Today, we will:

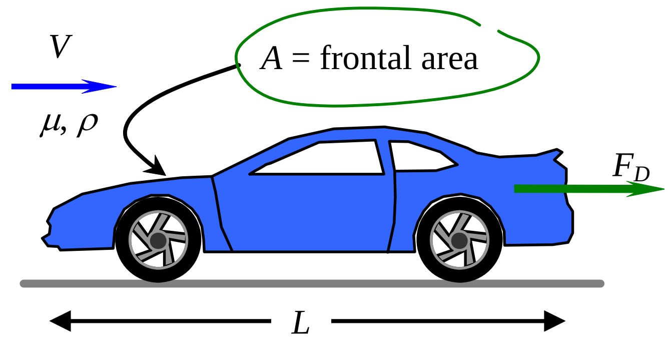
- Finish the example problem from last lecture.
- Do some more example problems – dimensional analysis
- Discuss experimental testing and incomplete similarity

Example: Dimensional analysis – Car drag

Given: The drag force F_D on a car is a function of four variables: air velocity V, air density ρ, air viscosity μ, and the length L of the car.

To do: Express this relationship in terms of nondimensional parameters.

Solution: We follow the six steps for the method of repeating variables.



See previous lecture. We were in the middle of step 5, and had

$$\Pi_1 = \text{ dependent Pi} = F_D V^a L^b \rho^c = F_D V^{-2} L^{-2} \rho^{-1} = \boxed{\frac{F_D}{\rho V^2 L^2}}$$

Modify or manipulate this Π if necessary to match one of the named established Π's (see Table 7-5)

Modified $\boxed{\Pi_1 = C_D = \text{drag coefficient} = \dfrac{F_D}{\frac{1}{2}\rho V^2 A}}$

$$\Pi_2 = \mu V^a L^b \rho^c$$

$$\{\Pi_2\} = \{1\} = \{m^0 L^0 t^0\} = \{\mu V^a L^b \rho^c\}$$

$$\left\{\left(\frac{m}{Lt}\right)\left(\frac{L}{t}\right)^a (L)^b \left(\frac{m}{L^3}\right)^c\right\}$$

$$\{m^0 L^0 t^0\} = \{m^1 L^{-1} t^{-1} L^a t^{-a} L^b m^c L^{-3c}\}$$

equate coefficients: m: $m^0 = m^1 m^c \rightarrow 0 = 1 + c \rightarrow \boxed{c = -1}$

t: $t^0 = t^{-1} t^{-a} \rightarrow 0 = -1 - a \rightarrow \boxed{a = -1}$

L: $L^0 = L^{-1} L^a L^b L^{-3c} \rightarrow 0 = -1 + a + b - 3c \rightarrow \boxed{b = -1}$

$$\boxed{\Pi_2 = \frac{\mu}{\rho V L}}$$

Guidelines for Manipulating the Π Parameters

There are several guidelines for manipulating the Π parameters. These guidelines are listed concisely in Table 7-4 in the text, as summarized below: See Table 7-4 for more details.

1. We may impose a constant (dimensionless) exponent on a Π or perform a functional operation on a Π.
2. We may multiply a Π by a pure (dimensionless) constant.
3. We may form a product (or quotient) of any Π with any other Π in the problem to replace one of the Π's.
4. We may use any of guidelines 1 to 3 in combination.
5. We may substitute a dimensional parameter in the Π with other parameter(s) of the same dimensions.

Here we sub. $\underset{\{L^2\}}{A}$ for $\underset{\{L^2\}}{L^2}$

The goal is to get each Π into a form that looks like one of the common *established* nondimensional parameters that are listed in Table 7-5 in the text. Some of the most popular and often-used ones are listed below. A more exhaustive list is given in the text.

TABLE 7–5

Some common established nondimensional parameters or Π's encountered in fluid mechanics and heat transfer*

Name	Definition	Ratio of Significance
Darcy friction factor	$f = \frac{8\tau_w}{\rho V^2}$ nondim constant	$\frac{\text{Wall friction force}}{\text{Inertial force}}$
Drag coefficient	$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$ our $\Pi_1 = \frac{F_D}{\rho V^2 L^2}$	$\frac{\text{Drag force}}{\text{Dynamic force}}$
Froude number	$\mathrm{Fr} = \frac{V}{\sqrt{gL}}$ $\left(\text{sometimes } \frac{V^2}{gL}\right)$	$\frac{\text{Inertial force}}{\text{Gravitational force}}$
Lift coefficient	$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A}$	$\frac{\text{Lift force}}{\text{Dynamic force}}$
Mach number	$\mathrm{Ma}\ (\text{sometimes } M) = \frac{V}{c}$	$\frac{\text{Flow speed}}{\text{Speed of sound}}$
Reynolds number	$\mathrm{Re} = \frac{\rho VL}{\mu} = \frac{VL}{\nu}$	$\frac{\text{Inertial force}}{\text{Viscous force}}$
Strouhal number	$\mathrm{St}\ (\text{sometimes S or Sr}) = \frac{fL}{V}$	$\frac{\text{Characteristic flow time}}{\text{Period of oscillation}}$

Reynolds number is the most important nondimensional parameter in fluid mechanics.

