Today, we will:

- Continue our discussion about dimensional analysis with pumps
- Do an example problem – dimensional analysis with pumps; the affinity laws
- Discuss turbines

From previous lecture…dimensional analysis of pump parameters:

$$C_H = \text{function}\left(C_Q, \text{Re}, \varepsilon/D\right) \quad \text{and} \quad C_P = \text{function}\left(C_Q, \text{Re}, \varepsilon/D\right)$$

where $\boxed{C_Q = \dfrac{\dot{V}}{\omega D^3}}$ Capacity coefficient, $\boxed{C_H = \dfrac{gH}{\omega^2 D^2}}$ Head coefficient, $\boxed{C_P = \dfrac{bhp}{\rho \omega^3 D^5}}$ Power coefficient

But for many pumps, effects of Re and roughness are small at high Re, and thus,

$$\boxed{C_H \approx \text{function}\left(C_Q\right) \quad \text{and} \quad C_P \approx \text{function}\left(C_Q\right)}$$

$\eta_{pump} \longrightarrow$ $\eta_{pump} = \dfrac{\rho \dot{V} g H}{bhp}$

$\dot{V} = \omega D^3 C_Q$

$gH = \omega^2 D^2 C_H$

$bhp = \rho \omega^3 D^5 C_p$

So, $\eta_{pump} = \dfrac{\cancel{\rho} \cancel{\omega} \cancel{D^3} C_Q \cancel{\omega^2} \cancel{D^2} C_H}{\cancel{\rho} \cancel{\omega^3} \cancel{D^5} C_p} = \boxed{\dfrac{C_Q C_H}{C_p} = \eta_{pump}}$

$\therefore \boxed{\eta_{pump} = \text{fnc}\left(C_Q\right)}$ also

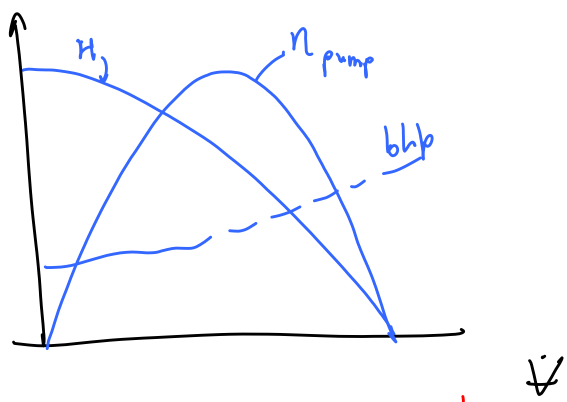
2. THE AFFINITY LAWS

If 2 pumps are geometrically similar & dynamically similar, then their pump performance curves fall on top of each other when plotted nondimensionally

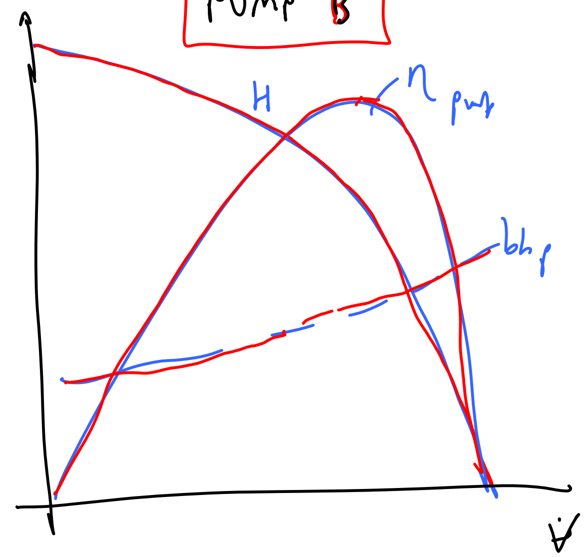
e.g. Pump A & Pump B

geometrically similar, but Pump B is bigger

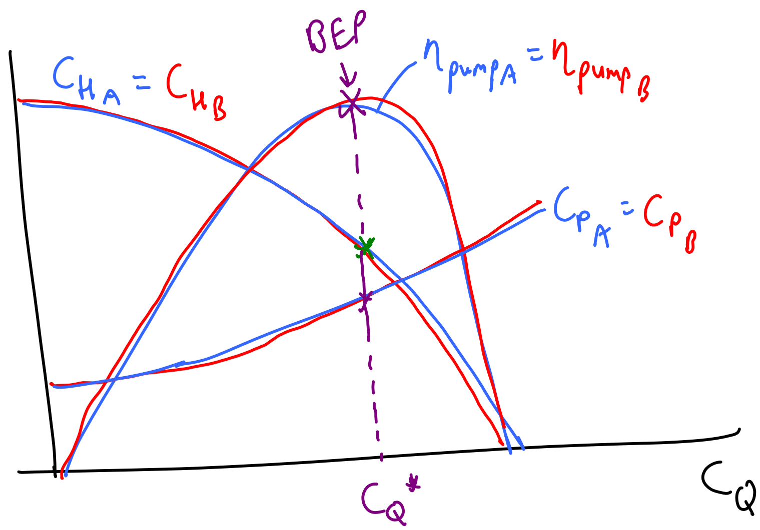
Pump A



Pump B



Plot both on same plot nondimensionally.



★ ONE PLOT HOLDS FOR A WHOLE FAMILY OF GEOMETRICALLY SIMILAR PUMPS!

