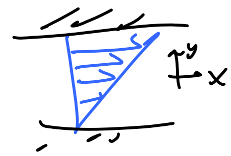
Today, we will:

- Continue Chapter 8 – flow in pipes
- Discuss the Darcy friction factor, the Moody Chart, and the Colebrook Equation
- Do some example problems – head losses in pipe flows

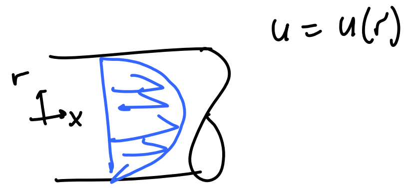
2. Wall shear stress

· Recall, for a simple shear flow $u = u(y)$.



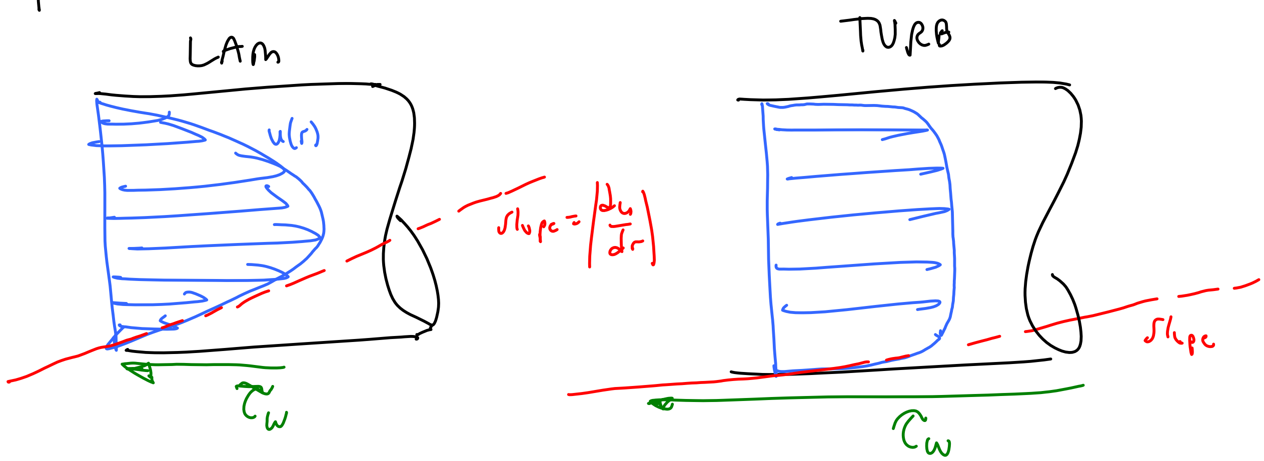
$$\tau = \text{shear stress} = \mu \frac{du}{dy}$$

· Here in fully developed pipe flow



$$\boxed{\tau = \mu \left| \frac{du}{dr} \right|}$$

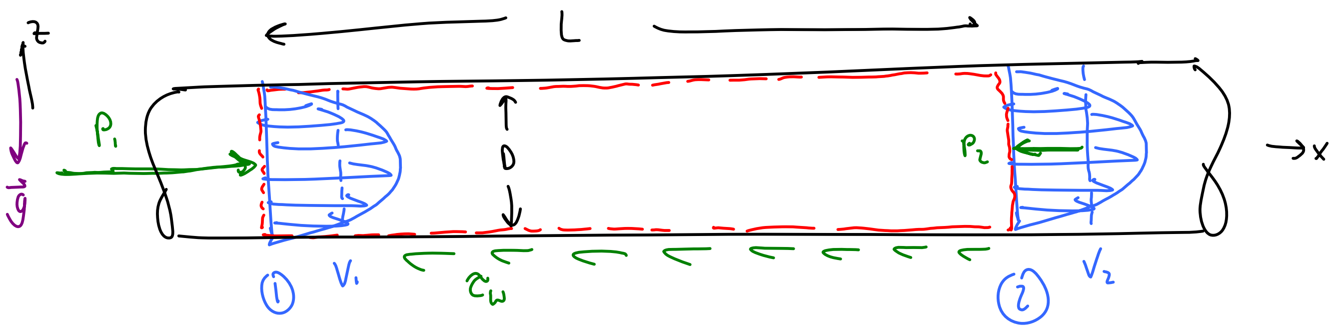
Compare laminar & turbulent



$\tau_{w_{turb}} > \tau_{w_{lam}}$ for the same flow rate, etc.

