Today, we will:

- Continue Chapter 3 – Pressure and Fluid Statics
- Discuss applications of fluid statics (barometers and U-tube manometers)
- Do some example problems (manometers)

D. Applications of Fluid Statics (continued)

1. Mercury barometer

\[ P_1 = P_2 = P_{\text{atm}} \]

\[ P_{\text{below}} = P_{\text{above}} + \rho g \Delta z \rightarrow P_2 = P_3 + \rho gh \Rightarrow P_{\text{atm}} = \rho gh \]

E.g. “29” of Hg” means

\[ P_{\text{atm}} = \rho gh = \left( 13.58 \, \text{lb/ft}^2 \right) \left( 9.807 \, \text{m/s}^2 \right) \left( 29.0 \, \text{in} \right) \left( \frac{0.0254 \, \text{m}}{\text{in}} \right) \left( \frac{N}{\text{kg} \cdot \text{m/s}^2} \right) \]

\[ P_{\text{atm}} = 98,099.7 \, \frac{N}{\text{m}^2} = 98,100 \, \text{Pa} \text{ or } 98.1 \, \text{kPa} \]

2. “Head” as a pressure measurement

\[ A \text{ Head} = \text{pressure expressed as the equivalent column height of a fluid} \]

\[ \text{E.g. here, the head associated with 98.1 kPa is 29” of mercury} \]
3. The U-tube manometer

**Purpose:** To measure an unknown pressure or pressure difference

**Example: Pressure measurement with a U-tube manometer**

**Given:** A U-tube manometer is used as an instrument to measure the pressure in a tank. The right leg of the manometer is open to atmospheric pressure.

(a) **To do:** Calculate the absolute and gage pressure $P_A$ and $P_{A,gage}$ for the general case in which $\rho_A$ is not small compared to $\rho_m$.

(b) **To do:** Simplify for the case in which $\rho_A << \rho_m$ (e.g., A is air and m is mercury).

**Solution:**

\[
\begin{align*}
\rho_{\text{man}} & = \rho_{\text{atm}} + \rho g (z_2 - z_1) \\
\rho_A & = \rho_{\text{atm}} + (\rho_m - \rho_A) g (z_2 - z_1) - \rho_A g (z_A - z_2) \\
\rho_{A,gage} & = P_A - P_{\text{atm}} = (\rho_m - \rho_A) g (z_2 - z_1) - \rho_A g (z_A - z_2)
\end{align*}
\]

\(\rho_m \gg \rho_A, \rho_m - \rho_A \approx \rho_m\)
Example: Pressure measurement with a U-tube manometer

Given: A U-tube manometer is used as a differential pressure measurement instrument to measure the pressure difference between two tanks. The two tanks are at the same elevation.

(a) To do: Calculate the pressure difference \( P_B - P_A \) for the general case in which \( \rho_A \) is not the same as \( \rho_B \) (they are different fluids).

(b) To do: Simplify for the case in which \( \rho_A = \rho_B \) (they are the same fluid).

Solution:

\[
\begin{align*}
\rho_m g h_A + \rho_A g h_A - \rho_B g h_A &= \rho_B g h_B - \rho_A g h_A + \rho_m g h_A + \rho_B g h_B \\
\therefore (\rho_m - \rho_B) g h_A &= (\rho_B - \rho_A) g h_B \\
\therefore P_B - P_A &= (\rho_m - \rho_B) g h_A
\end{align*}
\]
4. Some Notes about Manometry

The elevation difference $\Delta z$ in a U-tube manometer does not depend on the following:

1. **U-Tube diameter** (provided that the tube diameter is large enough that capillary effects are negligible). In the sketch below, for a given pressure in the tank, $\Delta z$ is the same in manometers A and B, even though the tube diameter of manometer B is larger than that of manometer A. Note that the amount of manometer liquid in each of the U-tube manometers has been adjusted such that the level of the interface between fluids 1 and 2 on the left side of each manometer is at the same elevation, for direct horizontal comparison.

2. **U-Tube length** (provided that the tubes are long enough to include elevation difference $\Delta z$). In the sketch, $\Delta z$ is the same in manometers A and C, even though manometer C is shorter than manometer A.

3. **U-Tube shape** (again provided that capillary effects are not important and the relative elevation is the same). In the sketch, $\Delta z$ is the same in manometers A and D, even though manometer D is oddly shaped. Can you think of an advantage of the “inclined manometer” configuration of manometer D?

Why?

$P_{\text{below}} = P_{\text{above}} + \rho g \Delta z \rightarrow \text{does not depend on diameter!}$

Inclined manometer can read to finer resolution

[More tick marks for $\Delta P$]
However, the elevation difference $\Delta z$ in a U-tube manometer does depend on the following:

1. **Manometer fluid.** For example, if we replace the blue manometer fluid in the above sketch with a higher density (gray colored) fluid, as in the sketch below, $\Delta z$ would decrease. In other words, $\Delta z_E < \Delta z_A$.

Which manometer (A or E) would have better resolution?

$\Delta z_E \quad \text{more tricky marker when h is higher} \quad \Delta z_A$

2. **Vertical location of the manometer.** For example, if we move manometer A to a lower elevation, all else being the same, and ignoring changes in atmospheric pressure (manometer A' in the above sketch), $\Delta z$ would increase, i.e., $\Delta z_A' > \Delta z_A$. Why?

The yellow fluid has more column height in A', i.e., pushes the manometer fluid down more.

$\therefore \rho_1' > \rho_1$, in this case
5. **Isobar** => a surface of constant pressure

Arbitrary shape container with a liquid in it

Hydrostatics:

\[ P_{\text{below}} = P_{\text{above}} + \rho g H \]

*Isobars*

= horizontal lines in hydrostatics

P increases as you go down

★ **Notice**: Isobars are perpendicular (⊥) to \( \vec{g} \)