Today, we will:
- Finish Chapter 7 – Dimensional Analysis
- Begin Chapter 8 – Internal Flow (flow in pipes)

D. Experimental Testing & Incomplete Similarity

It is not always possible to match all the independent Π's between a model & a prototype. (Incomplete Similarity)

Example → Drag on a car in a wind tunnel

- We had $\boxed{C_D = \text{fnc}(Re)}$ → if $Re_m = Re_p$, then $C_{D_m} = C_{D_p}$

- Suppose we use a $\frac{1}{16}$th scale model
  - Assume same air T & P for model & prototype
  - Want to simulate the real car (prototype) @ 60 mph
  - $$Re_m = \frac{\cancel{\rho_m} V_m L_m}{\cancel{\mu_m}} = Re_p = \frac{\cancel{\rho_p} V_p L_p}{\cancel{\mu_p}}$$

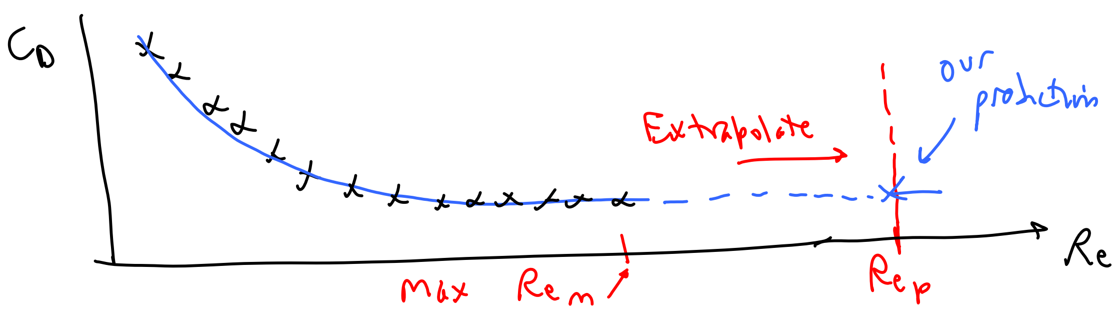
Solve for V_m = speed of wind tunnel $= V_p \frac{L_p}{L_m} = (60 \text{ mph})(16)$

$= \boxed{960 \text{ mph} = V_m}$

This is supersonic !! ⟶ ✗

We cannot match the Re in this problem.

Fortunately, Re-dependence often flattens out in these kinds of flows



- If we include c (speed of sound) in our dimensional analysis

We get a third Pi → $\boxed{Ma = \frac{V}{c}}$

$$\therefore \boxed{C_D = fnc(Re, Ma)}$$

- Eg Flow in rivers, boats, etc with a free surface

two important π's are typically Re & Fr

$$\boxed{Fr = \text{Froude \#} = \frac{V}{\sqrt{gL}}} \qquad \boxed{Re = \frac{\rho V L}{\mu}}$$

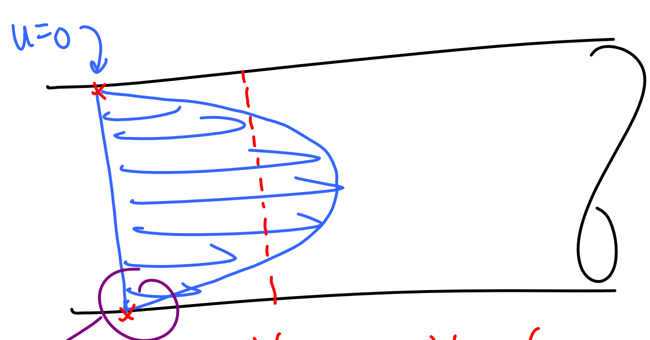
It is often hard or impossible to match both Fr & Re in a model test.