Manipulate → $\boxed{\Pi_2 = Re = \frac{\rho V L}{\mu}}$ = Reynolds #

I took $(\Pi_2)^{-1}$ to get Re

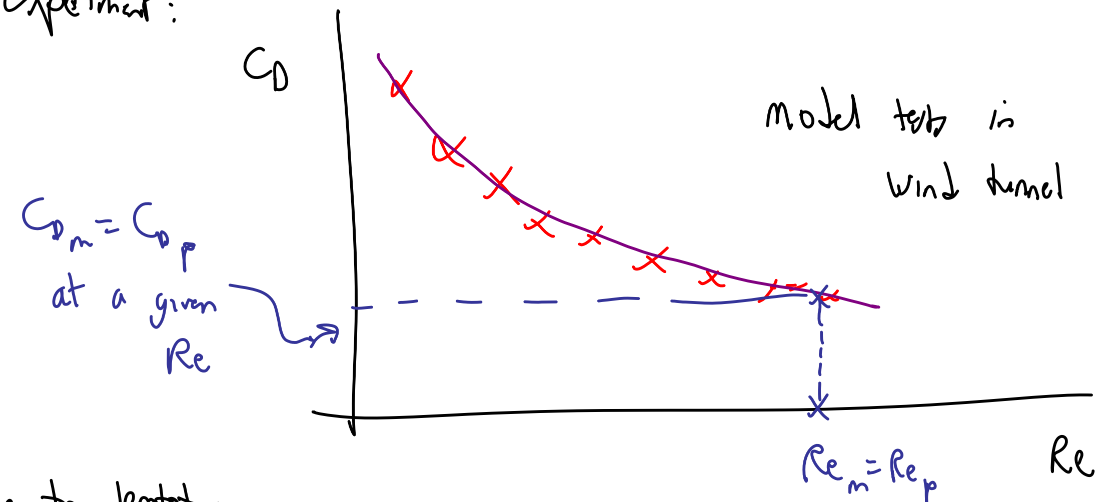
Step 6 → write the functional relationship: $\Pi_1 = fnc(\Pi_2, \Pi_3, \ldots)$

here $\Pi_1 = fnc(\Pi_2)$ or $\boxed{C_D = fnc(Re)}$ ☆

Originally $\boxed{F_D = fnc(V, L, \rho, \mu)}$ 5 parameters, 4 indep. variables

Now we have ☆ $\boxed{C_D = fnc(Re)}$ 2 parameters, 1 indep. variable

experiment:



Scale up to prototype

Given: Prototype car $V_p = 50$ mph, $L_p = 15$ ft, Predict F_{D_p}

air has ρ, μ

Model car $\frac{1}{5}$ scale → $L_m = 3$ ft same air properties

To do: (a) How fast to run the wind tunnel to achieve dynamic similarity? (V_m)

Soln: Need to have $\Pi_{2_m} = \Pi_{2_p} \longrightarrow Re_m = Re_p$

$$Re_m = \frac{\rho_m L_m V_m}{\mu_m} = Re_p = \frac{\rho_p L_p V_p}{\mu_p}$$

Solve $V_m = \underbrace{V_p}_{50\ \text{mph}} \underbrace{\frac{\cancel{\rho_p}}{\cancel{\rho_m}}}_{1} \underbrace{\frac{L_p}{L_m}}_{5} \underbrace{\frac{\cancel{\mu_m}}{\cancel{\mu_p}}}_{1} = \boxed{250\ \text{mph} = V_m}$

Since $\Pi_{2_m} = \Pi_{2_p}$, then $\Pi_{1_m} = \Pi_{1_p}$ (dynamic similarity)

Here, since $Re_m = Re_p$, then $C_{D_m} = C_{D_p}$

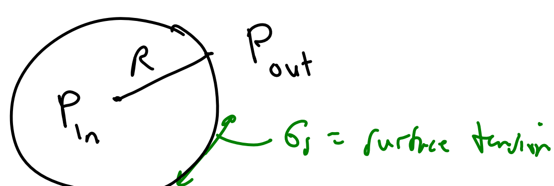
$$\frac{F_{D_p}}{\frac{1}{2}\rho_p V_p^2 A_p} = \frac{F_{D_m}}{\frac{1}{2}\rho_m V_m^2 A_m}$$

we measure this → Suppose it is 25.0 lbf

Solve $F_{D_p} = F_{D_m}\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{V_p}{V_m}\right)^2\left(\frac{A_p}{A_m}\right)$

$$(25.0\ \text{lbf})(1)\left(\frac{50\ \text{mph}}{250\ \text{mph}}\right)^2\left(5^2\right) = \boxed{25.0\ \text{lbf} = F_{D_p}}$$

