Today, we will:

- Review how to match a pump and a piping system.
- Do some example problems – matching pumps to systems
- Begin to discuss dimensionless parameters in pump performance

From previous lecture…the head form of the conservation of energy equation:

Available or Supply curve

$$\frac{P_1}{\rho_1 g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho_2 g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_L$$

Required or System or Demand curve

Solve for H = net pump head delivered = $h_{\text{pump,u}}$ = useful pump head:

$$\boxed{h_{\text{pump,u}}=\underbrace{\frac{P_2}{\rho_2 g}-\frac{P_1}{\rho_1 g}}_{\text{I}}+\underbrace{\frac{\alpha_2 V_2^2-\alpha_1 V_1^2}{2g}}_{\text{II}}+\underbrace{z_2-z_1}_{\text{III}}+\underbrace{h_L}_{\text{IV}}}\qquad(1)$$

I (+), (−), or 0   II (+), (−), 0   III (+), (−), 0   IV: ALWAYS > 0 (+)

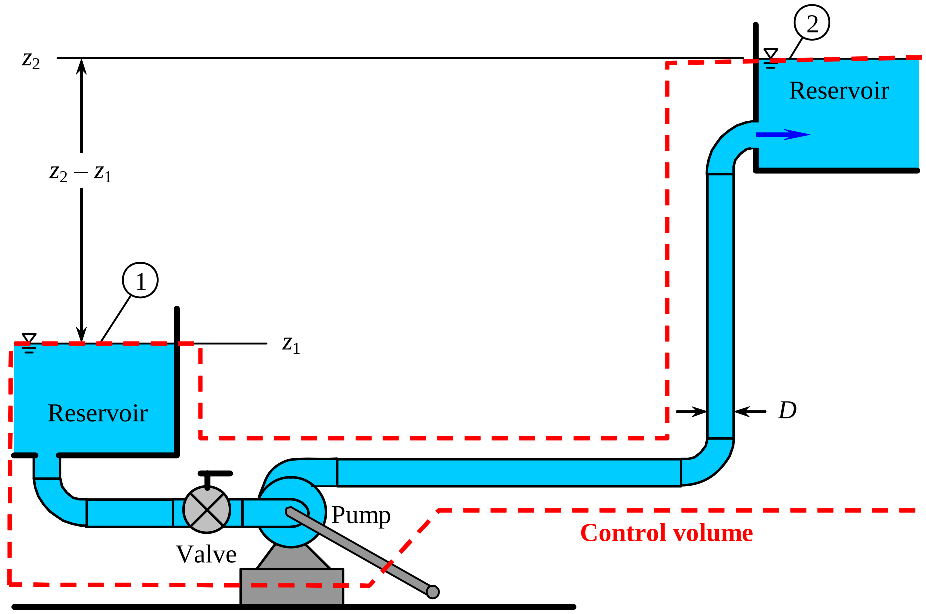
Consider a typical piping system with a pump that pumps water from a lower reservoir to a higher reservoir. In general, from Eq. (1), we see that the pump must do four things:

I Change the pressure in the flow from inlet to outlet

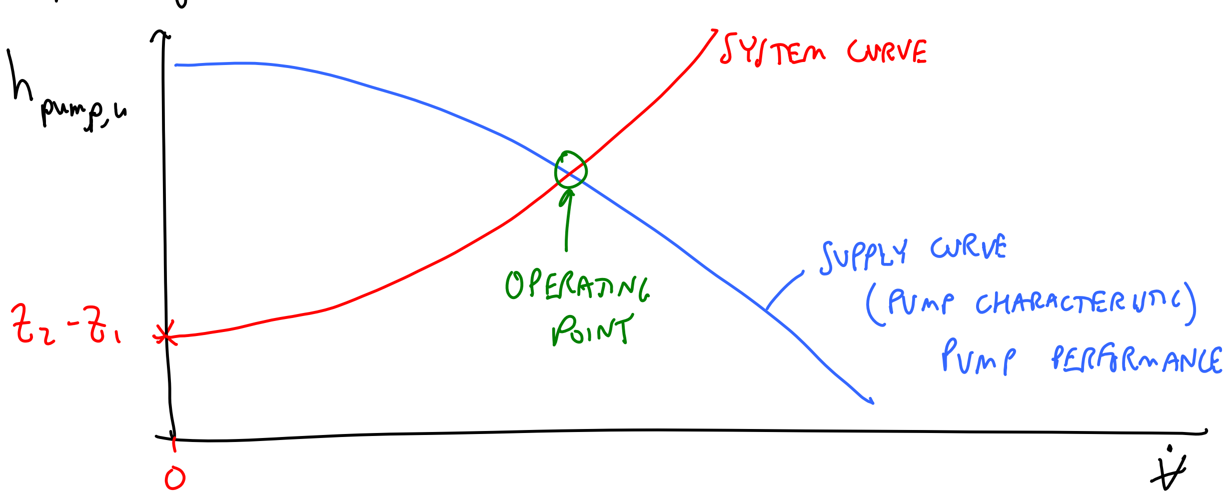
II Change the kinetic energy in the flow from inlet to outlet