The Affinity Laws:

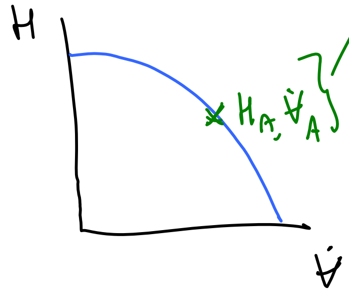
At some operating point for pump A

$(\dot{V}_A, H_A, \eta_A \ldots)$

Pump A

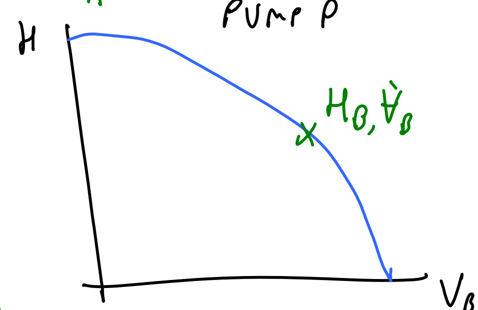
Calculate C_{Q_A} & C_{H_A} — apply these to pump B

Pump B



$\Longrightarrow$

$$\boxed{\begin{aligned} C_{Q_A} &= C_{Q_B} \\ C_{H_A} &= C_{H_B} \end{aligned}}$$



The two operating points for pump A & pump B are called

$\boxed{\text{Homologous Points}}$ ☆

i.e. if we set $C_{Q_A} = C_{Q_B}$, then $C_{H_A} = C_{H_B}$, $C_{P_A} = C_{P_B}$ etc.

$$C_{Q_A} = \frac{\dot{V}_A}{\omega_A D_A^3} = C_{Q_B} = \frac{\dot{V}_B}{\omega_B D_B^3} \Rightarrow \boxed{\frac{\dot{V}_B}{\dot{V}_A} = \frac{\omega_B}{\omega_A}\left(\frac{D_B}{D_A}\right)^3}$$

Do similarly for C_H & C_P:

THE AFFINITY LAWS: ☆

$$\boxed{\begin{aligned} \frac{\dot{V}_B}{\dot{V}_A} &= \left(\frac{\omega_B}{\omega_A}\right)^{1}\left(\frac{D_B}{D_A}\right)^3 \\ \frac{H_B}{H_A} &= \left(\frac{\omega_B}{\omega_A}\right)^{2}\left(\frac{D_B}{D_A}\right)^2 \\ \frac{bhp_B}{bhp_A} &= \left(\frac{\omega_B}{\omega_A}\right)^{3}\left(\frac{D_B}{D_A}\right)^5 \frac{\rho_B}{\rho_A} \end{aligned}}$$

"Jingle" → "Very Hard Problems are as easy as 1, 2, 3!"

Volume flow rate — Head — Power — exponent on ω ratio

Holds for cases where all you change is the RPM (ω)

(For a given pump)

E.g. Suppose you double the RPM of a pump: $\frac{\omega_B}{\omega_A} = 2$

$\dot{V} \uparrow$ by a factor of $2^1 = 2$

$H \uparrow$ .. ... — $2^2 = 4$

bhp $\uparrow$ .. .. .. — $2^3 = 8$

} at Homologous points

Example: Given: Pump A = existing $D_A = 6.5$ cm fluid = water

RPM $\longrightarrow \dot{n}_A = 1500$ rpm

@ BEP, $\dot{V}_A = 455 \frac{cm^3}{s}$ $H_A = 1.44$ m

To do: Design a bigger pump (B) with $D_B = 8.2$ cm (geometrically similar)

rotate @ $\dot{n}_B = 1750$ rpm Same fluid ($\rho_B = \rho_A$)

(a) Predict $\dot{V}_B$ & H_B @ BEP

Soln. Pump B is geometrically similar to Pump A & we operate both pumps @ BEP. Use affinity laws

$$\dot{V}_B = \dot{V}_A \left(\frac{\omega_B}{\omega_A}\right)\left(\frac{D_B}{D_A}\right)^3 = \dot{V}_A \left(\frac{\dot{n}_B}{\dot{n}_A}\right)\left(\frac{D_B}{D_A}\right)^3$$

$$455 \frac{cm^3}{s} \left(\frac{1750 \cancel{rpm}}{1500 \cancel{rpm}}\right)\left(\frac{8.2 \cancel{cm}}{6.5 \cancel{cm}}\right) = 1065.76 \frac{cm^3}{s}$$

$$\boxed{\dot{V}_B = 1070 \frac{cm^3}{s}}$$

(more than twice as much!)

Similarly $$H_B = H_A \left(\frac{\dot{n}_B}{\dot{n}_A}\right)^2 \left(\frac{D_B}{D_A}\right)^2 = 3.1193 \text{ m}$$

$$\boxed{H_B = 3.12 \text{ m}}$$ (more than 2× as much)

(b) To estimate the % increase in required bhp

Sol'n $$\frac{bhp_B}{bhp_A} = \cancel{\frac{\rho_B}{\rho_A}} \left(\frac{\dot{n}_B}{\dot{n}_A}\right)^3 \left(\frac{D_B}{D_A}\right)^5 = 5.0739$$

$$\% \text{ increase} = \frac{bhp_B - bhp_A}{bhp_A} \times 100\% = \left(\frac{bhp_B}{bhp_A} - 1\right) \times 100\%$$

$$\boxed{\% \text{ increase} = 407\%}$$

TURBINES — See pdf file on website

$\boxed{\text{Turbine\_lecture.pdf}}$