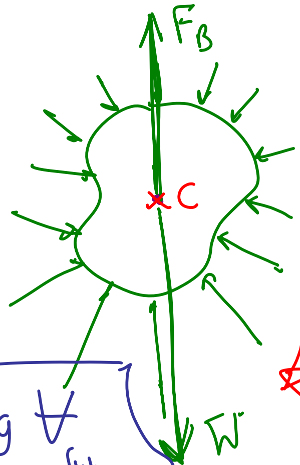
Today, we will:

- Discuss buoyancy and stability in hydrostatics
- Discuss fluids in rigid-body motion (linear acceleration and solid-body rotation)
- If time, begin Chapter 4 – Fluid Kinematics

$\boxed{P_{below} = P_{above} + \rho g |\Delta z|}$

E. Hydrostatic Forces on Submerged Surfaces (continued)

3. Buoyancy and Stability (Sec. 3-6)



F_B = Buoyant force on the body.

Acts at the centroid

$\boxed{F_B = \rho_f \, g \, \forall_{submerged}}$

Archimedes' Principle → F_B = the weight of the fluid that is displaced by the body & it acts through the centroid of the displaced volume & is upward

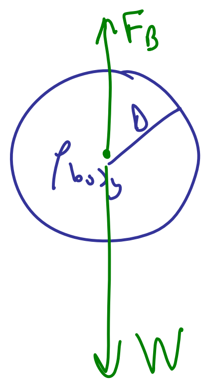
Example: Buoyancy

Given: A sphere of diameter D = 0.0550 m and density ρ_{body} = 1700 kg/m^3 falls into a tank of water (ρ_f = 1000 kg/m^3).

To do: Calculate the net body force on the sphere due to gravity.

Solution:

FBD.



ρ_f

$$F_B = \rho_f \, g \, \frac{\pi D^3}{6} \quad \text{(up)}$$

$$W = \rho_{body} \, g \, \frac{\pi D^3}{6} \quad \text{(down)}$$

Net body force $F_{net\ \underline{down}} = W - F_B$

$$\boxed{F_{net\ down} = \left(\rho_{body} - \rho_f\right) g \, \frac{\pi D^3}{6}}$$

#'s → get 0.598 N downward.

Stability of a Boat

Author: John M. Cimbala, Penn State University
Latest revision: 10 September 2012

b. Stability

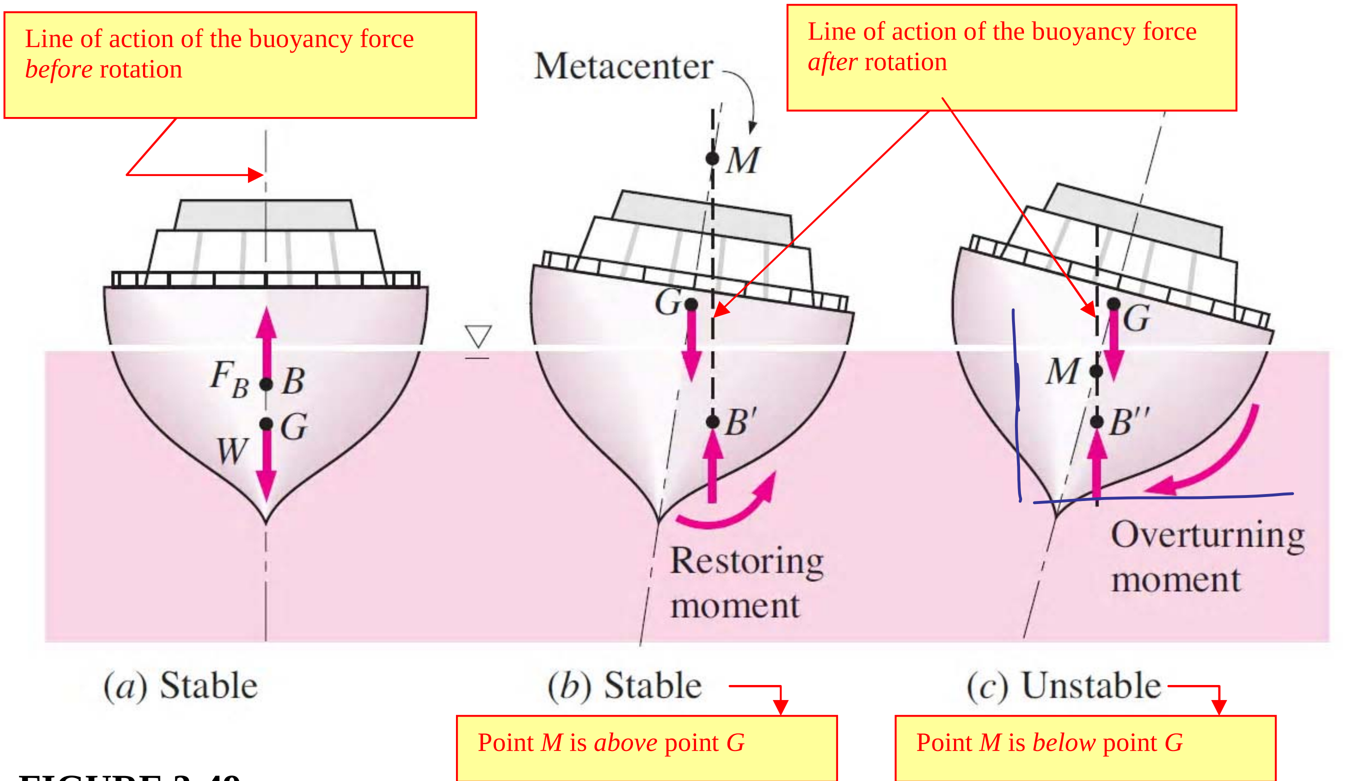


FIGURE 3-49

A floating body is *stable* if the body is bottom-heavy and thus the center of gravity *G* is below the centroid *B* of the body, or if the metacenter *M* is above point *G*. However, the body is *unstable* if point *M* is below point *G*.

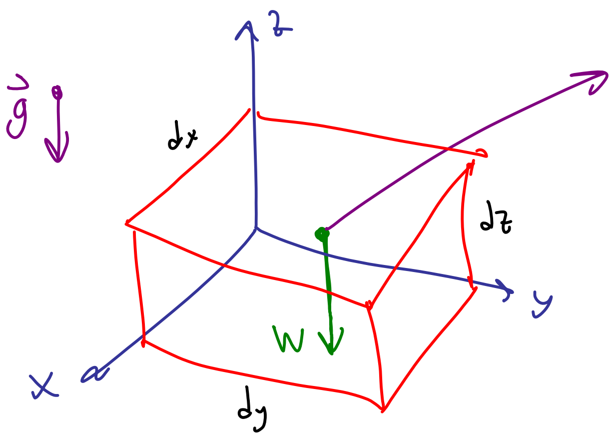
M = the **metacenter** = the point where the lines of action of the buoyancy force *before* and *after* rotation intersect

F. Fluids in Rigid-Body Motion (sec. 3-7)

Fluid behaves like a solid body → ∴ No SHEAR STRESS

1. Equations recall our FBD of a fluid element

There are body forces (gravity) & Pressure forces



$\vec{a}$ = acceleration vector

For statics ($\vec{a}=0$),

$$\boxed{\sum \vec{F} = 0}$$

For this case ($\vec{a} \neq 0$)

$$\boxed{\sum \vec{F} = m\vec{a}}$$

$$\sum \vec{F} = \sum \vec{F}_{grav.} + \sum \vec{F}_{pressure} = m\vec{a} \qquad m = \rho\, dx\,dy\,dz$$

$$\rho \vec{g}\, dx\,dy\,dz - \underbrace{\left(\frac{\partial P}{\partial x}\vec{i} + \frac{\partial P}{\partial y}\vec{j} + \frac{\partial P}{\partial z}\vec{k}\right)}_{\vec{\nabla}P \,=\, \text{gradient of } P} dx\,dy\,dz = \rho\, dx\,dy\,dz\, \vec{a}$$