III Change the elevation in the flow from inlet to outlet

IV Overcome irreversible head losses



When we plot eq. (1), we get the system curve for the piping system

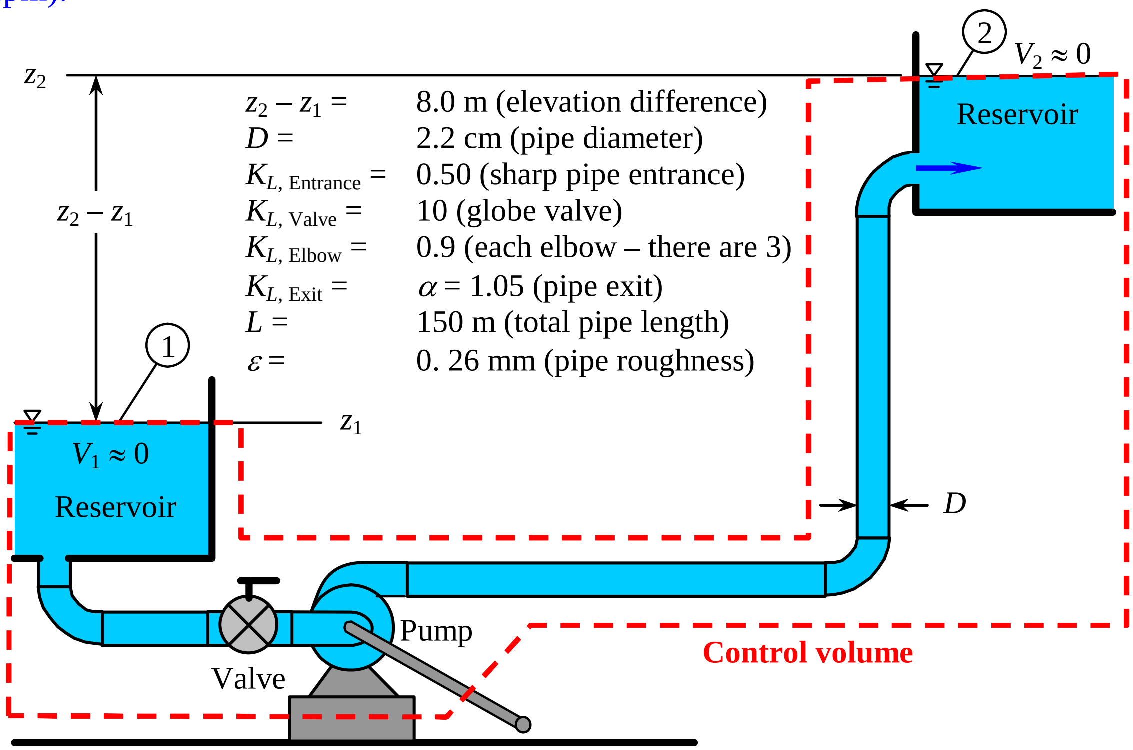


Example Problem – Matching a Pump to a Piping System

Given: Water (ρ = 1000 kg/m^3, μ = 1.00 × 10^{-3} kg/m·s) is pumped from one large reservoir to another large reservoir that is at a higher elevation. The free surfaces of both reservoirs are exposed to atmospheric pressure, as sketched. The dimensions and minor loss coefficients are provided in the figure. The pipe is 2.2 cm I.D. cast iron pipe. The total pipe length is 150.0 m. The entrance and exit are sharp. There are three regular threaded 90-degree elbows, and one fully open threaded globe valve. The pump's performance (supply curve) is approximated by the expression

$$H_{\text{available}} = h_{\text{pump, u supply}} = H_0 - a\dot{V}^2$$

where shutoff head H_0 = 20.0 m of water column, coefficient a = 0.072 m/Lpm^2, available pump head $H_{\text{available}}$ is in units of meters of water column, and volume flow rate $\dot{V}$ is in units of liters per minute (Lpm).



To do: Calculate the volume flow rate through this piping system.