VI. INTERNAL FLOWS (FLOW IN PIPES) (Ch. 8)

A. Intro

1. Average velocity in a pipe

Actual flow in a pipe has the no-slip condition @ pipe wall



$\dot{m} = \rho V A$

use V_{avg} here

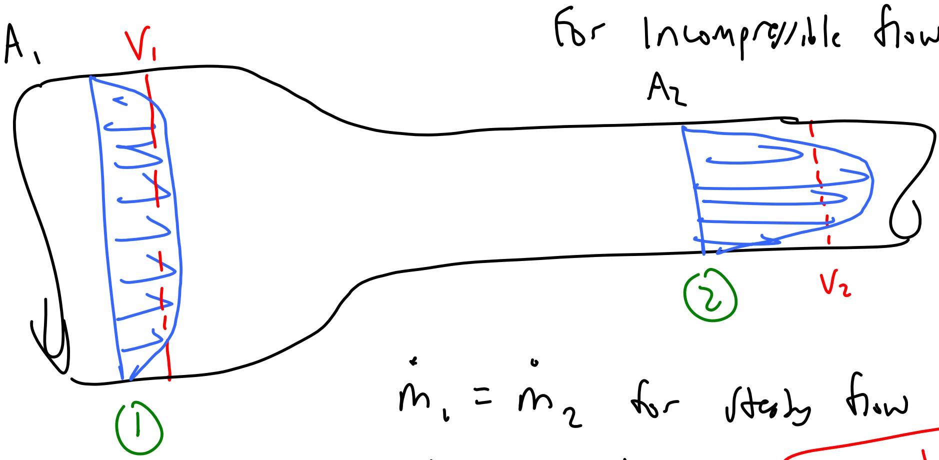
$V_{avg} \rightarrow V$ (drop subscript)

Shear @ wall

leads to friction along the pipe wall

For pipe flows with varying diameter,

For Incompressible flow



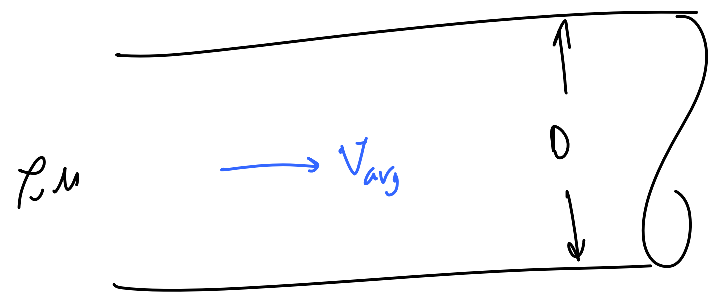
$\dot{m}_1 = \dot{m}_2$ for steady flow

$\underbrace{\cancel{\rho} V_1 A_1 = \cancel{\rho} V_2 A_2} \rightarrow \boxed{V_2 = V_1 \dfrac{A_1}{A_2}}$ $V_2 \uparrow$

2. Laminar vs Turbulent

→ (see typed-up comparison on pg. 6) →

Critical Reynolds # for pipe flow



$$\boxed{Re = \frac{\rho V_{avg} D}{\mu}}$$

or $Re = \dfrac{V_{avg} D}{\nu}$

[ν = kinematic viscosity = μ / ρ]

For a round pipe flow $\boxed{Re_{critical} \simeq 2300}$ ☆

actual value of $Re_{critical}$ depends on:
- roughness
- vibrations
- upstream disturbances
- etc

☆
- if $Re \lesssim 2300$ → flow is laminar
- if $2300 \lesssim Re \lesssim 4000$ → transitional (in between)
- if $Re \gtrsim 4000$ → flow is turbulent

c. <u>Hydraulic Diameter</u> For non-round pipes, replace D with D_h

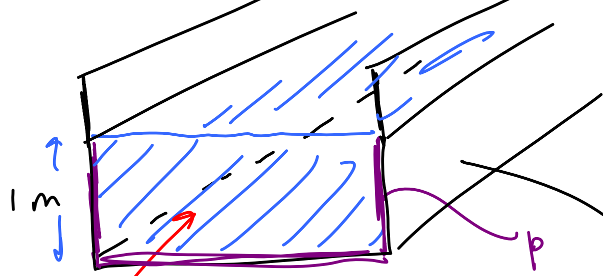
D_h = hydraulic dia. = "equivalent" dia of a round pipe

$$\boxed{D_h = \frac{4A_c}{p}}$$

where A_c = cross-sectional area

p = wetted perimeter (portion of the duct in contact with the fluid.