3. Pressure drop in fully developed pipe flow

Approx:
- Steady (in the mean)
- incompressible
- horizontal pipe
- fully developed



Cons. of mass. $\dot{m}_1 = \dot{m}_2 = \dot{m}$ $\underline{\dot{m} = \text{const}}$

$$\rho \dot{\forall}_1 = \rho \dot{\forall}_2 = \rho \dot{\forall} = \text{const.} \qquad \underline{\dot{\forall} = \text{const}}$$

$$\rho V_1 \frac{\pi D^2}{4} = \rho V_2 \frac{\pi D^2}{4} \rightarrow \boxed{V_1 = V_2}$$

Cons. of x-mom:

$$\sum F_x = \cancel{\sum F_{x_{grav}}}^{0} + \underbrace{\sum F_{x_{visc}}}_{-\tau_w \cdot \pi D L} + \sum F_{x_{pressure}} + \cancel{\sum F_{x_{other}}}^{0}$$

$$P_1 \frac{\pi D^2}{4} - P_2 \frac{\pi D^2}{4}$$

$$= \underbrace{\sum_{out} \beta u \dot{m}}_{\cancel{\beta_2 \dot{m} V_2}} - \underbrace{\sum_{in} \beta u \dot{m}}_{\cancel{\beta_1 \dot{m} V_1}}$$

$\beta_1 = \beta_2$
$V_1 = V_2$
$\dot{m} = \text{const}$

cancel

X-mom eq becomes $\left(P_1 - P_2\right) \frac{\pi D^2}{4} = \tau_w \pi D L$

$$\boxed{P_1 - P_2 = 4 \tau_w \frac{L}{D}} \qquad (1)$$

• Energy (head form)

$\alpha_1 = \alpha_2$ (fully dev.)
$V_1 = V_2$

cancel

$$\frac{P_1}{\rho g} + \cancel{\alpha_1 \frac{V_1^2}{2g}} + \cancel{z_1} + \cancel{h_{pump,u}}^{0} = \frac{P_2}{\rho g} + \cancel{\alpha_2 \frac{V_2^2}{2g}} + \cancel{z_2} + \cancel{h_{turbine,e}}^{0} + h_L$$

$$\boxed{P_1 - P_2 = \rho g h_L} \qquad (2)$$

Combine (1) & (2) $\rightarrow$ $4\tau_w \frac{L}{D} = \rho g h_L$

or $\boxed{h_L = \frac{4\tau_w}{\rho g} \frac{L}{D}}$ (3)

4. The Darcy friction factor, f

Use Dimensional analysis to find τ_w

$$\boxed{\tau_w = \text{fnc}(\rho, V, \mu, D, \varepsilon)}$$

ε = avg. roughness height of the inner pipe wall

$\{\varepsilon\} = \{L\}$

pick ρ, V, D as repeating vars

Sol'n:

$$\Pi_1 = \boxed{f = \frac{8\tau_w}{\rho V^2}} = \text{Darcy friction factor}$$

$$\Pi_2 = \boxed{Re = \frac{\rho V D}{\mu}}$$

$$\Pi_3 = \boxed{\varepsilon/D = \text{roughness factor}}$$

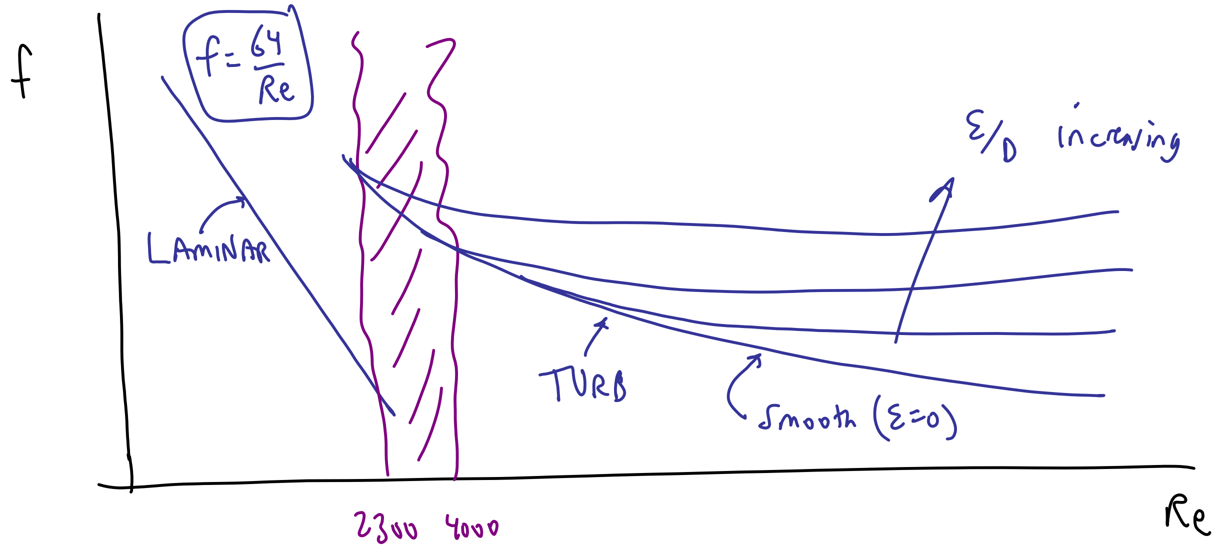
$$\boxed{f = \text{fnc}(Re, \varepsilon/D)}$$

