes in atmospheric pressure (manometer A′ in the above sketch), Δ*z* would *increase*, i.e., $\Delta z_{A'} > \Delta z_A$. Why?

The yellow fluid has more column height in A′, ∴ pushes the manometer fluid down more,

∴ $P_1' > P_1$ in this case

5. <u>Isobar</u> $\Rightarrow$ $\boxed{\text{a surface of constant pressure}}$ ☆



Arbitrary shape container with a liquid in it

Hydrostatics: $P_{below} = P_{above} + \rho g |\Delta z|$

Isobars = horizontal lines in hydrostatics ☆

P increases as you go down

☆ NOTICE: ISOBARS ARE PERPENDICULAR ($\perp$) to $\vec{g}$