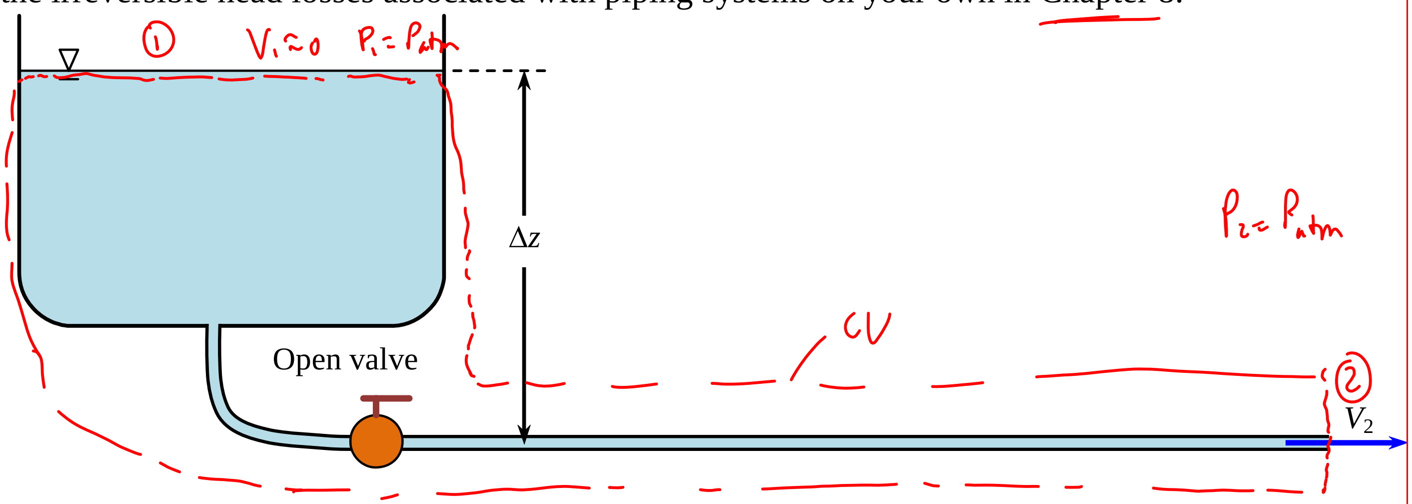
Today, we will:

- Do another example problem – head form of the energy equation
- Discuss grade lines – energy grade line and hydraulic grade line
- Derive and discuss the Bernoulli equation

5. Examples (continued)

Example – Water draining from a tank

Given: Water drains by gravity from a tank exposed to atmospheric pressure. The vertical distance from the pipe outlet to the surface of the water in the tank is Δz = 0.500 m. The irreversible head losses in the piping system (due to friction in the pipe, losses through the valve, elbow, etc.) are estimated as h_L = 0.400 m of equivalent water column height. *Note*: You will learn how to calculate the irreversible head losses associated with piping systems on your own in Chapter 8.



To do: Calculate the average velocity at the outlet, V_2.

Solution: Draw a CV

From previous lecture…use the head form of the conservation of energy equation:

$$\boxed{\left(\cancel{\frac{P_1}{\rho_1 g}}+\cancel{\alpha_1\frac{V_1^2}{2g}}+z_1\right)+\cancel{\sum h_{\text{pump,u}}}=\left(\cancel{\frac{P_2}{\rho_2 g}}+\alpha_2\frac{V_2^{\,2}}{2g}+z_2\right)+\cancel{\sum h_{\text{turbine, e}}}+h_L}$$

cancel

$(V_1 \approx 0)$ $(P_1 = P_2 = P_{atm})$ $[\Delta z = z_1 - z_2]$

Solve for V_2

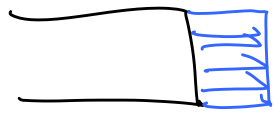
$$\boxed{V_2 = \left\{\frac{2g}{\alpha_2}\left[\Delta z - h_L\right]\right\}^{1/2}}$$

Ans.

What is α_2?

Numbers: 3 cases

1) Ignore α @ outlet (approx. outlet as uniform flow)



V_2 $\qquad \alpha_2 = 1$

$$V_2 = \left\{ \frac{2\,(9.807\ \mathrm{m/s^2})}{1.00} \left[0.50\ \mathrm{m} - 0.40\ \mathrm{m} \right] \right\}^{1/2} = \boxed{1.40\ \mathrm{m/s}}$$

X Not good answer because we know $\alpha_2 \neq 1$

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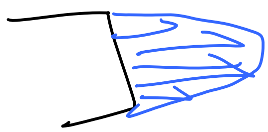
2) Assume fully developed laminar pipe flow — $\boxed{\alpha_2 = 2.0}$



get $\boxed{V_2 = 0.990\ \mathrm{m/s}}$

[41% difference from first case!]

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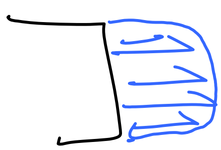
3) Fully developed & turbulent → $\boxed{\alpha_2 \cong 1.05}$

↓



$\boxed{V_2 = 1.37\ \mathrm{m/s}}$

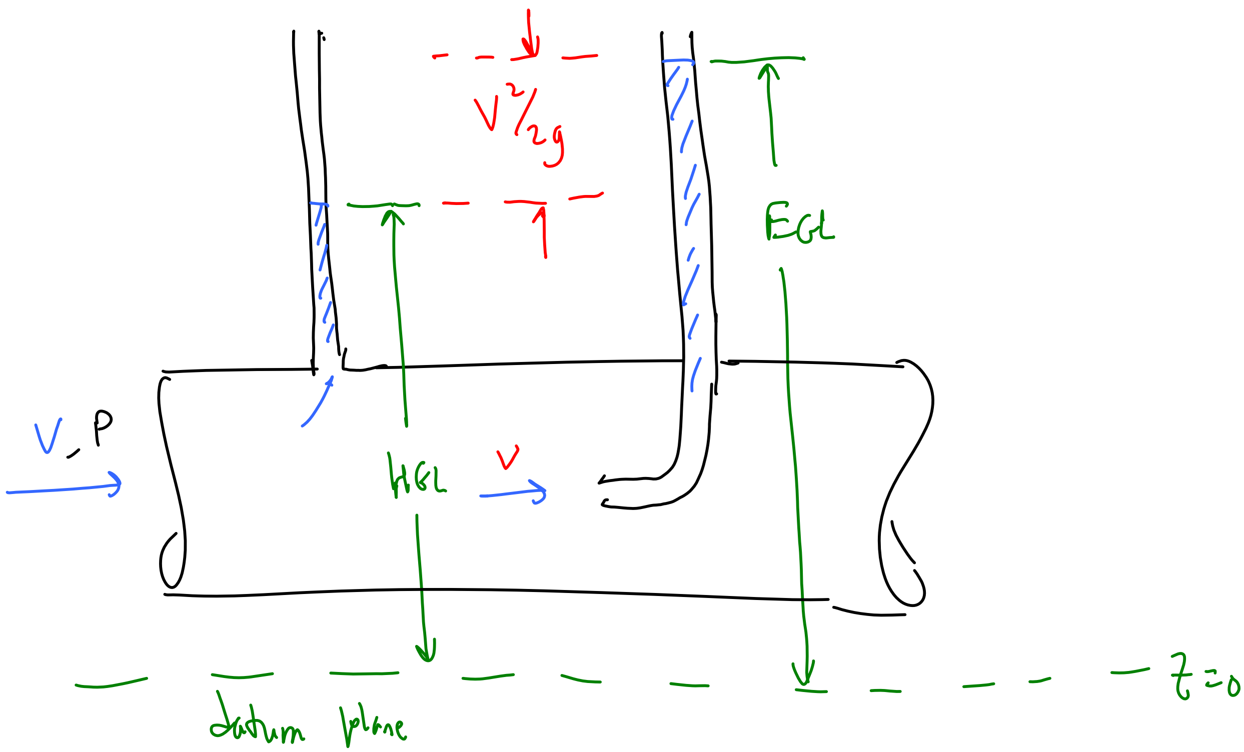
How do we know whether the outlet is laminar or turbulent