Solution:

- First we draw a control volume, as shown by the dashed line. We cut through the pump shaft and through the surface of both reservoirs (inlet 1 and outlet 2), where we know that the velocity is nearly zero and the pressure is atmospheric. The rest of the control volume simply surrounds the piping system.
- We apply the head form of the energy equation from the inlet (1) to the outlet (2):

$P_1 = P_2 = P_{\text{atm}}$

$V_1 = V_2 \approx 0$

$$\cancel{\frac{P_1}{\rho g}} + \alpha_1 \cancel{\frac{V_1^2}{2g}} + z_1 + h_{\text{pump,u}} = \cancel{\frac{P_2}{\rho g}} + \alpha_2 \cancel{\frac{V_2^2}{2g}} + z_2 + \cancel{h_{\text{turbine,e}}} + h_L$$

cancel

We will call this $h_{\text{pump, u, system}}$ since it is the required pump head for the given piping system.

The rest of this problem will be solved in class.

$$h_{pump,u,sys} = h_L + (z_2 - z_1)$$

$$= \frac{V^2}{2g}\left(f\frac{L}{D} + \sum K_L\right) + (z_2 - z_1)$$

$\sum K_L = 0.5 + 10.0 + 3(0.9) + 1.05$

inlet valve 3 elbows exit w/ $\alpha = 1.05$

$\sum K_L = 14.25$

Also, $\dot{V} = V\frac{\pi D^2}{4}$ → Solve for V & plug in to get Re

we get

$$\boxed{h_{pump,u,sys} = \frac{8\dot{V}^2}{\pi^2 g D^4}\left(f\frac{L}{D} + \sum K_L\right) + (z_2 - z_1)}$$

Eq. for system curve (required pump head)

The pump supply curve is given → $\boxed{H = h_{pump,u,supply} = H_0 - a\dot{V}^2}$

supply curve

Equate the system & supply curve to find the operating point

$$\underbrace{H_0 - a\dot{V}^2}_{\text{SUPPLY}} = \underbrace{\frac{8\dot{V}^2}{\pi^2 g D^4}\left(f\frac{L}{D} + \sum K_L\right) + (z_2 - z_1)}_{\text{SYSTEM or DEMAND}}$$

Some iteration is required since $f = \text{fnc}(Re, \varepsilon/D)$, & $Re = \text{fnc of } \dot{V}$

Results:

$$h_{pump,u} = 11.64 \text{ m}$$

$$\dot{V} = 1.796 \times 10^{-4} \text{ m}^3/\text{s} = 10.78 \frac{\text{L}}{\text{min}} \quad (\text{LPM})$$

This is a great problem for EES → see website

C. Dimensionless Parameters in Pump Performance (Sec 14.3)

1. Nondimensional groups or Pi's

Use method of repeating variables for pump performance

repeating variables

group g & H together

$$gH = \text{fnc}\left(\dot{V}, D, \varepsilon, \omega, \rho, \mu\right)$$

D: pump dia

ε: roughness

ω: rotational speed (rad/s)

Result:

$$\boxed{\frac{gH}{\omega^2 D^2} = \text{fnc}\left(\frac{\dot{V}}{\omega D^3}, \underbrace{\frac{\rho \omega D^2}{\mu}}_{Re}, \underbrace{\frac{\varepsilon}{D}}_{\text{roughness parameter}}\right)} \qquad (1)$$

Similarly, $bhp = \text{fnc}$ (same 6 variables)

$$\boxed{\frac{bhp}{\rho \omega^3 D^5} = \text{fnc}\left(\frac{\dot{V}}{\omega D^3}, \frac{\rho \omega D^2}{\mu}, \frac{\varepsilon}{D}\right)} \qquad (2)$$

Turbomachinery people use Q for volume flow rate. We use $\dot{V}$

Other parameters:

$$C_H = \frac{gH}{\omega^2 D^2} = \text{head coefficient}$$

$$C_Q = \frac{Q}{\omega D^3} = \frac{\dot{V}}{\omega D^3} = \text{capacity coefficient}$$

$$C_P = \frac{bhp}{\rho \omega^3 D^5} = \text{Power coefficient}$$

Eqs (1) & (2) become

$$\boxed{\begin{aligned} C_H &= \text{fnc}\left(C_Q,\ Re,\ \varepsilon/D\right) \\ C_p &= \text{fnc}\left(C_Q,\ Re,\ \varepsilon/D\right) \end{aligned}}$$ ☆

Simplification: if Re is high enough, we approach Reynolds number independence

Also, ε/D is usually not very critical

Approx:

$$\boxed{\begin{aligned} C_H &\approx \text{fnc}\left(C_Q\right) \\ C_p &\approx \text{fnc}\left(C_Q\right) \end{aligned}}$$ ☆

These are approximations that are valid for
- high Re
- ε/D not critically important

Can plot: