Today, we will:

- Discuss dimensional analysis of turbines
- Do an example problem – dimensional analysis with turbines
- Discuss piping networks – how to deal with pipes in series or in parallel

b. Dimensionless parameters in turbine performance

We perform exactly the same dimensional analysis for turbines as we did for pumps. Result:

Dimensionless Parameters: $\boxed{C_Q = \frac{\dot{V}}{\omega D^3}}$ $\boxed{C_H = \frac{gH}{\omega^2 D^2}}$ $\boxed{C_P = \frac{bhp}{\rho \omega^3 D^5}}$

Capacity coefficient Head coefficient Power coefficient

Same Π's as for a pump.

Difference → $\boxed{\eta_{turbine} = \frac{bhp}{\rho \dot{V} g H} = \frac{C_P}{C_Q C_H}}$

Difference → for turbines we typically plot against C_P instead of C_Q

$$\boxed{\begin{aligned} C_Q &= fnc(C_P) \\ C_H &= fnc(C_P) \\ \eta_{turbine} &= fnc(C_P) \end{aligned}}$$

✩ AFFINITY LAWS ARE THE SAME

Example: Scaling up a hydroturbine

Given: An existing hydroturbine (A): Fluid is water at 20°C, D_A = 1.95 m, $\dot{n}_A = 120$ rpm, bhp_A = 220 MW, and $\dot{V}_A = 335$ m³/s at $H_A = 72.4$ m. We are designing a new turbine (B) that is geometrically similar, still uses water at 20°C, and $\dot{n}_B = 120$ rpm, but $H_B = 97.4$ m. [Dam B has a higher gross head available than Dam A.]

To do: (a) Calculate D_B and $\dot{V}_B$ for operation of turbine B at a homologous point.

(b) Calculate bhp_B and estimate the turbine efficiency of both turbines.

Solution:

(a) At homologous points, the two turbines are dynamically similar. Apply the affinity laws:

$$C_{H,A} = \frac{gH_A}{\omega_A{}^2 D_A{}^2} = C_{H,B} = \frac{gH_B}{\omega_B{}^2 D_B{}^2} \rightarrow \text{solve for } D_B = D_A\left(\frac{\omega_A}{\omega_B}\right)\sqrt{\frac{H_B}{H_A}} = D_A\left(\frac{\dot{n}_A}{\dot{n}_B}\right)\sqrt{\frac{H_B}{H_A}}$$

Plug in numbers: $\boxed{D_B = 2.26 \text{ m}}$

Similarly, $$C_{Q,A} = \frac{\dot{V}_A}{\omega_A D_A{}^3} = C_{Q,B} = \frac{\dot{V}_B}{\omega_B D_B{}^3} \rightarrow \dot{V}_B = \dot{V}_A\left(\frac{\omega_B}{\omega_A}\right)\left(\frac{D_B}{D_A}\right)^3 = \dot{V}_A\left(\frac{\dot{n}_B}{\dot{n}_A}\right)\left(\frac{D_B}{D_A}\right)^3$$

Plug in numbers: $\boxed{\dot{V}_B = 523. \text{ m}^3/\text{s}}$

(b) Similarly, $$C_{P,A} = \frac{bhp_A}{\rho_A \omega_A{}^3 D_A{}^5} = C_{P,B} = \frac{bhp_B}{\rho_B \omega_B{}^3 D_B{}^5} \rightarrow bhp_B = bhp_A\left(\frac{\rho_B}{\rho_A}\right)\left(\frac{\dot{n}_B}{\dot{n}_A}\right)^3\left(\frac{D_B}{D_A}\right)^5$$

Plug in numbers: $\boxed{bhp_B = 462. \text{ MW}}$ (more than 2X bhp_A)

Finally, the efficiency is calculated for each turbine:

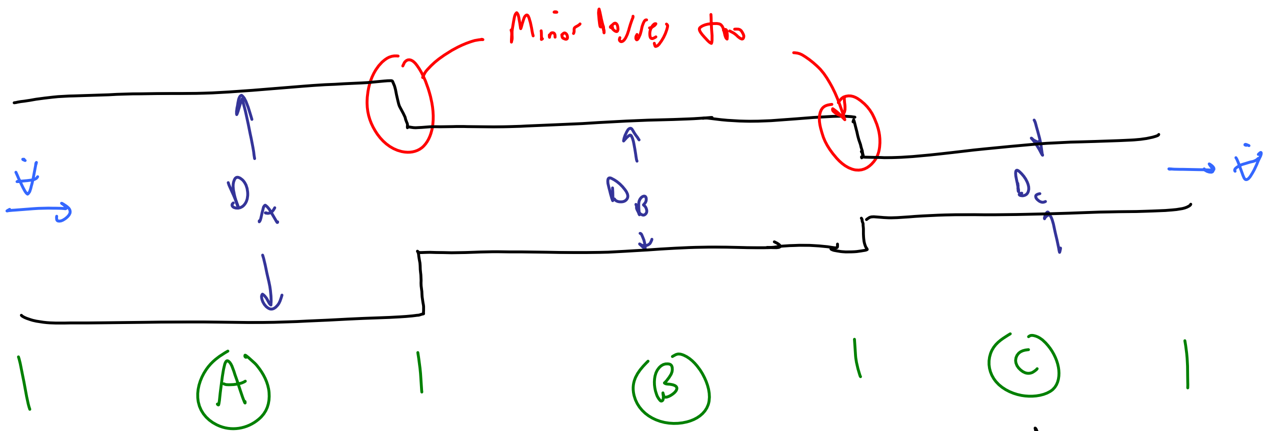
$$\eta_{\text{turbine},A} = \frac{bhp_A}{\rho_A g H_A \dot{V}_A} = \frac{220{,}000{,}000\ \text{W}}{\left(1000\,\dfrac{\text{kg}}{\text{m}^3}\right)\left(9.81\,\dfrac{\text{m}}{\text{s}^2}\right)72.4\ \text{m}\left(335\,\dfrac{\text{m}^3}{\text{s}}\right)}\left(\frac{\text{N}\cdot\text{m}}{\text{W}\cdot\text{s}}\right)\left(\frac{\text{kg}\cdot\text{m}}{\text{s}^2\cdot\text{N}}\right) = \boxed{92.5\%}$$

$$\eta_{\text{turbine},B} = \frac{bhp_B}{\rho_B g H_B \dot{V}_B} = \frac{461{,}820{,}979\ \text{W}}{\left(1000\,\dfrac{\text{kg}}{\text{m}^3}\right)\left(9.81\,\dfrac{\text{m}}{\text{s}^2}\right)97.4\ \text{m}\left(522.728\,\dfrac{\text{m}^3}{\text{s}}\right)}\left(\frac{\text{N}\cdot\text{m}}{\text{W}\cdot\text{s}}\right)\left(\frac{\text{kg}\cdot\text{m}}{\text{s}^2\cdot\text{N}}\right) = \boxed{92.5\%}$$

Same!

F. Piping Networks (Ch. 8)

1. Pipes in Series (Easy - we already know how to solve this)



Cons. of mass (assume incompressible), $\boxed{\dot{V}_A = \dot{V}_B = \dot{V}_C}$

But $V_C > V_B > V_A$

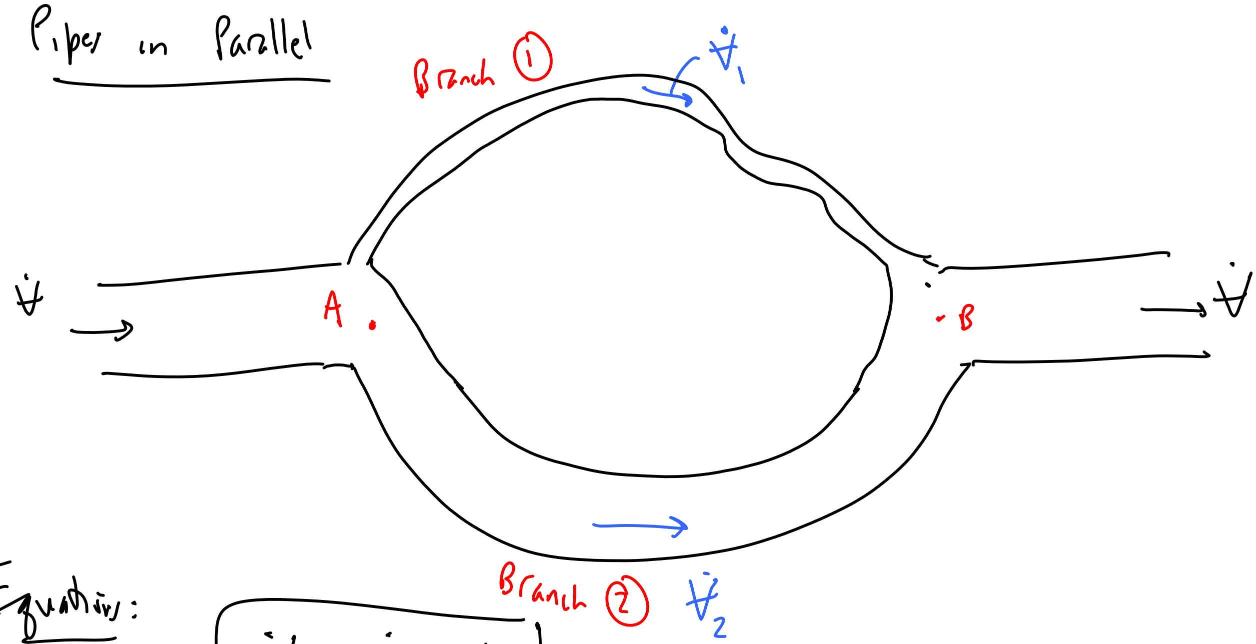
$Re_A \neq Re_B \neq Re_C$ & $\left(\frac{\varepsilon}{D}\right)_A \neq \left(\frac{\varepsilon}{D}\right)_B \neq \left(\frac{\varepsilon}{D}\right)_C$

$\therefore\ f_A \neq f_B \neq f_C$ → Need to use Moody chart 3 times. (or Colebrook eq.)

h_L term in energy eq. is:

$$\boxed{h_L = \sum_{i\ A,B,C} f_i \frac{L_i}{D_i}\frac{V_i^2}{2g} + \sum_{j\ A,B,C} K_{L,j}\frac{V_j^2}{2g}} \ \star$$

2. Pipes in Parallel



Equations:

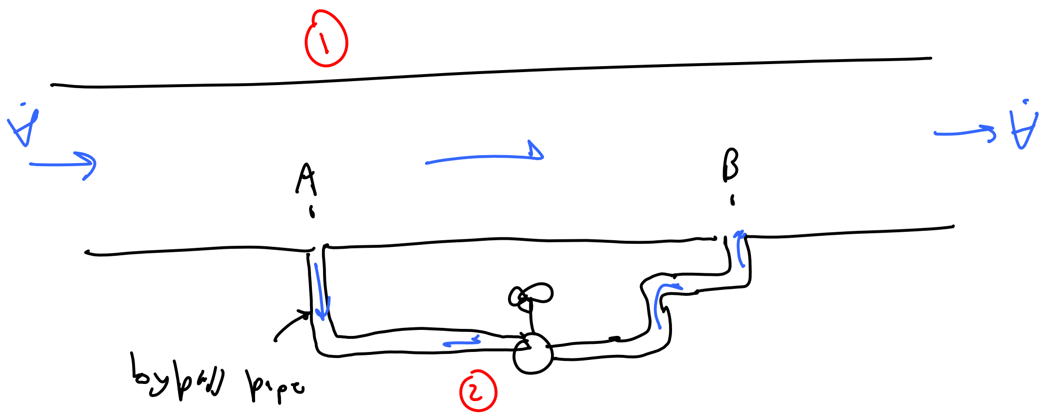
$$\boxed{\dot{V} = \dot{V}_1 + \dot{V}_2}$$

But, $P_A - P_B$ = same for either branch

$$\therefore \boxed{h_{L_1} = h_{L_2}} \; *$$

e.g.

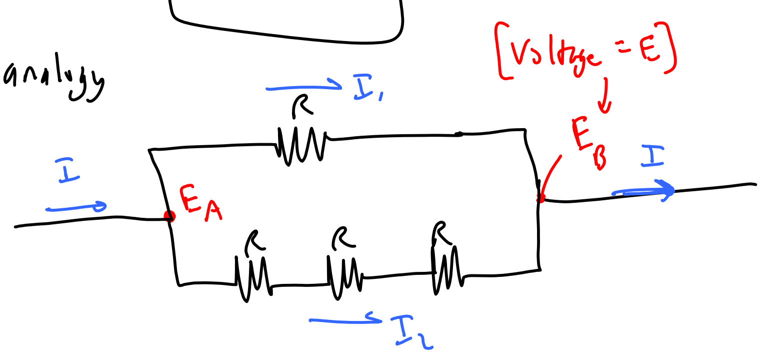
Given:



$P_A - P_B$ is the same regardless of which branch we are considering

$\therefore \boxed{h_{L_1} = h_{L_2}}$ But $\dot{V}_2 << \dot{V}_1$

Electrical analogy

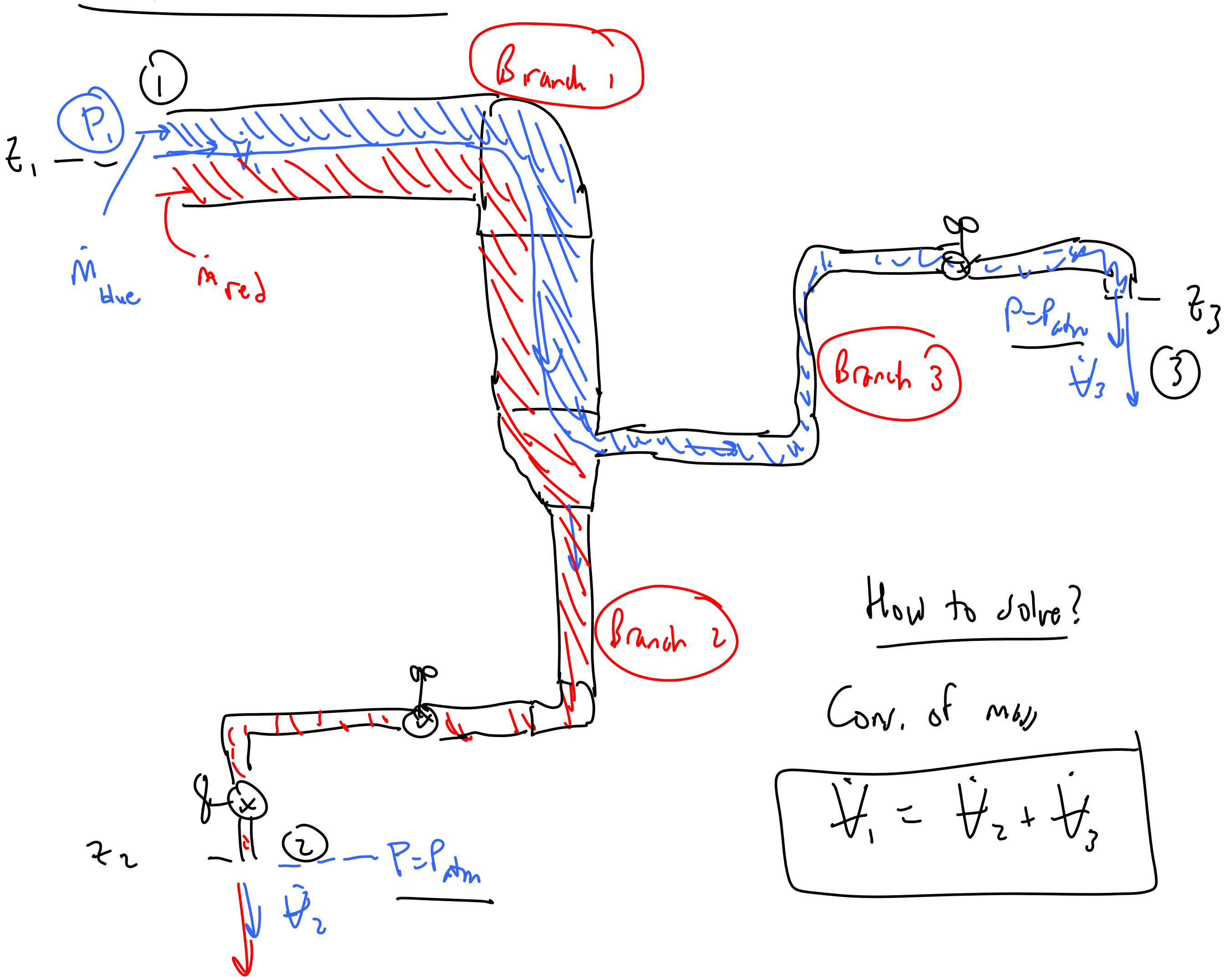


$I = I_1 + I_2$

$E_A - E_B$ = same !

But $I_2 < I_1$

3. Complex Piping Networks



How to solve?

Cons. of mass

$$\boxed{\dot{\forall}_1 = \dot{\forall}_2 + \dot{\forall}_3}$$

- Now apply energy eq. in head form
- Pretend or imagine that blue & red fluid enters ①

  Only the blue fluid goes through pipe ③

  " " " red " " " " " ②

From ① to ② (red fluid only)

$$\boxed{\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + \cancel{h_{pump,u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + \cancel{h_{turbine,e}} + h_{L_{1 to 2}}}$$

From (1) to (3) (blue fluid),

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + \cancel{h_{pump,u}} = \frac{P_3}{\rho g} + \alpha_3 \frac{V_3^2}{2g} + z_3 + \cancel{h_{turbine}} + h_{L,1 to 3}$$

Also, we know that

$$\boxed{h_{L,1 to 2} = h_{L_1} + h_{L_2}}$$

$$\boxed{h_{L,1 to 3} = h_{L_1} + h_{L_3}}$$

How to solve? Simultaneous equation solving:

Here, unknowns:

- Re_1, Re_2, Re_3
- f_1, f_2, f_3
- $(\varepsilon/D)_1$ $(\varepsilon/D)_2$ $(\varepsilon/D)_3$
- $\dot{V}_1$ $\dot{V}_2$ $\dot{V}_3$
- V_1 V_2 V_3, etc (lots of unknowns!)

$\dot{V}_1 = \frac{\pi D_1^2}{4} V_1$, etc

Equations:

- the 2 energy eq.'s above + defn's for Re + Colebrook eq or Moody chart (3 of them) etc. (lots of equations)

$\boxed{\text{\# unknowns = \# equations}}$ → solve a set of simultaneous eq's

How? → EES, Matlab, ... or with calculator (Hard!)

EES is specifically intended for solving simultaneous equations