↳ see Ch. 8

6. Grade Lines

a. Hydraulic grade line (HGL) ≡ the height to which a liquid will rise from a pressure tap normal to the flow

→ (Static pressure tap)



b. Energy Grade Line (EGL) = the height to which a liquid will rise in a total pressure probe aligned into the flow

$$\boxed{HGL = \frac{P}{\rho g} + z}$$

$$\boxed{EGL = \frac{P}{\rho g} + \frac{V^2}{2g} + z}$$

∴ difference can be used to measure V!

$$EGL - HGL = \frac{V^2}{2g}$$

$$\boxed{V = \sqrt{2g\,(EGL - HGL)}}$$

For a case with no pump & no turbine,

$$EGL_1 = EGL_2 + h_L \text{ along a streamline}$$

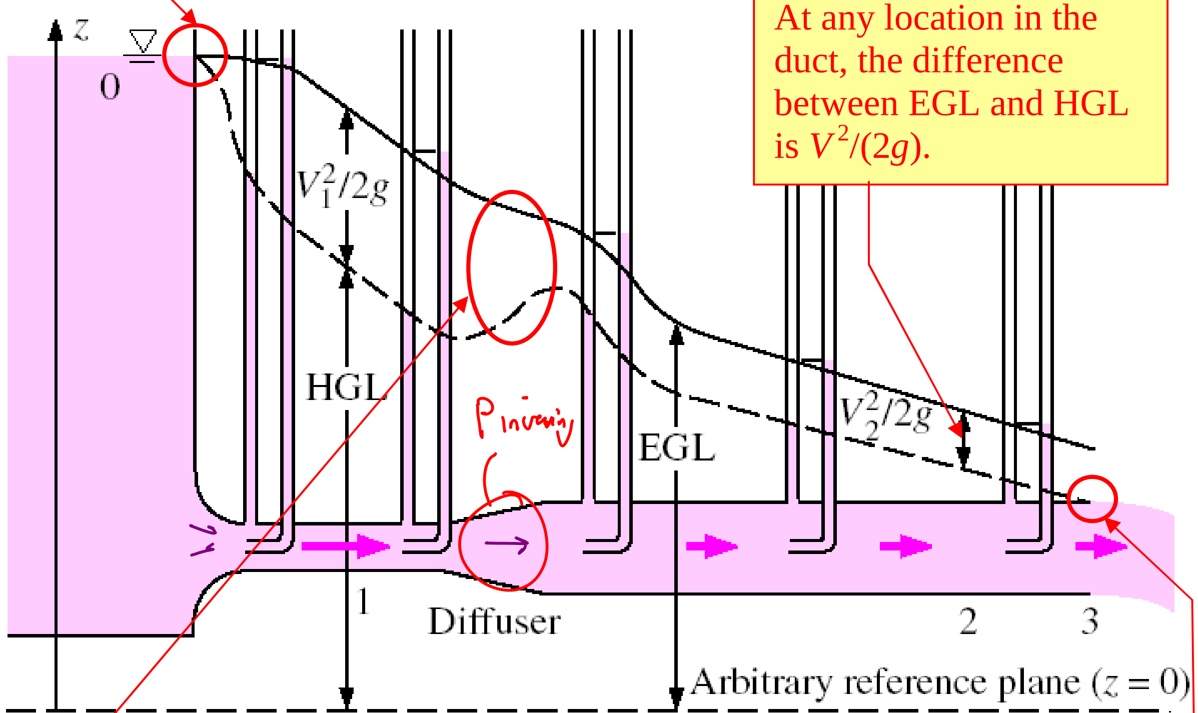
Example of Grade Lines in a Fluid Flow

At point 0, HGL = EGL inside the tank, since the fluid is at rest (V = 0). Neither EGL or HGL can rise above this value unless work is added to the flow (e.g., with a pump).

FIGURE 5-35

The *hydraulic grade line* (HGL) and the *energy grade line* (EGL) for free discharge from a reservoir through a horizontal pipe with a diffuser.

At any location in the duct, the difference between EGL and HGL is $V^2/(2g)$.



EGL continually falls due to irreversible losses, but HGL can rise or fall. Overall, however, HGL also must fall. In fact, HGL can *never* rise above EGL.

Since the jet exits at atmospheric pressure at the outlet of the pipe, $P_3 = P_{atm}$, and HGL is equal to the height of the free surface of the liquid.

D. The Bernoulli Equation

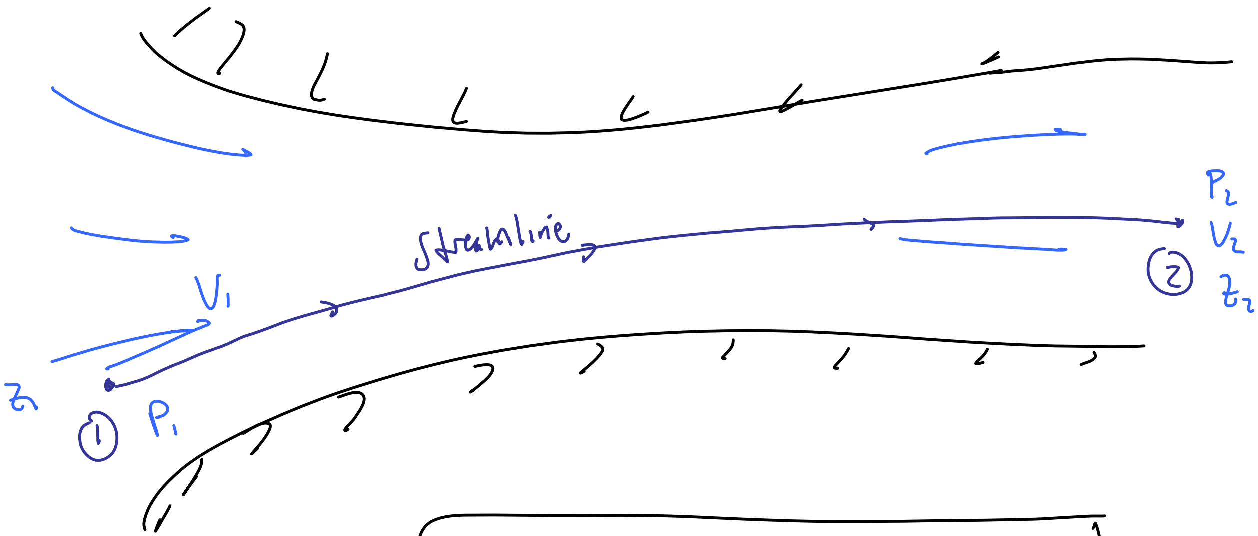
1. Derivation

Begin with the head form of the conservation of energy equation along a streamline:

$$\boxed{\left(\frac{P_1}{\rho_1 g}+\frac{V_1^2}{2g}+z_1\right)+\cancel{h_{\text{pump,u}}}=\left(\frac{P_2}{\rho_2 g}+\frac{V_2^2}{2g}+z_2\right)+\cancel{h_{\text{turbine, e}}}+\underbrace{\cancel{h_L}}_{\approx 0}}$$

Assume:
- No pump
- No turbine
- Negligible irreversible losses
- Incompressible $\longrightarrow (\rho_1 = \rho_2)$

We write Bernoulli along a streamline.



Bernoulli Eq.

$$\boxed{\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2}$$ along a streamline

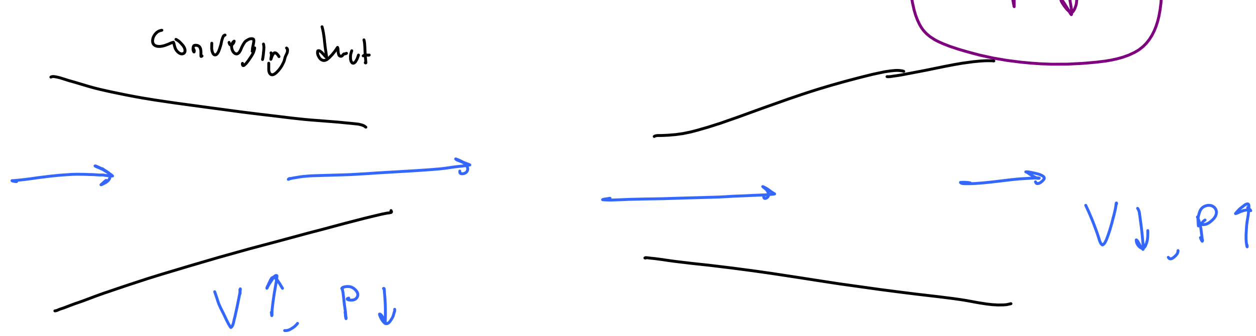
"The beloved Bernoulli eq."

It is a degenerate form of the energy eq.

For gases or cases in which Δz is insignificant

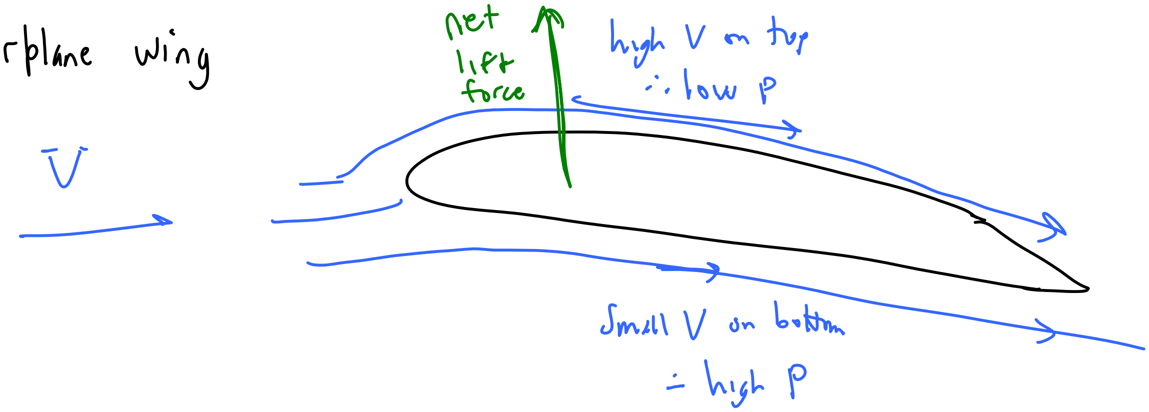
$$\boxed{P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2}$$

as $V \uparrow$
$P \downarrow$

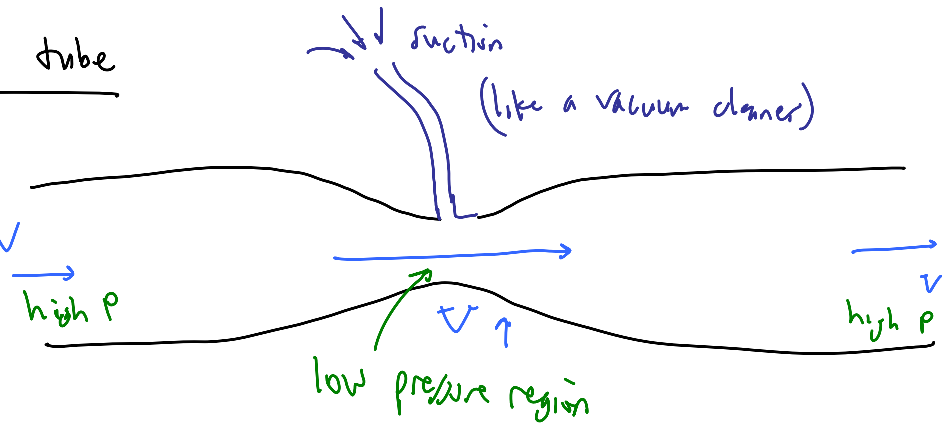


Applications of Bernoulli eq.

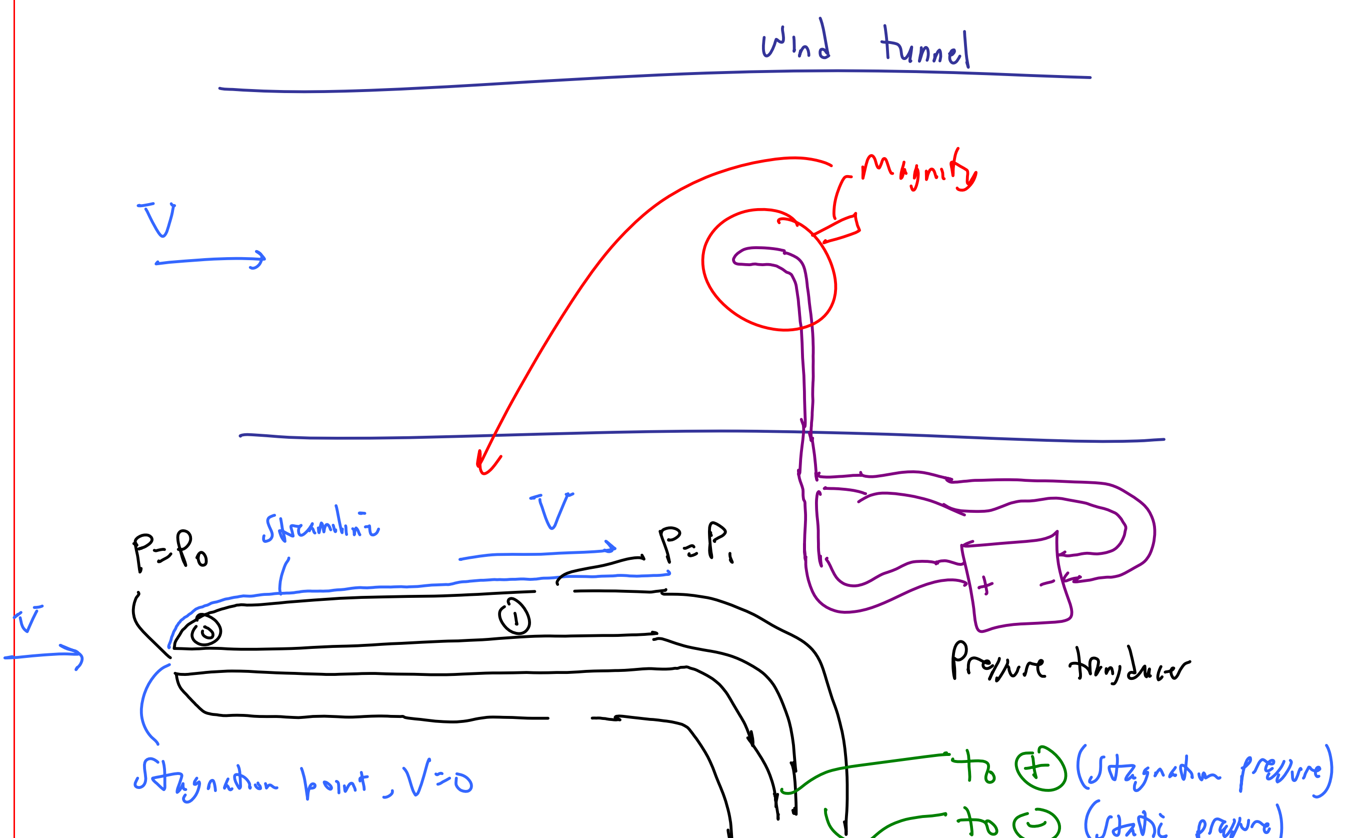
e.g Airplane wing



eg Venturi tube



eg Pitot-static probe



- Assume negligible friction losses
- No Δz

$\therefore$ Use Bernoulli eq.

Streamline from 0 to 1 $\rightarrow$ $P_0 + \frac{1}{2}\rho \cancel{V_0^2} = P_1 + \frac{1}{2}\rho V_1^2$

$V_0 = 0$ @ stagnation pt.

$V_1 = V$

Solve for $V = V_1$

$\rightarrow$ $\boxed{V = \sqrt{\frac{2\Delta P}{\rho}} \quad \text{where } \Delta P = P_0 - P_1}$

☆

Use this to measure velocity in a flow stream

measure this with the pressure transducer (or a manometer)

<u>Final Comment:</u> There are many useful applications of the Bernoulli equation, but be careful – it is not always appropriate to use it.