$\vec{\nabla}$ = del operator

$$\boxed{\vec{\nabla} = \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)}$$

a vector operator

$$☆ \quad \boxed{\vec{\nabla}P = \rho\underbrace{(\vec{g} - \vec{a})}_{\text{call this } \vec{G}}} \qquad (1)$$

For solid body acceleration (rigid body motion)

let $\vec{G}$ = modified gravity vector

$$\boxed{\vec{G} = \vec{g} - \vec{a}}$$

$\therefore$ (1) becomes $\boxed{\vec{\nabla}P = \rho\vec{G}}$ ☆ (2)

If $\vec{a} = 0$, $\vec{G} = \vec{g}$ & (2) reduces to $\boxed{\vec{\nabla}P = \rho\vec{g}}$

= eq. of hydrostatics!

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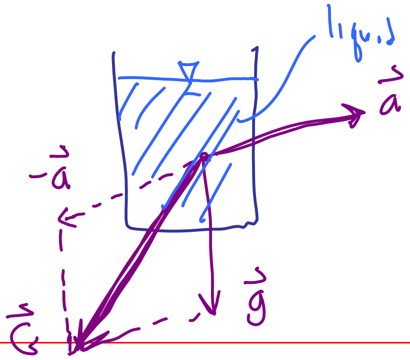
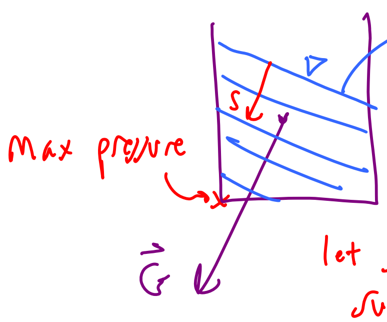
2. Uniform linear acceleration

Isolate on this liquid:





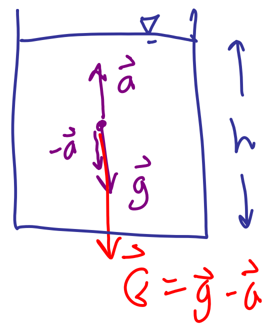
surface tilts as shown $\perp$ to $\vec{G}$

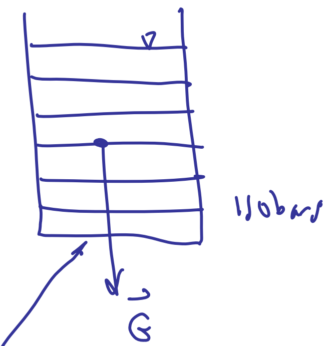
let s = distance down from surface // to $\vec{G}$

Modified eq. for P → ☆ $\boxed{P_{\text{"below"}} = P_{\text{"above"}} + \rho G|\Delta s|}$

Examples:

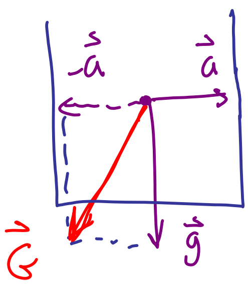
· accelerate up





at bottom, $\boxed{P = P_{atm} + \rho G h}$

· accel. to the right



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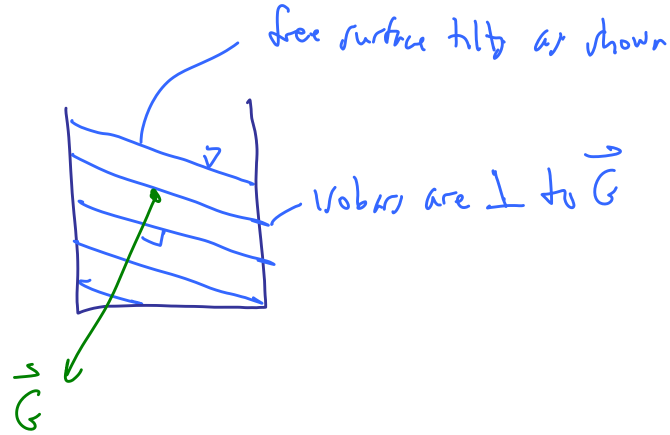
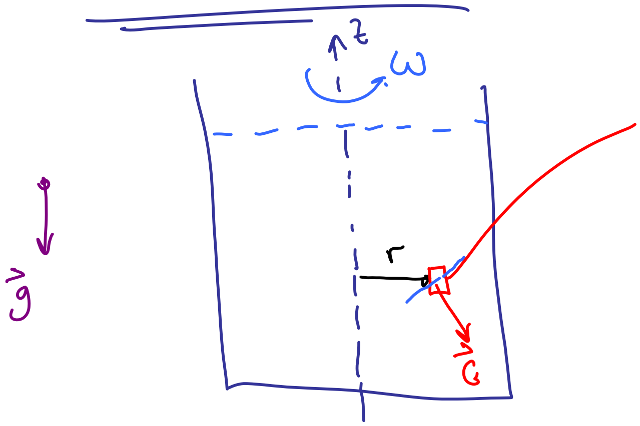


