Today, we will:

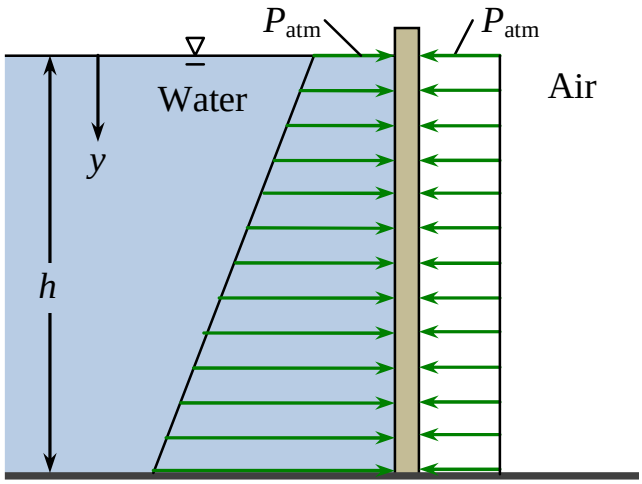
- Discuss hydrostatic forces on submerged surfaces
- Do some example problems – hydrostatic forces on submerged surfaces

E. Hydrostatic Forces on Submerged Surfaces ←

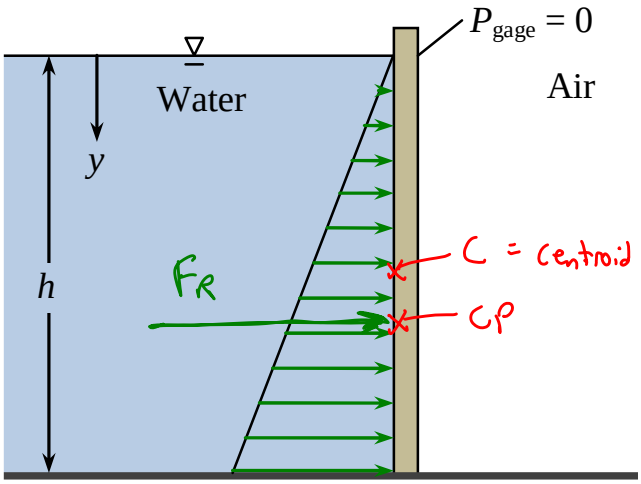
1. Plane (flat) surfaces

$$P_{below} = P_{above} + \rho g |\Delta z|$$

Example – the vertical wall of a rectangular container with a liquid in it.



Absolute pressure distribution



Gage pressure distribution

F_R = resultant net force = $\int_A P_{gage}\, dA$

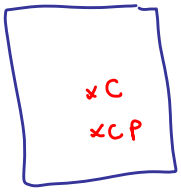
We need to calculate F_R and its location

Define CP = Center of Pressure ≡ location where the resultant force acts

For a vertical plate, $F_R = P_{gage,avg} \cdot A$ — A = area of the plate (one side); $P_{gage,avg}$ = average pressure on the plate (gage pressure)

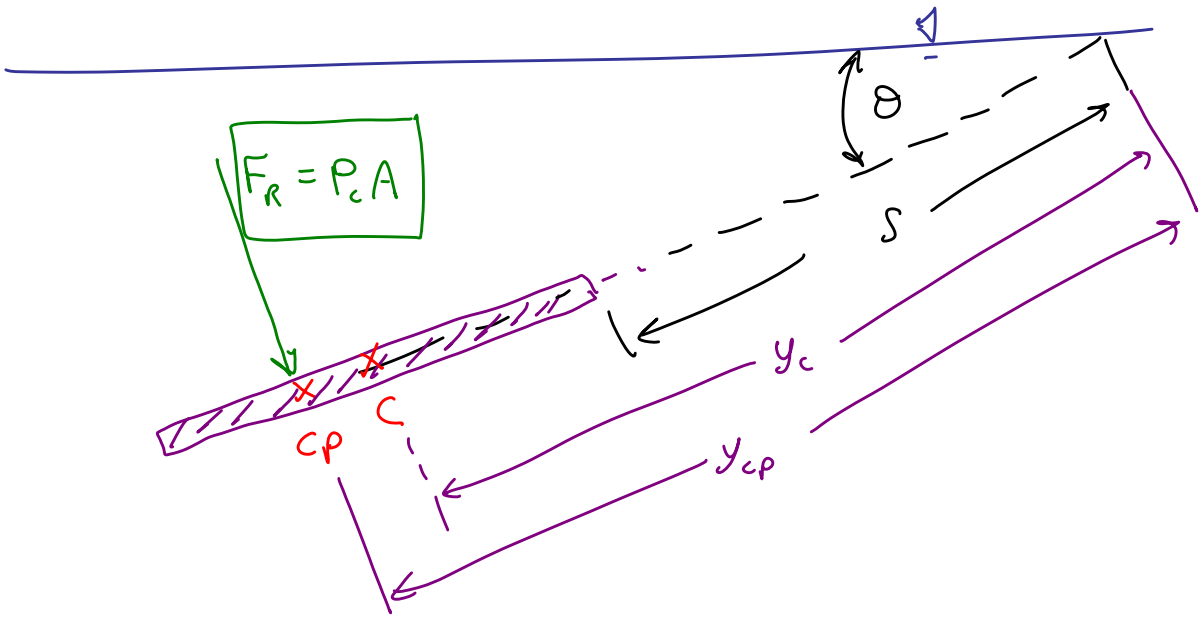
$P_{gage,avg} = P_{gage,C}$ at the centroid

CP is below C since P increases with depth



For a plate like the one shown above, CP is 2/3 down from the top ($@\ y = \frac{2h}{3}$)

General case → plate can be at an angle
& plate can be submerged



* See pdf file in book for details & equations

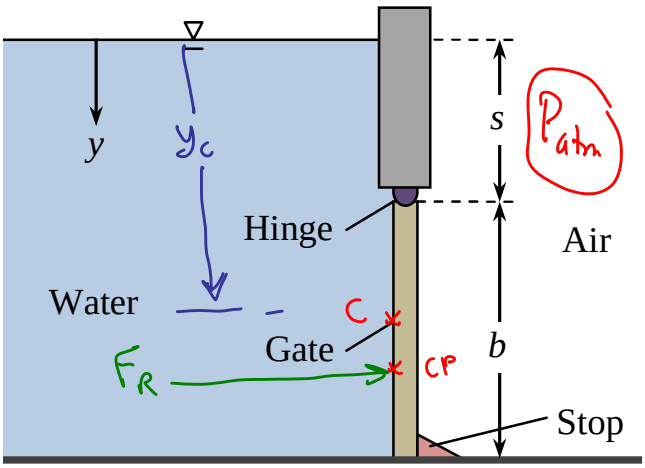
Also, see some examples in the book, & you will have some HW problems too

Example: Force on a submerged gate

Given: A rectangular gate of height *b* and width *a* (into the page) holds back water in a reservoir. (The gate can swing open to let some water out when necessary.) The height from the water surface to the hinge is *s*.

To do: Calculate the resultant force on the gate and its location.

Solution:



See Fig. 3-30 → Centroid is at the center of the plate

$$y_c = s + \frac{b}{2}$$

@ C, $P_c = P_{atm} + \rho g y_c = P_{atm} + \rho g (s + b/2)$

$F_R = F_c A$

$F_R = [P_{atm} + \rho g (s + \frac{b}{2})] ab$ on the left side

$- P_{atm} ab$ on the right side

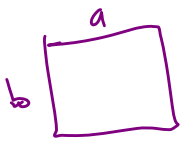
Net force $F_R = \rho g (s + b/2) ab$ to the right ★

Location of CP: Eq. (3-22 b)

$$y_p = [y_{cp}] = y_c + \frac{I_{xx,c}}{y_c A} \quad (*)$$

(Centroidal moment)

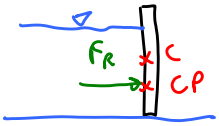
Fig. 3-30 for a rectangular plate, $I_{xx,c} = ab^3/12$



Plug this into (*) →

$$y_p = s + \frac{b}{2} + \frac{b^2}{12(s + b/2)} \quad ★ \quad (**)$$

Special case → if $s = 0$

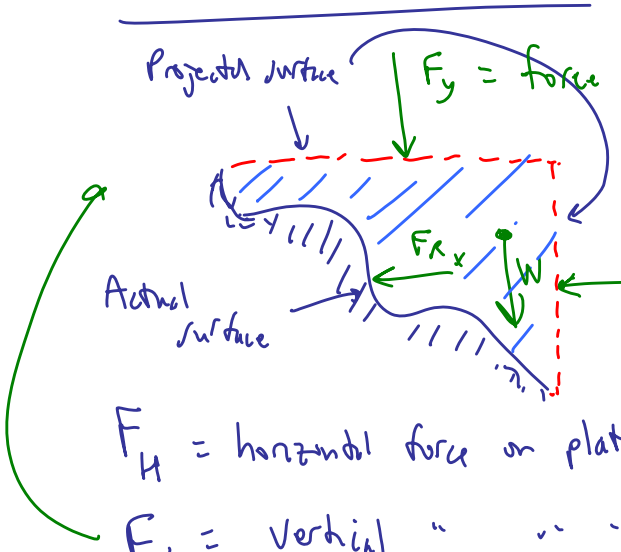


$F_{net} = \rho g \frac{b}{2} ab$

from (**), $y_p = \frac{2}{3} b$

2. Forces on curved surfaces

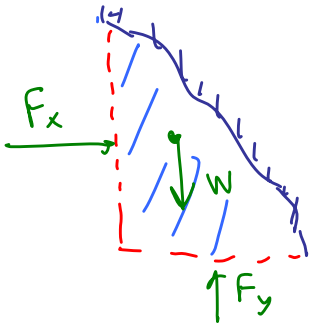
$$F = \int p \, dA$$



F_H = horizontal force on plate = F_x (force on projected area)

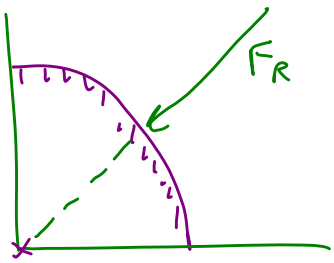
F_V = vertical " " " = $F_y + W$ (if the surface is below the projected surface)

F_V = " " = $F_y - W$ (if the surface is above the projected surface)



Where does the force act?

For circular arcs, the line of action of the resultant force is always through the center of the circular arc



See Example 3-9 in text & on pdf file.