Today, we will:

- Continue Chapter 5 – Conservation of mass for control volumes
- Do some example problems, conservation of mass
- Begin to discuss conservation of energy

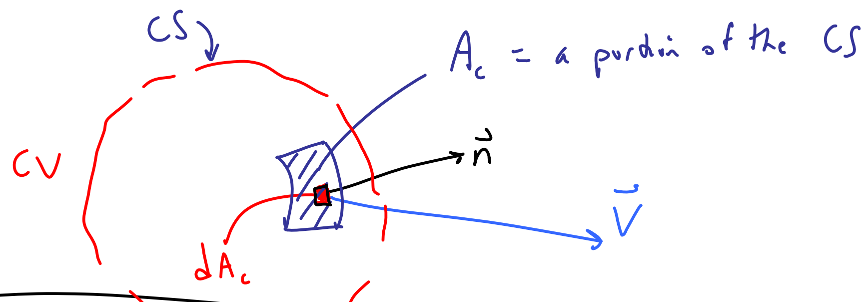
B. Conservation of Mass (continued)

1. Equations and definitions (continued)

Last lecture, we derived the **control volume equation for conservation of mass**:

$$\boxed{\frac{d}{dt}\int_{CV}\rho dV + \int_{CS}\rho\left(\vec{V}\cdot\vec{n}\right) dA = 0} \qquad (1)$$

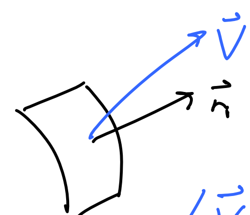
• Mass flow rate:



Define $\boxed{\dot{m}_{A_c} = \int_{A_c}\rho\left(\vec{V}\cdot\vec{n}\right) dA_c}$ — The signs take care of themselves

mass flow rate out of area A_c

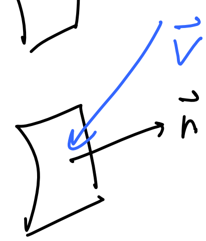
- If mass flows out, $\vec{V}\cdot\vec{n} > 0$



OUTLET

$\dot{m}_{A_c} > 0$

- If mass flow in, $\vec{V}\cdot\vec{n} < 0$



INLET

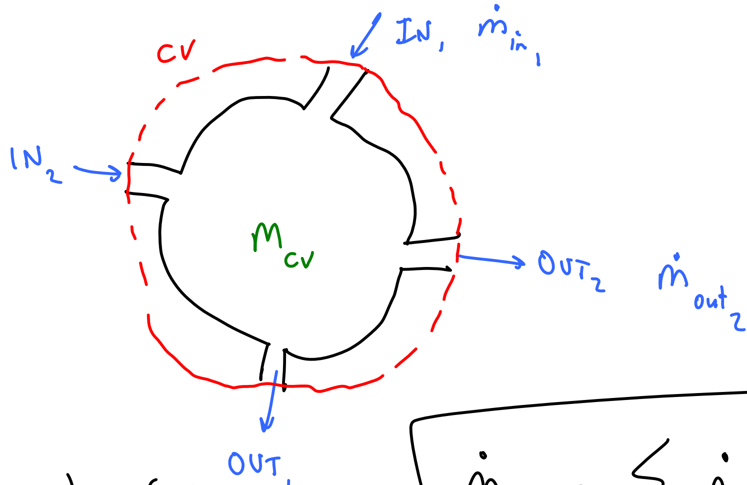
$\dot{m}_{A_c} < 0$

Now integrate over the entire CS, we get $\boxed{\dot{m}_{CS} = \int_{CS}\rho\left(\vec{V}\cdot\vec{n}\right) dA}$ (2)

[2nd term in Eq. (1)]

$\dot{m}_{CS}$ = net rate of mass flow out of the CV.

In most problems, we have well-defined inlets & outlets



Eq. (2) can be simplified to

$$\boxed{\dot{m}_{cs} = \sum_{out} \dot{m} - \sum_{in} \dot{m}}$$

Eq (1) becomes

$$\boxed{\frac{d}{dt}\underbrace{\int_{cv} \rho \, d\forall}_{= m_{cv}} = \sum_{in} \dot{m} - \sum_{out} \dot{m}}$$

or

$$\boxed{\frac{d}{dt} m_{cv} = \sum_{in} \dot{m} - \sum_{out} \dot{m}}$$

rate of change of mass within the CV = net rate of mass flow into the CV through the inlets & outlets

- For steady flow

$$\left(\frac{d}{dt} m_{cv} = 0\right)$$

$$\boxed{\sum_{in} \dot{m} = \sum_{out} \dot{m}}$$

"What goes in must come out!"

- VOLUME FLOW RATE we can define

$$\boxed{\dot{\forall}_{A_c} = \int_{A_c} (\vec{V} \cdot \vec{n}) \, dA_c}$$

- Average velocity through A_c

$$V_{avg,A_c} = \frac{1}{A_c}\int_{A_c}(\vec{V}\cdot\vec{n})\,dA_c = \boxed{\frac{\dot{\forall}_{A_c}}{A_c} = V_{avg,A_c}}$$

- So, we can write an approximation:

$$\boxed{\dot{m}_{A_c} \approx \rho_{avg} V_{avg} A_c}$$

- We typically drop the "avg" subscript $\rightarrow$ it is understood to be an average

We typically write $\boxed{\dot{m}_{A_c} = \rho\dot{\forall}_{A_c} = \rho V A_c}$

If the flow is <u>incompressible</u> $[\rho \approx \text{const}]$ & <u>steady</u> $\left[\frac{\partial}{\partial t} = 0\right]$

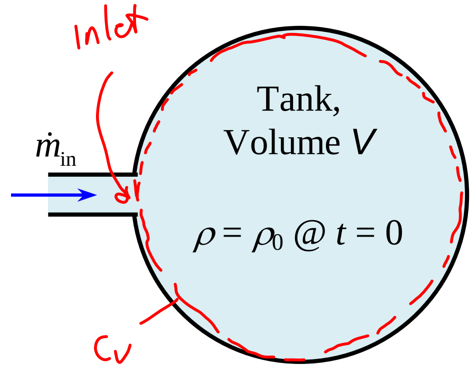
Cons. of mass reduces to

$$\boxed{\sum_{in}\dot{\forall} = \sum_{out}\dot{\forall}}$$

Now we are ready to do some examples:

Example: Unsteady conservation of mass (flow into a tank)

Given: Air is pumped into a rigid tank of volume V. The mass flow rate of the air entering the tank is constant, $\dot{m}_{in}$. We assume that the process is slow enough that the air in the tank remains at the same temperature (isothermal conditions).



To do: Generate an equation for density ρ in the tank as a function of time.

Solution: • Draw a CV = first step Always

• Apply cons. of mass eq. (Fixed CV.)

$$\frac{d}{dt}\int_{CV} \rho \, d\forall = \sum_{in} \dot{m} - \cancel{\sum_{out} \dot{m}}$$

assume $\rho = \rho(t)$ only [ρ is const. inside the whole tank at any instant in time]

$$\frac{d\rho}{dt}\int_{CV} d\forall = \dot{m}_{in}$$

$\forall$ = constant.

