Today, we will:

- Continue discussing conservation of energy for a control volume
- Do an example problem – energy equation with a pump
- Discuss the kinetic energy correction factor
- If time, begin derivation of the "head" form of the energy equation

From previous lecture…the conservation of energy equation for a fixed control volume:

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}} = \frac{d}{dt}\int_{CV} e\rho dV + \int_{CS}\left(u + \frac{V^2}{2} + gz\right)\rho\left(\vec{V}\cdot\vec{n}\right)dA \qquad (1)$$

[Note: For convenience later (hindsight), we keep e as e in the volume integral, but expand it in the area integral.]

$$\dot{W}_{\text{net in}} = \dot{W}_{\text{shaft, net in}} + \dot{W}_{\text{pressure, net in}} + \cancel{\dot{W}_{\text{viscous, net in}}} + \cancel{\dot{W}_{\text{other, net in}}}$$

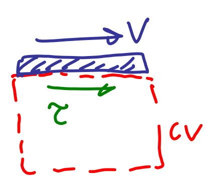
See text for derivation

$$\dot{W}_{\text{pressure, net in}} = -\int_{CS}\frac{P}{\rho}\rho\left(\vec{V}\cdot\vec{n}\right)dA$$

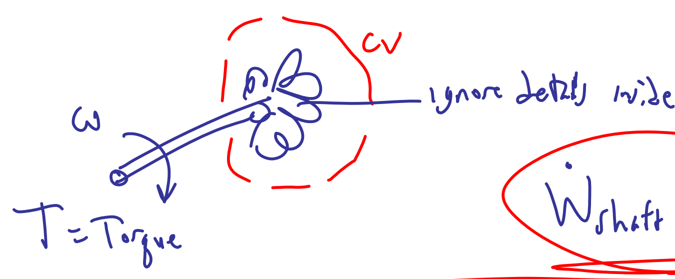
⊕ve term @ an inlet (push fluid into the CV)
⊖ve @ an outlet (fluid pushes into the surroundings)

$\dot{W}_{\text{viscous, net in}} \cong 0$ Usually zero

$\dot{W}_{\text{other, net in}} = 0$ all other power inputs (electromagnetic, etc.)



$\dot{W}_{\text{shaft, net in}}$: Power from rotating shafts that cut through the CS



$$\dot{W}_{\text{shaft}} = \omega T$$

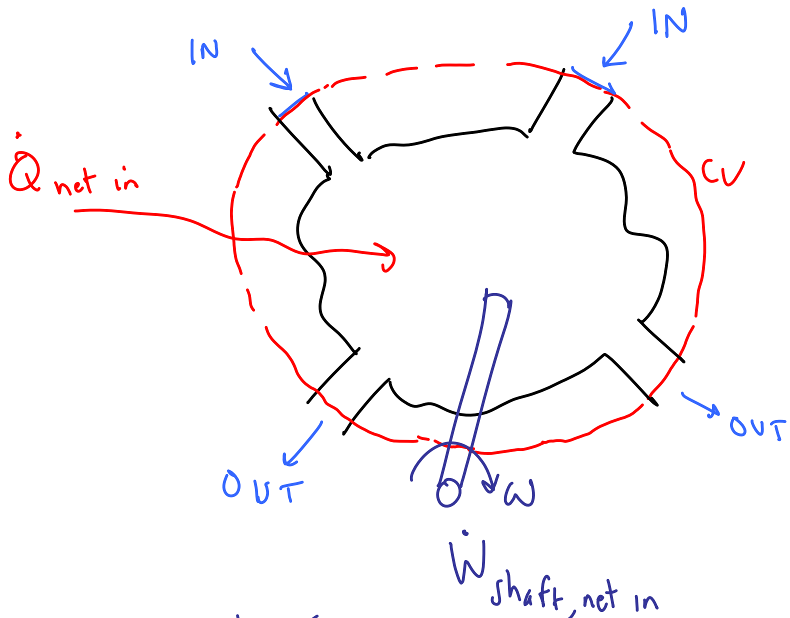
⊕ve for a pump
⊖ve for a turbine

(1) becomes

$$\dot{Q}_{\text{net, in}} + \dot{W}_{\text{shaft, net in}} = \frac{d}{dt}\int_{CV} e\rho dV + \int_{CS}\left(\left(u + \frac{P}{\rho}\right) + \frac{V^2}{2} + gz\right)\rho\left(\vec{V}\cdot\vec{n}\right)dA \qquad (2)$$

$u + \frac{P}{\rho}$: specific enthalpy, h

Simplification for well-defined inlets & outlets:



We can approximate Eq. (2),

$$\dot{Q}_{net\ in} + \dot{W}_{shaft,\ net\ in} = \frac{d}{dt}\int_{CV} e\rho\, d\forall + \sum_{out} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{in} \dot{m}\left(h + \frac{V^2}{2} + gz\right) \tag{3}$$

h, V, ρ, etc. are average values

- For steady flow, the $\frac{d}{dt}$ term drops out

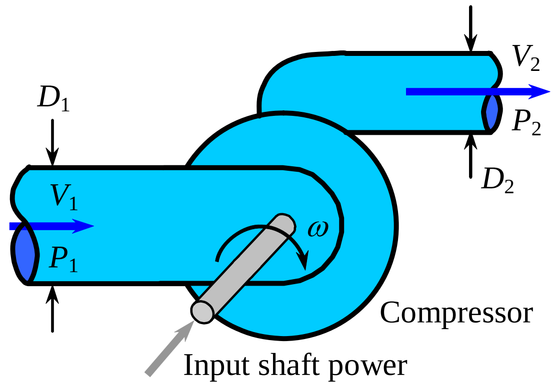
steady-state, steady flow energy eq. for a CV

SSSF eq. of thermo.

$$\dot{Q}_{net\ in} + \dot{W}_{shaft,\ net\ in} = \sum_{out} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{in} \dot{m}\left(h + \frac{V^2}{2} + gz\right) \tag{4}$$

Example: Control volume energy equation applied to an air compressor

Given: A large air compressor takes in air at absolute pressure P_1 = 14.0 psia, at temperature T_1 = 80°F (539.67 R), and with mass flow rate $\dot{m}$ = 20.0 lbm/s. The diameter of the compressor inlet is D_1 = 24.5 inches. At the outlet, P_2 = 70.0 psia and T_2 = 500°F (959.67 R). The diameter of the compressor outlet is D_2 = 7.50 inches. The shaft driving the compressor supplies 3100 horsepower to the compressor.



(a) **To do**: Calculate the average velocity of the air entering the compressor.

Solution: At the inlet, $\dot{m} \approx \rho_{1,\text{ avg}} V_{1,\text{ avg}} A_1 = \rho_1 V_1 A_1$ where the subscripts "avg" have been dropped for convenience. Thus, $V_1 = \dfrac{\dot{m}}{\rho_1 A_1} = \dfrac{4\dot{m}}{\pi \rho_1 D_1^2} = \dfrac{4RT_1\dot{m}}{\pi P_1 D_1^2}$, where we have used the ideal gas law $P = \rho RT$ to calculate the density of the air. Substitution of the values yields

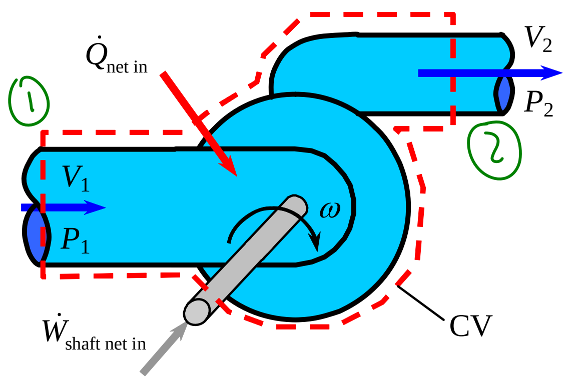
$$V_1 = \frac{4RT_1\dot{m}}{\pi P_1 D_1^2} = \frac{4\left(53.34\dfrac{\text{ft}\cdot\text{lbf}}{\text{lbm}\cdot\text{R}}\right)(539.67\text{ R})\left(20.0\dfrac{\text{lbm}}{\text{s}}\right)}{\pi\left(14.0\dfrac{\text{lbf}}{\text{in}^2}\right)(24.5\text{ in})^2} = 87.229\frac{\text{ft}}{\text{s}} \approx \mathbf{87.2}\,\frac{\textbf{ft}}{\textbf{s}}$$

(b) **To do**: Calculate the average velocity of the air leaving the compressor.

Solution: Similarly, using the pressure, temperature, and diameter at the compressor outlet, we get V_2 = 331.051 ft/s, or $\boldsymbol{V_2}$ **= 331. ft/s** (to three significant digits of precision)

(c) **To do**: Calculate the net rate of heat transfer from the air compressor into the room in units of Btu/hr.

Solution: First we choose a control volume. We draw the control volume around the entire compressor, cutting through the shaft, and cutting through the inlet and outlet, as sketched. Note that we draw the net rate of heat transfer $\dot{Q}_{\text{net in}}$ *into* the control volume to keep the signs straight. We expect a negative value since the compressor will actually give off heat into the room.



Next, we apply the approximate form of the control volume energy equation,

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft net in}} = \underbrace{\cancel{\frac{d}{dt}\int_{CV} e\rho dV}}_{\text{steady}} + \cancel{\sum_{\text{out}}}\dot{m}\left(h + \frac{V^2}{2} + \cancel{gz}\right) - \cancel{\sum_{\text{in}}}\dot{m}\left(h + \frac{V^2}{2} + \cancel{gz}\right)$$

(Ignore potential energy changes for gases)

and we solve for $\dot{Q}_{\text{net in}}$.

get rid of Σ's since 1 inlet & 1 outlet

Solution to be completed in class.

Solve for $\dot{Q}_{\text{net in}}$

$$\dot{Q}_{net\ in} = -\dot{W}_{shaft,\ net\ in} + \dot{m}\left(h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}\right)$$

average velocities

for an ideal gas $h_2 - h_1 = C_p(T_2 - T_1)$

use avg. C_p between T_1 & T_2

Plug in all #'s

unit goal

$$\dot{Q}_{net\ in} = (-3100\ hp)\left(\frac{2544.5\ Btu}{hp\cdot hr}\right) + \underbrace{\left(20.0\ \frac{lbm}{s}\right)}_{\dot{m}}\left[\underbrace{\left(6019\ \frac{ft^2}{s^2\,\cancel{R}}\right)}_{c_p}(959.67 - 539.67)\cancel{R} + \frac{(331.051\ ft/s)^2 - (87.229\ ft/s)^2}{2}\right]\left(\frac{lbf\cdot s^2}{32.174\ lbm\cdot ft}\right)\left(\frac{1\ Btu}{778.169\ lbf\cdot ft}\right)\left(\frac{3600\ s}{hr}\right)$$

$$\boxed{\dot{Q}_{net\ in} = -4.71\times 10^5\ Btu/hr}$$

Notice $\dot{Q}$ is negative as expected

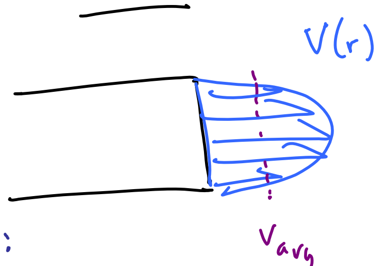
Typical room air conditioner $\approx 5000\ \frac{Btu}{hr}$

$\approx$ 100 room air conditioners

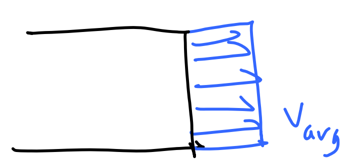
3. The kinetic energy correction factor, α

outlet

EXACT



APPROX



k.e. term:

$$\frac{1}{2}\rho\int V^3\,dA \underset{>}{\neq} \rho V_{avg} A \frac{V_{avg}^2}{2}$$

Introduce α = kinetic energy correction factor

So that $$\frac{1}{2}\rho\int_A V^3\,dA = \alpha\,\frac{V_{avg}^2}{2}\,\rho V_{avg} A$$

i.e., $$\boxed{\alpha = \frac{1}{A}\int_A \left(\frac{V}{V_{avg}}\right)^3 dA}$$

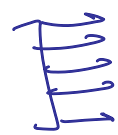
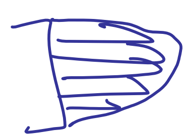
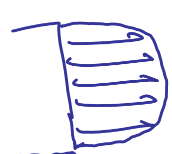
For any real velocity profile @ an inlet or outlet, $\alpha > 1$

En eq for a CV becomes

$$\dot{Q}_{net\ in} + \dot{W}_{shaft,\ net\ in} = \frac{d}{dt}\int_{CV} e\rho\, d\forall + \sum_{out} \dot{m}\left(h + \alpha\frac{V^2}{2} + gz\right) - \sum_{in} \dot{m}\left(h + \alpha\frac{V^2}{2} + gz\right)$$

More accurate version of the CV energy eq.

Some common values of α:

- Uniform flow  $\alpha = 1$
- Fully developed laminar pipe flow  $\alpha = 2$
- Fully " turbulent " "  $\alpha \simeq 1.04$ to 1.11

Let's use $\alpha = 1.05$ as a reasonable approximation for turbulent pipe flow.

E.g. our same compressor problem with $\alpha_1 = 1.05$ @ inlet

$\quad$ & $\alpha_2 = 1.05$ @ outlet

get $\boxed{\dot{Q}_{net\ in} = -4.64 \times 10^5 \ Btu/hr}$ ☆

we had with $\alpha = 1$ $\rightarrow$ $\dot{Q}_{net\ in} = 4.71 \times 10^5 \ Btu/hr$

(ignored α)

Bottom line → When we include α in our calculations, its effect is not enormous, but we get more accurate results without significant increase in effort. Therefore, I recommend that we always include α in our calculations