Today, we will:

- Derive the "head" form of the energy equation
- Discuss pumps and turbines and their efficiencies
- Do some example problems – energy equation with pumps and turbines
- Discuss grade lines – energy grade line and hydraulic grade line

C. Conservation of Energy (continued)

4. The "head" form of the energy equation

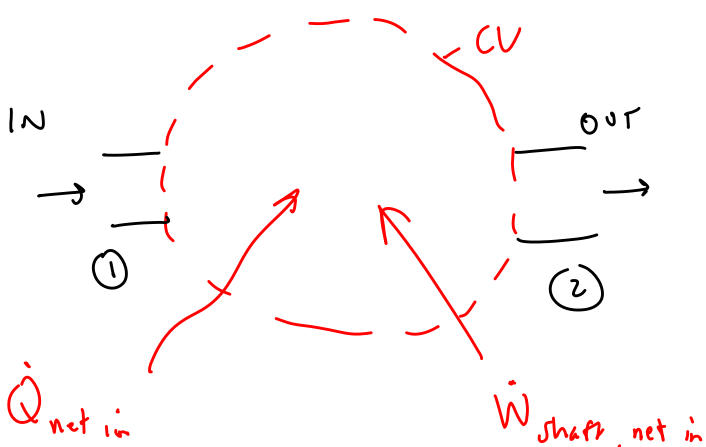
From previous lecture…the Steady-State Steady-Flow (SSSF) conservation of energy equation for a fixed control volume:

$\dot m_1 = \dot m_2 = \dot m$

$$\boxed{\dot Q_{\text{net in}} + \dot W_{\text{net shaft in}} = \cancel{\sum_{\text{outlets}}} \dot m \left(h + \alpha \frac{V^2}{2} + gz \right) - \cancel{\sum_{\text{inlets}}} \dot m \left(h + \alpha \frac{V^2}{2} + gz \right)} \quad \textbf{(1)}$$

THE "HEAD" FORM OF THE ENERGY EQ. IS THE MOST USEFUL FORM IN FLUIDS ☆

Assumptions:
- Steady (SSSF)
- Only 1 inlet (call it ①), & 1 outlet (call it ②)



(1) Becomes $\dot Q_{\text{net in}} + \dot W_{\text{shaft\_net in}} = \dot m \left(h + \alpha \frac{V^2}{2} + gz \right)_2 - \dot m \left(h + \alpha \frac{V^2}{2} + gz \right)_1$

To get dimensions of "head" (length)
÷ all terms by $\dot m g$

let $h = u + \frac{P}{\rho}$

$$\boxed{\frac{\dot W_{\text{shaft\_net in}}}{\dot m g} + \left(\frac{P}{\rho g} + \alpha \frac{V^2}{2g} + z \right)_1 = \left(\frac{P}{\rho g} + \alpha \frac{V^2}{2g} + z \right)_2 + \underbrace{\frac{u_2 - u_1}{g} - \frac{\dot Q_{\text{net in}}}{\dot m g}}_{h_L}} \quad (2)$$

h_L = irreversible head loss

$$\boxed{h_L = \frac{u_2 - u_1}{g} - \frac{\dot{Q}_{net\ in}}{\dot{m} g}}$$

Ideal flow (no irreversibilities), $\boxed{h_L = 0}$

Real flow (there are irreversibilities) e.g., friction, turbulent mixing

$$\frac{u_2 - u_1}{g} > \frac{\dot{Q}_{net\ in}}{\dot{m} g} \rightarrow \therefore \boxed{h_L > 0}$$

☆ h_L cannot be negative (would violate 2nd law of thermo)

Comments: · Every term in the head eq. has dimensions of {length}

[equiv. column height of the fluid that is flowing in the CV]

· Split the shaft power term into · pumps (add power)
· turbines (extract power)

$$\boxed{\dot{W}_{shaft,\ net\ in} = \sum \dot{W}_{pump} - \sum \dot{W}_{turbines}}$$

· Separate losses in the pumps & turbines from the other irreversible losses in the C.V. → treat them separately

(2) becomes

pump head (useful) turbine head (extracted)

☆ $$\left(\frac{P}{\rho g} + \alpha \frac{V^2}{2g} + z\right)_1 + h_{pump,u} = \left(\frac{P}{\rho g} + \alpha \frac{V^2}{2g} + z\right)_2 + h_{turbine,e} + h_L$$

✪ The above eq is (5-77), the most useful form of the energy eq.

* "Head" form of energy eq.

Where, $h_{pump,u}$ ≡ useful head supplied by the pump

shaft power of the pump

$$\boxed{h_{pump,u} = \eta_{pump} \frac{\dot{W}_{pump}}{\dot{m} g}}$$

η_{pump} = pump efficiency

$$\boxed{0 < \eta_{pump} < 1}$$

$$\boxed{\eta_{pump} = \frac{\text{useful power supplied to the fluid}}{\text{actual shaft power through the CV}} = \frac{\text{"water horsepower"}}{\text{"brake horsepower"}}}$$

$$\boxed{\eta_{turbine} = \frac{\text{actual shaft power through the CV}}{\text{power extracted from the turbine}} = \frac{\text{"brake horsepower"}}{\text{"water horsepower"}}}$$

$$\boxed{0 < \eta_{turbine} < 1}$$

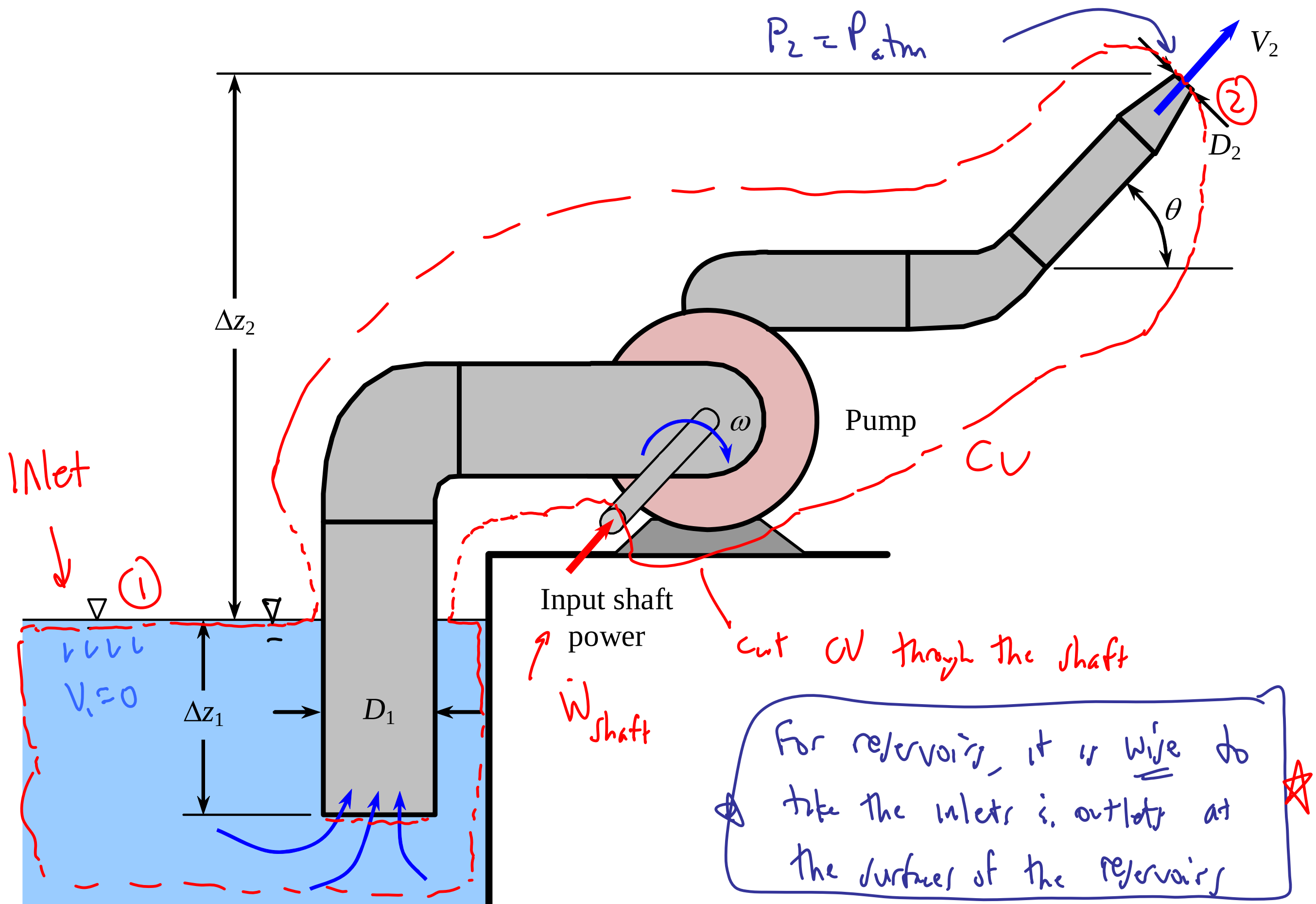
$h_{turbine\_e}$ = extracted head removed by the turbine

✪ $$\boxed{h_{turbine\_e} = \frac{1}{\eta_{turbine}} \frac{\dot{W}_{turbine}}{\dot{m} g}}$$

5. Examples (See Ex. 5.11 - 5.15)

Example – Fire-fighting water pump

Given: A self-priming pump is used to draw water from a lake and shoot it through a nozzle, as sketched. The diameter of the pump inlet is D_1 = 12.0 cm. The diameter of the nozzle outlet is D_2 = 2.54 cm, and the average velocity at the nozzle outlet is V_2 = 65.8 m/s. The pump efficiency is 80%. The vertical distances are Δz_1 = 1.00 m and Δz_2 = 2.00 m. The irreversible head losses in the piping system (not counting inefficiencies associated with the pump itself) are estimated as h_L = 4.50 m of equivalent water column height. *Note*: Later on, in Chapter 8, you will learn how to calculate the irreversible head losses associated with piping systems on your own. For now, they are given.



(**a**) **To do**: Calculate the volume flow rate of the water in m^3/hr and gallons per minute (gpm).

Solution: At the outlet, $\dot{V} = V_{2,\text{ avg}} A_2 = V_2 \dfrac{\pi D_2^{\,2}}{4} = \left(65.8 \dfrac{\text{m}}{\text{s}}\right) \dfrac{\pi (0.0254 \text{ m})^2}{4} = 0.033341 \dfrac{\text{m}^3}{\text{s}}$, where we have dropped the subscript "avg" for convenience. We convert to the required units as follows:

$\dot{V} = 0.033341 \dfrac{\text{m}^3}{\text{s}} \left(\dfrac{3600 \text{ s}}{\text{hr}}\right) = \mathbf{120.\ \dfrac{m^3}{hr}}$ and $\dot{V} = 0.033341 \dfrac{\text{m}^3}{\text{s}} \left(\dfrac{15{,}850 \text{ gpm}}{\text{m}^3/\text{hr}}\right) = \mathbf{528.\ gpm}$, where both answers are given to three significant digits of precision.

(**b**) **To do**: Calculate the power delivered by the pump to the water, i.e. calculate the ***water horsepower*** $\dot{W}_{\text{water horsepower}}$ in units of kW.

(**c**) **To do**: Calculate the required shaft power to the pump, i.e. calculate the ***brake horsepower*** bhp in units of kW.

Solutions for parts (b) and (c) to be completed in class.

(b) · First step: Draw a CV (see sketch 1) [We picked a "wise" CV.]

· Energy eq. in head form:

$$\cancel{\frac{P_1}{\rho g}} + \alpha_1 \underbrace{\cancel{\frac{V_1^2}{2g}}}_{\text{(wise CV)}} + z_1 + h_{pump,u} = \cancel{\frac{P_2}{\rho g}} + \alpha_2 \frac{V_2^2}{2g} + z_2 + \underbrace{\cancel{h_{turbine}}}_{\text{(none)}} + h_L$$

$P_1 = P_2 = P_{atm}$ (cancel)

· Simplify as much as possible.

At outlets exposed to atmospheric pressure $P_{outlet} = P_{atm}$. ☆

$$\therefore \boxed{h_{pump\_u} = \alpha_2 \frac{V_2^2}{2g} + \underbrace{z_2 - z_1}_{\Delta z} + h_L}$$ ☆

increase ke | increase pe | overcome irreversibilities

☆ $$\boxed{\dot{W}_{water\ horsepower} = \underbrace{\dot{m}}_{=\rho\dot{V}} g\, h_{pump,u} = \rho \dot{V} g \left[\alpha_2 \frac{V_2^2}{2g} + \Delta z + h_L\right]}$$

☆ Answer in variable form

· Numbers → $\boxed{\dot{W}_{water\ horsepower} = 77.8\ \text{kW}}$

(c) Sol'n:

$$bhp = \frac{\dot{W}_{water\ horsepower}}{\eta_{pump}} = \frac{77.756\ \text{kW}}{0.80} = \boxed{97.2\ \text{kW} = bhp}$$ ☆