e.g. <u>open channel</u>



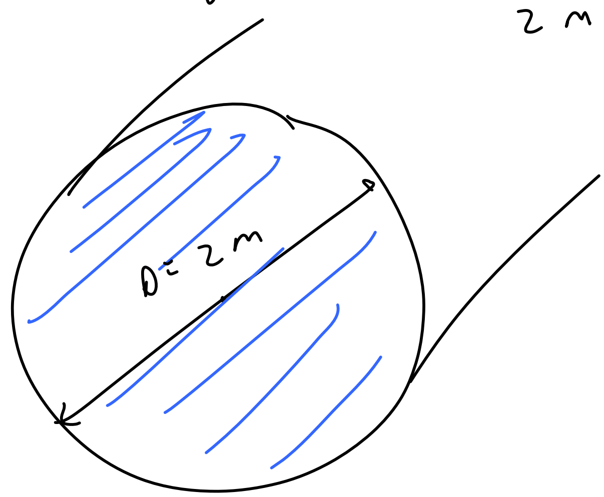
$p = 1 + 2 + 1 \text{ m}$
$= 4 \text{ m}$

$A_c = 1 \text{ m} \times 2 \text{ m} = 2 \text{ m}^2$

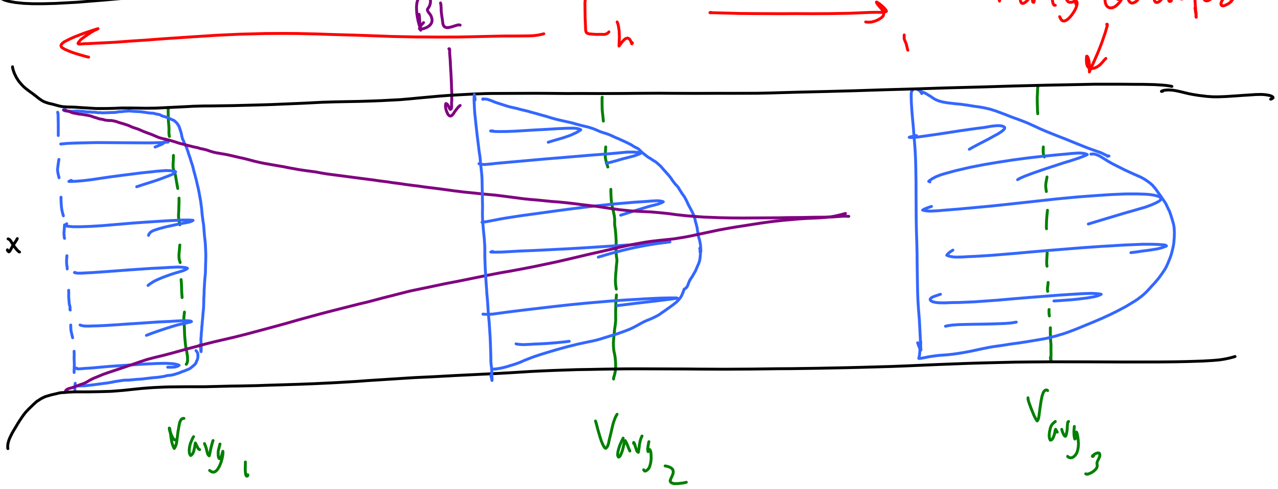
$$D_h = \frac{4A_c}{p} = \frac{4(2 \text{ m}^2)}{4 \text{ m}} = 2 \text{ m}$$

$$\boxed{D_h = 2 \text{ m}}$$

∴ This channel is "equivalent" to a pipe of 2 m diameter



d. Entrance region



$V_{avg_1} = V_{avg_2} = V_{avg_3}$ for incompressible flow

L_h = entrance length

$$L_h = fnc(\rho, \mu, D, V_{avg})$$

Dim anal. → $\boxed{\frac{L_h}{D} = fnc(Re)}$

Laminar	$\frac{L_h}{D} = 0.05\,Re$
Turbulent	$\frac{L_h}{D} \approx 10$

☆

B. Fully Developed Pipe Flow

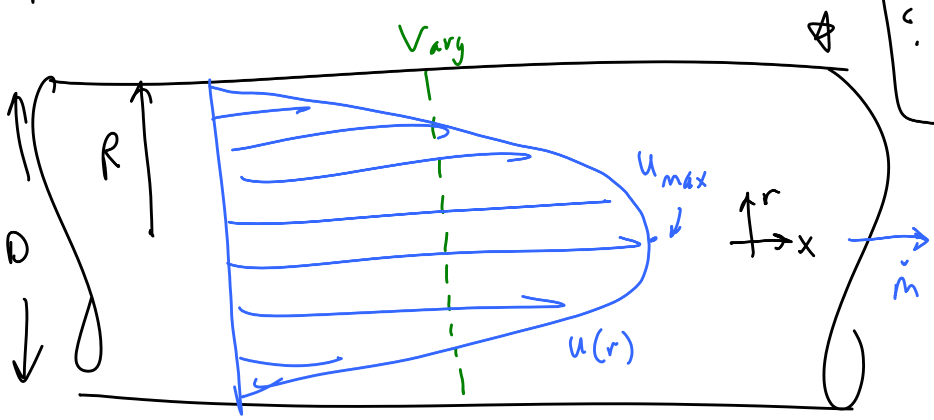
1. Comparison between laminar & turbulent

$$\boxed{u(r) = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right)}$$

$\boxed{\text{\&}\ V_{avg} = \frac{1}{2} u_{max}}$ ☆

exact

LAMINAR



Laminar Versus Turbulent Flow – A Comparison
(Section 8-2, Çengel and Cimbala)

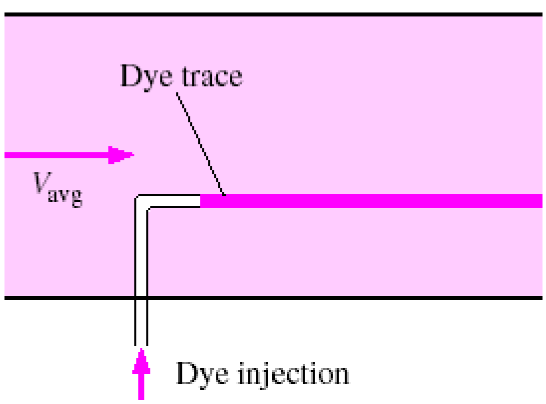
Laminar Flow

Can be steady or unsteady.

(Steady means that the flow field at any instant in time is the same as at any other instant in time.)

Can be one-, two-, or three-dimensional.

Has regular, *predictable* behavior



Analytical solutions are possible (see Chapter 9).

Occurs at *low* Reynolds numbers.

Turbulent Flow

Is always *unsteady*.

Why? There are always random, swirling motions (vortices or eddies) in a turbulent flow.

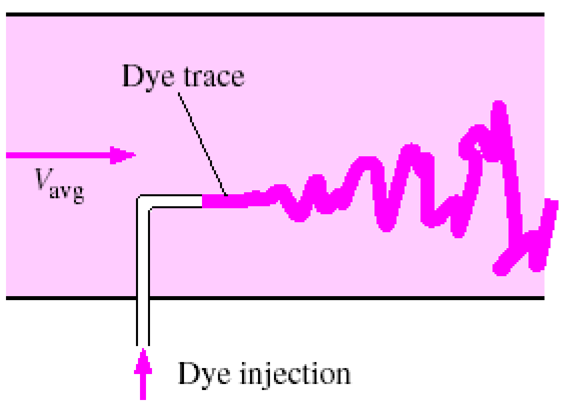
Note: However, a turbulent flow can be steady *in the mean*. We call this a *stationary turbulent flow*.

Is always *three-dimensional*.

Why? Again because of the random swirling eddies, which are in all directions.

Note: However, a turbulent flow can be 1-D or 2-D *in the mean*.

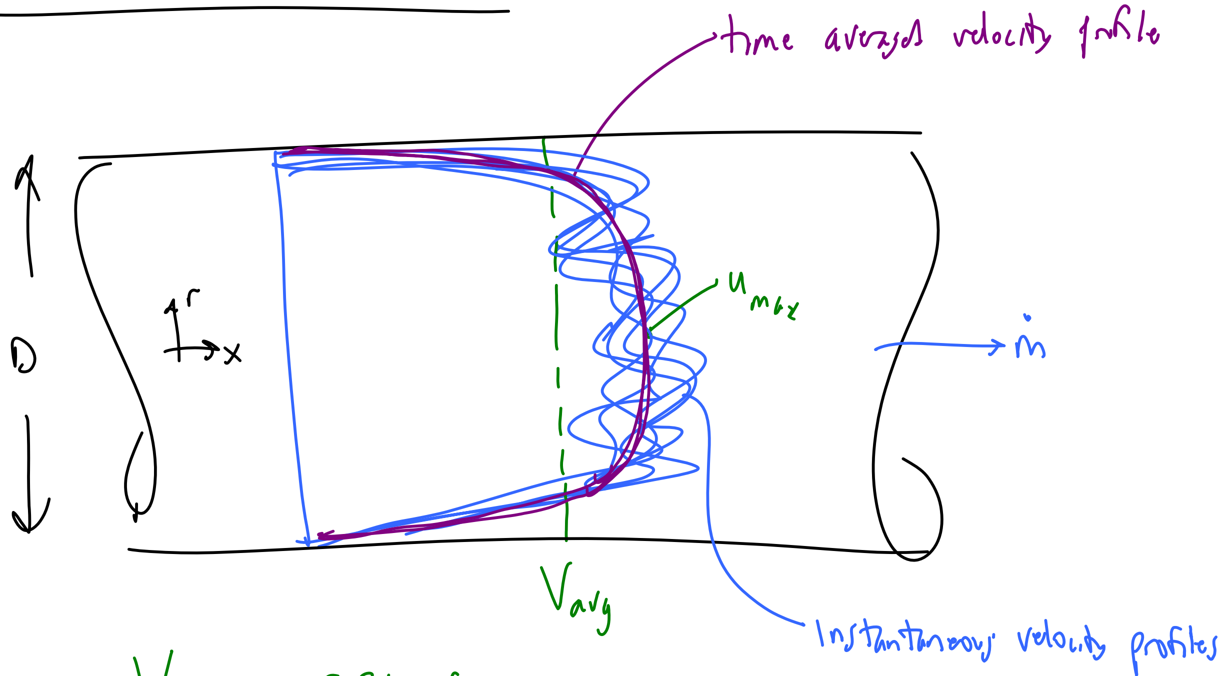
Has irregular or *chaotic* behavior (cannot predict exactly – there is some randomness associated with any turbulent flow.



No analytical solutions exist! (It is too complicated, again because of the 3-D, unsteady, chaotic swirling eddies.)

Occurs at *high* Reynolds numbers.

Turbulent fully developed pipe flow



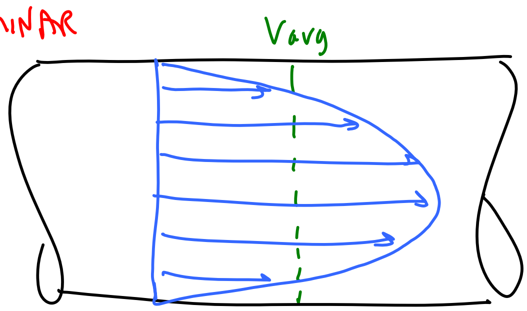
- $V_{avg} \approx 85\%$ of u_{max}
- No exact equation → see text for semi-empirical equations for the profile shape

  (log law, $1/7^{th}$ power law, etc)

- The turbulent profile is "fuller" compared to the laminar profile.
- In other words it is closer to a "plug flow" or uniform flow.

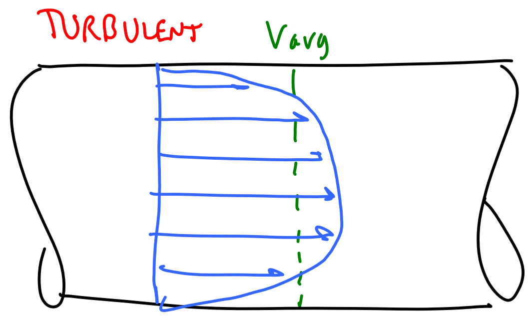
  For the same V_{avg}, [That is why α is much larger for laminar flow than for turbulent flow]

LAMINAR



$\alpha = 2$

TURBULENT



$\alpha \approx 1.05$