Example Soap Bubble



Given:

$\Delta P = P_{in} - P_{out} = \text{fnc}(\sigma_s, R)$

To do:

Find the functional relationship between ΔP & σ_s & R

Step 1 $\Delta P = \text{fnc}(\sigma_s, R)$ $\quad n=3$

Step 2 $\left\{\frac{m}{t^2 L}\right\}$ $\left\{\frac{m}{t^2}\right\}$ $\{L\}$

Step 3 find the reduction j Try $j=3$ since I have m, L, t

$\therefore$ We expect $k = n-j = 3-3 = 0$ π's ??? X impossible!

Let's try instead $j = j-1 \rightarrow j=2$, $k = n-j = 3-2 = 1\ \pi$

Step 4 Select j repeating variables $\rightarrow$ pick σ_s & R

Step 5 Solve for the π's $\boxed{\pi_1 = \Delta P\, \sigma_s^{a} R^{b}}$

$$\{m^0 L^0 t^0\} = \left\{\frac{m}{t^2 L}\left(\frac{m}{t^2}\right)^a (L)^b\right\}$$

m: $0 = 1 + a \rightarrow a = -1$

L: $0 = -1 + b \rightarrow b = 1$ agree

t: $0 = -2 - 2a \rightarrow a = -1$

$$\pi_1 = \Delta P\, \sigma_s^{-1} R^{1} \rightarrow \boxed{\pi_1 = \frac{\Delta P\, R}{\sigma_s}}$$

$\rightarrow$ Here, no "manipulation" necessary

Step 6 fnc'l relationship $= \pi_1 = \text{fnc}(\text{nothing})!$ $\quad \pi_1 = \text{constant}$

$$\therefore \boxed{\Delta P = \frac{\text{const.}\ \sigma_s}{R}}$$

[Dim. anal. is not able to give us the value of the constant. Need an experiment or analysis]

$$\boxed{\Delta P = \frac{4\sigma_s}{R}}$$

recall the constant $= 4$

Example: Dimensional analysis – shaft power

Given: The output power $\dot{W}$ of a spinning shaft is a function of torque T and angular velocity ω. [NOTE: I DID THIS PROBLEM AFTER CLASS]

To do: Express the relationship between $\dot{W}$, T, and ω in dimensionless form.

Solution:

Note: radian is dimensionless $\{\omega\} = \left\{\frac{\text{rad}}{s}\right\} = \left\{\frac{1}{t}\right\}$

Step 1: List: $\dot{W} = \text{fnc}(T, \omega)$ $\boxed{n=3}$

Step 2: Dim's $\left\{\frac{mL^2}{t^3}\right\}$ $\left\{\frac{mL^2}{t^2}\right\}$ $\left\{\frac{1}{t}\right\}$

Step 3: pick $j=3$, expect $k = n-j = 3-3 = 0$ Π's ✗ impossible!

pick $\boxed{j=2}$ instead. expect $k = n-j = 3-2 = \underline{\underline{1\ \Pi}}$

Step 4: pick 2 repeating variables → pick T & ω

Calculate Π's:

$$\Pi_1 = \dot{W}\,T^a\,\omega^b$$

Step 5: $$\{\Pi_1\} = \{m^0 L^0 t^0\} = \left\{\left(\frac{mL^2}{t^3}\right)\left(\frac{mL^2}{t^2}\right)^a\left(\frac{1}{t}\right)^b\right\}$$

equate exponents:
m: $1 + a = 0 \rightarrow \underline{a = -1}$
L: $2 + 2a = 0 \rightarrow a = -1$ agree ☺
t: $-3 - 2a - b = 0 \rightarrow \underline{b = -1}$

So, $$\boxed{\Pi_1 = \frac{\dot{W}}{T\omega}}$$

Step 6: functional relationship → $\Pi_1 = \text{fnc}(\Pi_2, \Pi_3 \ldots \Pi_k)$

Here $\Pi_1 = \text{fnc}(\text{nothing}) = \text{constant}.$

$$\therefore \boxed{\dot{W} = \text{const}\ \omega T}$$

[The constant turns out to = 1, but we can't know that from Dim. Anal.]

Our answer is correct to within an unknown constant, without knowing any physics!