$$\Rightarrow \frac{d\rho}{dt}\forall = \dot{m}_{in}$$

$$\frac{d\rho}{dt} = \frac{\dot{m}_{in}}{\forall} = \text{const}$$

Integrate w.r.t. time from $t=0$ to $t=t$

$$\boxed{\rho = \rho_0 + \frac{\dot{m}_{in}}{\forall} t}$$

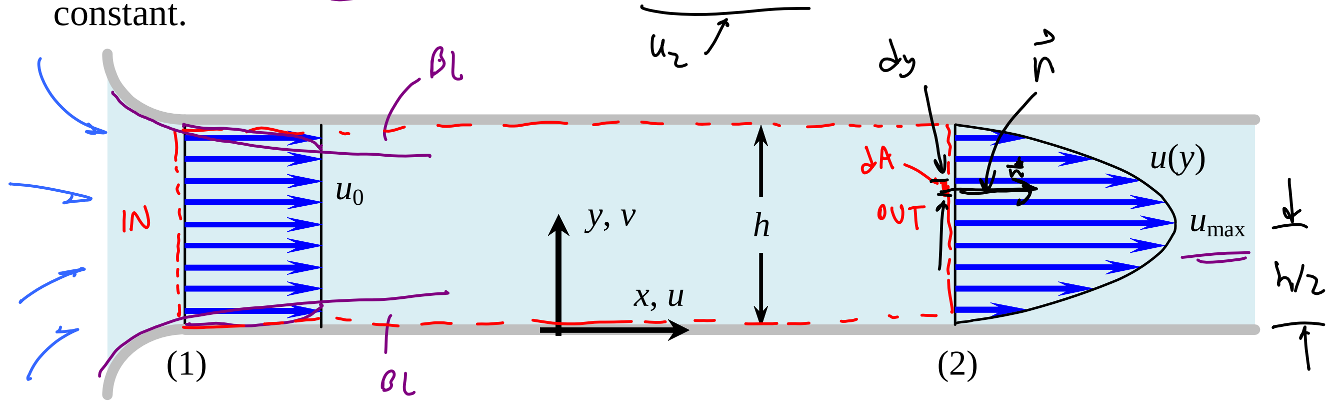
Answer

[$\rho_0 = \rho$ in tank @ $t=0$]

Example: Velocity profiles in 2-D channel flow

Given: Consider steady, incompressible, two-dimensional flow of a liquid between two very long parallel plates as sketched. At the inlet (1) there is a nice bell mouth, and the velocity is nearly uniform (except for a very thin boundary layer, not shown).

- At (1), $u = u_0$ = constant, $v = 0$, and $w = 0$.
- At (2), the flow is fully developed, and $u = ay(h - y)$, $v = 0$, and $w = 0$, where a is a constant.



To do: Generate expressions for constant a and speed u_{max} in terms of the given variables.

[b = width into the page]

Solution: · Draw a CV.

· Cons. of mass

$$\sum_{in} \dot{V} = \sum_{out} \dot{V}$$

@ ① @ ②

$$u_0 h b = \int_{A_2} \vec{V}\cdot\vec{n}\, dA$$

$\vec{V}\cdot\vec{n} \to u_2$, $dA = b\,dy$

$$u_0 h \cancel{b} = \cancel{b}\int_{y=0}^{y=h} u_2\, dy$$

given → $\boxed{u_2(y) = ay(h-b)}$

} algebra

$$u_0 h = \frac{ah^3}{6} \rightarrow \text{solve for } a \qquad \boxed{a = \frac{6u_0}{h^2}}$$

· U_{max} occurs in the middle @ $y = h/2$

$$u = a\,y\,(h - y)$$

$$U_{max} = \frac{6u_0}{h^2}\,\frac{h}{2}\left(h - \frac{h}{2}\right) = \frac{3u_0}{2}$$

$$\boxed{U_{max} = \frac{3}{2}u_0}$$

C. Conservation of Energy

1. Eqs & Definitions

- From RTT, (see previous notes)

$$\boxed{\dot{Q}_{net\ in} + \dot{W}_{net\ in} = \frac{d}{dt}\int_{CV} e\rho\, d\forall + \int_{CS} e\rho(\vec{V}\cdot\vec{n})\,dA} \quad (3)$$

Net rate of heat transfer to the CV + Net rate of work (i.e., Power) done on the CV = rate of change of total energy inside the CV + net rate of energy flow out of the CV through the CS

e = total specific energy $= \frac{E}{m}$

$$\boxed{e = \underset{\text{internal}}{u} + \underset{\text{potential}}{ke} + \underset{\text{kinetic}}{pe} = u + \frac{V^2}{2} + gz}$$

So Eq (3) becomes

$$\boxed{\dot{Q}_{net_{in}} + \dot{W}_{net\ in} = \frac{d}{dt}\int_{CV}\left(u + \frac{V^2}{2} + gz\right)\rho\, d\forall + \int_{CS}\left(u + \frac{V^2}{2} + gz\right)\rho(\vec{V}\cdot\vec{n})\,dA}$$