3. Rigid-Body Rotation

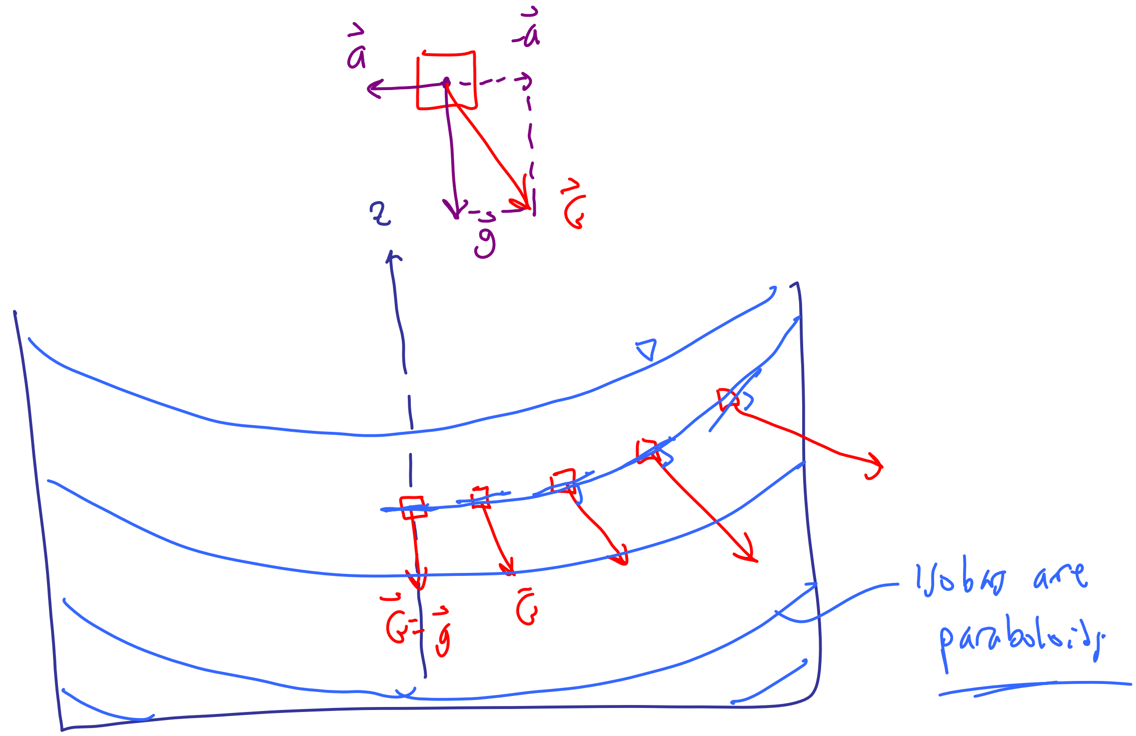
ω = angular velocity up (+z direction)



fluid element @ some radius r

Eq. $\boxed{\vec{\nabla} P = \rho(\vec{g} - \vec{a})}$

FBD of the element:



See eq.s in book

See pdf file → mercury & glass mirrors

Class demo — water in solid body rotation