Eq (3) was $h_L = \frac{4\tau_w}{\rho g} \frac{L}{D} \rightarrow \boxed{h_L = f \frac{L}{D} \frac{V^2}{2g}}$ ✰

Laminar flow - we can solve for f exactly $\rightarrow$ $\boxed{f = 64/Re}$ not dependent on ε/D
(ch. 9)

Turbulent flow - we use empirical data & curve fits & charts

5. The Moody Chart [f vs Re & ε/D]



$\boxed{\text{See Fig A-12}}$, also see pdf file

6. Empirical Eqs in place of the Moody chart

The Colebrook Eq. (Eq 8-50)

$$\boxed{\frac{1}{\sqrt{f}} = -2.0 \log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)} \quad \star$$

an implicit eq → must iterate to find f.

7. Examples

Given: $D = 5.00$ cm

$L = 100$ m of fully dev. pipe

$\dot{V} = 0.0100\ m^3/s$ of water

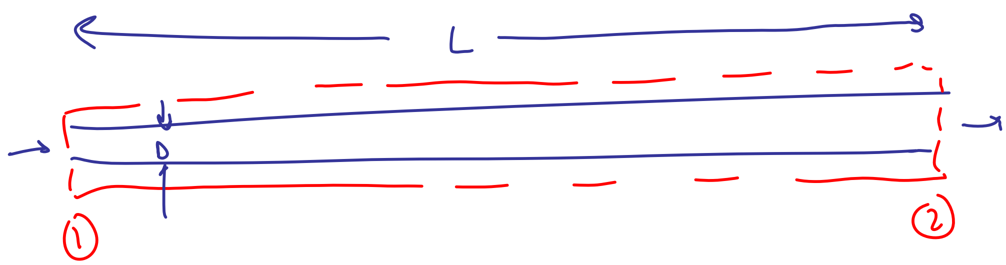
$T = 10.0°C \longrightarrow$ look Table A-3 $\quad \rho = 999.7\ kg/m^3$

$\mu = 1.307 \times 10^{-3}\ kg/m \cdot s$

To do: (a) Calc pressure drop in this pipe if $\varepsilon = 0.0050$ cm

(b) " " " " $\varepsilon = 0$ (smooth pipe)

Soln,



Energy eq. in head form

cancel

$$\frac{P_1}{\rho g} + \cancel{\alpha_1 \frac{V_1^2}{2g}} + \cancel{z_1} + \cancel{h_{pump,u}}^{0} = \frac{P_2}{\rho g} + \cancel{\alpha_2 \frac{V_2^2}{2g}} + \cancel{z_2} + \cancel{h_{turbine,e}}^{0} + h_L$$

cancel

$$\boxed{\Delta P = P_1 - P_2 = \rho g h_L} \qquad (1)$$

We also know $\boxed{h_L = f \frac{L}{D} \frac{V^2}{2g}}$ (2)

$$V = \frac{\dot{V}}{A} = \frac{4\dot{V}}{\pi D^2} = \frac{4(0.0100\ m^3/s)}{\pi (0.050\ m)^2} \Bigg\} \quad \underline{V = 5.092958\ m/s}$$

$$Re = \frac{\rho V D}{\mu} = \frac{(999.7 \text{ kg/m}^3)(5.092958 \text{ m/s})(0.05 \text{ m})}{1.307 \times 10^{-3} \text{ kg/m·s}} = 1.9477 \times 10^5$$

$(Re >> 2300) \longrightarrow$ Definitely turbulent

$\varepsilon / D = 0.001$

$f \rightarrow$ Moody chart @ $Re, \varepsilon/D \rightarrow f = 0.0210$

Excel gives $f = 0.02107$

$$\Delta P = \rho \frac{V^2}{2} f \frac{L}{D}$$

Plug in #'s

$$\boxed{\Delta P = 546. \text{ kPa}}$$

Comment: We typically give answers to 3 sig. digits, but keep in mind that the Moody chart & Colebrook eq. are accurate to only about ±10% → [they claim ±15% over the entire chart]

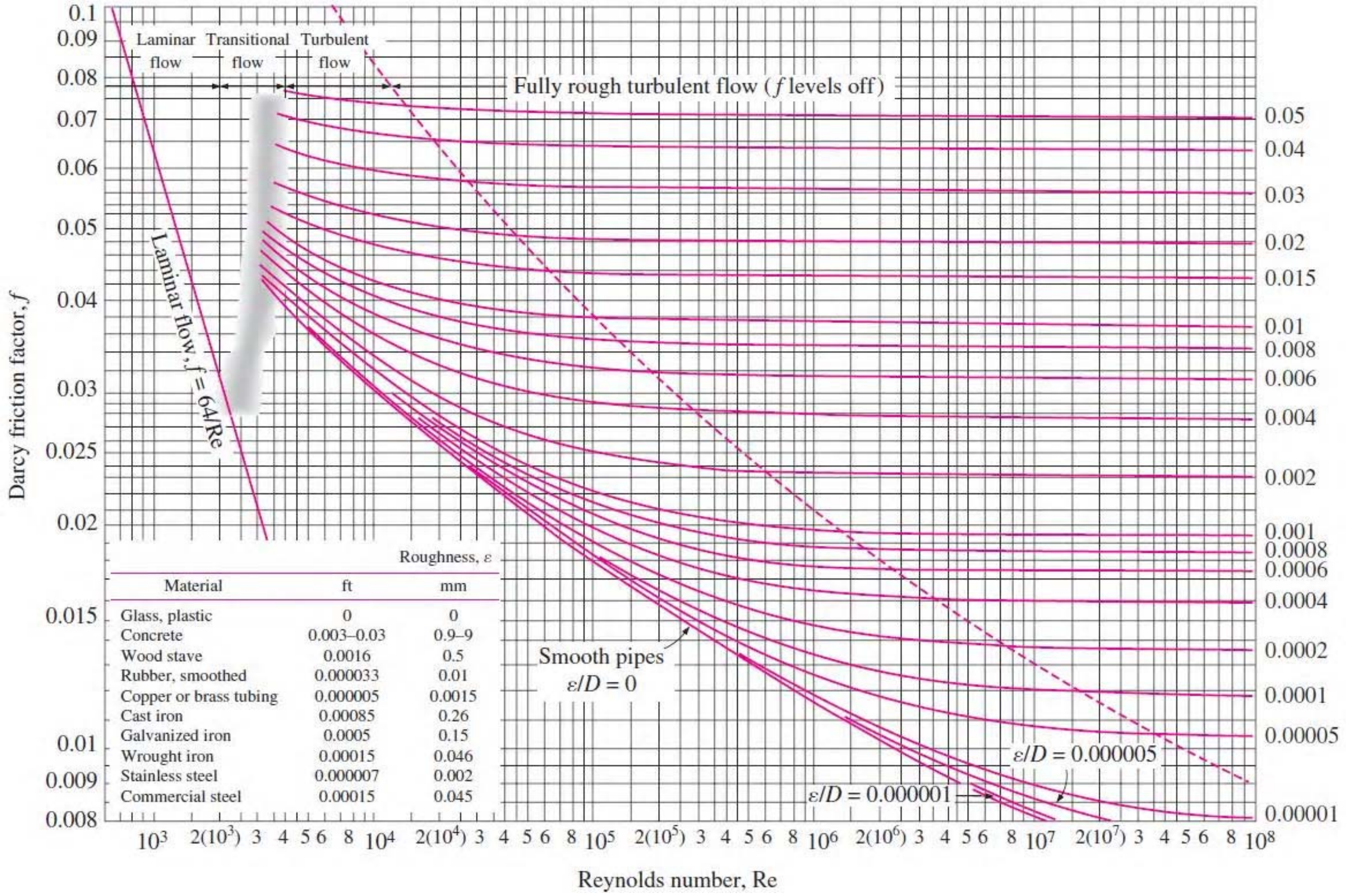
(b) Repeat for $\varepsilon = 0$ (smooth)

Do it on your own for practice } get $\boxed{\Delta P = 408. \text{ kPa}}$

Significantly lower!

A little roughness makes a big difference!

The Moody Chart (From Appendix, Figure A-12, in the textbook)



Material	ft	mm
Glass, plastic	0	0
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

Fully Developed Pipe Flow Equations

There are empirical equations available to use in place of the Moody chart. The most useful one (in fact, the equation with which the turbulent portion of the Moody chart is drawn) is:

The Colebrook equation

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \qquad \text{(turbulent flow)} \qquad \mathbf{(8\text{–}50)}$$

Note: This is $\log_{10}$, not the natural log, ln.

Unfortunately, the Colebrook equation is ***implicit*** in *f* (since *f* appears on both sides of the equation), and the equation must be solved by iteration. An approximation to the Colebrook equation was created by Haaland, accurate to ±2% compared to the Colebrook equation:

The Haaland equation

$$\frac{1}{\sqrt{f}} \cong -1.8\log\left[\frac{6.9}{\mathrm{Re}}+\left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right] \qquad \mathbf{(8\text{–}51)}$$

Also $\log_{10}$, not ln.

Finally, Swamee and Jain generated some approximations that can be used directly in place of the Colebrook equation when solving problems of certain types. These are also accurate to ±2% compared to iterative solutions using the Colebrook equation: