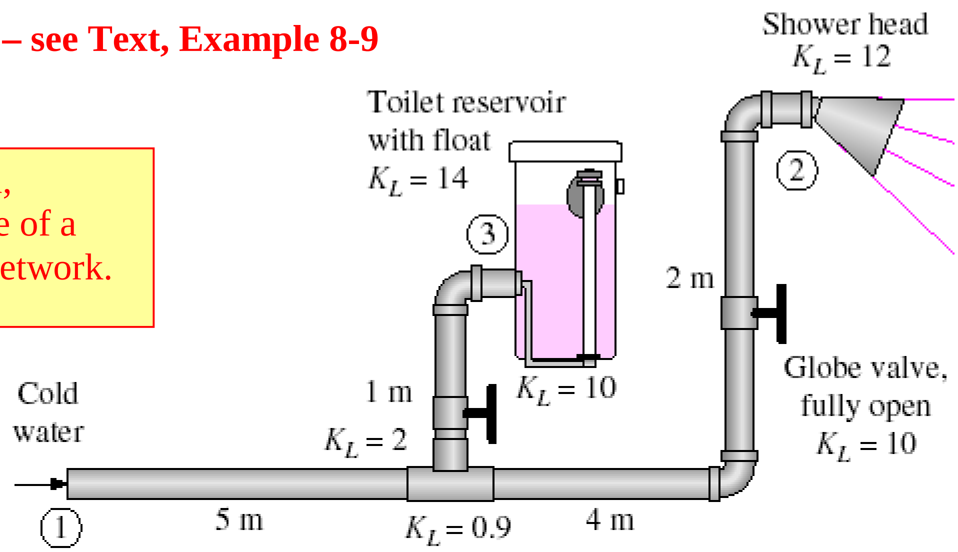
Today, we will:

- Do some examples of complex piping networks (multiple pipes with branches, etc.)
- Briefly mention flow meters and velocity measurement
- Begin Chapter 9 – Differential Analysis of Fluid Flows

Toilet Flushing Example – see Text, Example 8-9

This is a practical, everyday example of a complex piping network.



EXAMPLE 8–9 **Effect of Flushing on Flow Rate from a Shower**

The bathroom plumbing of a building consists of 1.5-cm-diameter copper pipes with threaded connectors, as shown in Fig. 8–49. (*a*) If the gage pressure at the inlet of the system is 200 kPa during a shower and the toilet reservoir is full (no flow in that branch), determine the flow rate of water through the shower head. (*b*) Determine the effect of flushing of the toilet on the flow rate through the shower head. Take the loss coefficients of the shower head and the reservoir to be 12 and 14, respectively.

SOLUTION The cold-water plumbing system of a bathroom is given. The flow rate through the shower and the effect of flushing the toilet on the flow rate are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The flow is turbulent and fully developed. **3** The reservoir is open to the atmosphere. **4** The velocity heads are negligible.

This is a simplifying assumption that may or may not be valid. We should check the validity later.

We consider only the cold water line. The hot water line is separate, and is not connected to the toilet, so the volume flow rate of hot water through the shower remains constant. The cold water, however, is affected by flushing the toilet.

Properties The properties of water at 20°C are $\rho = 998\ \text{kg/m}^3$, $\mu = 1.002 \times 10^{-3}\ \text{kg/m} \cdot \text{s}$, and $\nu = \mu/\rho = 1.004 \times 10^{-6}\ \text{m}^2/\text{s}$. The roughness of copper pipes is $\varepsilon = 1.5 \times 10^{-6}$ m.

Analysis This is a problem of the second type since it involves the determination of the flow rate for a specified pipe diameter and pressure drop. The solution involves an iterative approach since the flow rate (and thus the flow velocity) is not known.

Part (a) is not a parallel system since no flow through the toilet.

(*a*) The piping system of the shower alone involves 11 m of piping, a tee with line flow ($K_L = 0.9$), two standard elbows ($K_L = 0.9$ each), a fully open globe valve ($K_L = 10$), and a shower head ($K_L = 12$). Therefore, $\Sigma K_L = 0.9 + 2 \times 0.9 + 10 + 12 = 24.7$. Noting that the shower head is open to the atmosphere, and the velocity heads are negligible, the energy equation for a control volume between points 1 and 2 simplifies to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump}, u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine}, e} + h_L$$

$$\rightarrow \quad \frac{P_{1,\text{ gage}}}{\rho g} = (z_2 - z_1) + h_L$$

We neglect the velocity heads in the energy equation. Alternatively, if $V_1 = V_2$, then these two terms cancel each other out.

$P_2 = P_{\text{atm}}$, and therefore $P_1 - P_2 = P_{1,\text{gage}}$.

Therefore, the head loss is

$$h_L = \frac{200{,}000 \text{ N/m}^2}{(998 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} - 2 \text{ m} = 18.4 \text{ m}$$

Also,

$$h_L = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g} \quad \rightarrow \quad 18.4 = \left(f\frac{11 \text{ m}}{0.015 \text{ m}} + 24.7\right)\frac{V^2}{2(9.81 \text{ m/s}^2)}$$

since the diameter of the piping system is constant. The average velocity in the pipe, the Reynolds number, and the friction factor are

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} \quad \rightarrow \quad V = \frac{\dot{V}}{\pi(0.015 \text{ m})^2/4}$$

$$\text{Re} = \frac{VD}{\nu} \quad \rightarrow \quad \text{Re} = \frac{V(0.015 \text{ m})}{1.004 \times 10^{-6} \text{ m}^2/\text{s}}$$

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

Colebrook equation

$$\rightarrow \quad \frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{1.5 \times 10^{-6} \text{ m}}{3.7(0.015 \text{ m})} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

This is a set of four equations with four unknowns, and solving them with an equation solver such as EES gives

$$\dot{V} = 0.00053\ \text{m}^3/\text{s}, \quad f = 0.0218, \quad V = 2.98\ \text{m/s}, \quad \text{and} \quad \text{Re} = 44{,}550$$

Therefore, the flow rate of water through the shower head is **0.53 L/s.**

Answer to part (a) – no toilet flushing

(*b*) When the toilet is flushed, the float moves and opens the valve. The discharged water starts to refill the reservoir, resulting in parallel flow after the tee connection. The head loss and minor loss coefficients for the shower branch were determined in (*a*) to be $h_{L,2} = 18.4$ m and $\Sigma K_{L,2} = 24.7$, respectively. The corresponding quantities for the reservoir branch can be determined similarly to be

$$h_{L,3} = \frac{200{,}000\ \text{N/m}^2}{(998\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} - 1\ \text{m} = 19.4\ \text{m}$$

$$\Sigma K_{L,3} = 2 + 10 + 0.9 + 14 = 26.9$$

The relevant equations in this case are

$$\dot{V}_1 = \dot{V}_2 + \dot{V}_3$$

$$h_{L,2} = f_1 \frac{5\ \text{m}}{0.015\ \text{m}} \frac{V_1^2}{2(9.81\ \text{m/s}^2)} + \left(f_2 \frac{6\ \text{m}}{0.015\ \text{m}} + 24.7\right) \frac{V_2^2}{2(9.81\ \text{m/s}^2)} = 18.4$$

$$h_{L,3} = f_1 \frac{5\ \text{m}}{0.015\ \text{m}} \frac{V_1^2}{2(9.81\ \text{m/s}^2)} + \left(f_3 \frac{1\ \text{m}}{0.015\ \text{m}} + 26.9\right) \frac{V_3^2}{2(9.81\ \text{m/s}^2)} = 19.4$$

$$V_1 = \frac{\dot{V}_1}{\pi(0.015\ \text{m})^2/4}, \quad V_2 = \frac{\dot{V}_2}{\pi(0.015\ \text{m})^2/4}, \quad V_3 = \frac{\dot{V}_3}{\pi(0.015\ \text{m})^2/4}$$

$$\text{Re}_1 = \frac{V_1(0.015\ \text{m})}{1.004 \times 10^{-6}\text{m}^2/\text{s}}, \quad \text{Re}_2 = \frac{V_2(0.015\ \text{m})}{1.004 \times 10^{-6}\text{m}^2/\text{s}}, \quad \text{Re}_3 = \frac{V_3(0.015\ \text{m})}{1.004 \times 10^{-6}\text{m}^2/\text{s}}$$

$$\frac{1}{\sqrt{f_1}} = -2.0 \log\left(\frac{1.5 \times 10^{-6}\ \text{m}}{3.7(0.015\ \text{m})} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right)$$

$$\frac{1}{\sqrt{f_2}} = -2.0 \log\left(\frac{1.5 \times 10^{-6}\ \text{m}}{3.7(0.015\ \text{m})} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right)$$

$$\frac{1}{\sqrt{f_3}} = -2.0 \log\left(\frac{1.5 \times 10^{-6}\ \text{m}}{3.7(0.015\ \text{m})} + \frac{2.51}{\text{Re}_3\sqrt{f_3}}\right)$$

This is where EES comes in handy – solving all these simultaneous equations!

Now we need three Colebrook equations – one for each branch!

Solving these 12 equations in 12 unknowns simultaneously using an equation solver, the flow rates are determined to be

$$\dot{V}_1 = 0.00090 \text{ m}^3/\text{s}, \quad \dot{V}_2 = 0.00042 \text{ m}^3/\text{s}, \quad \text{and} \quad \dot{V}_3 = 0.00048 \text{ m}^3/\text{s}$$

Therefore, the flushing of the toilet **reduces the flow rate of cold water through the shower by 21 percent** from 0.53 to 0.42 L/s, causing the shower water to suddenly get very hot (Fig. 8–50).

Discussion If the velocity heads were considered, the flow rate through the shower would be 0.43 instead of 0.42 L/s. Therefore, the assumption of negligible velocity heads is reasonable in this case.

Note that a leak in a piping system will cause the same effect, and thus an unexplained drop in flow rate at an end point may signal a leak in the system.

FIGURE 8–50
Flow rate of cold water through a shower may be affected significantly by the flushing of a nearby toilet.

EES Solution – Toilet Flushing Example Problem

Part (a) EES Equation window:

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"Example 8.9 - Toilet Flushing Problem, Part (a)"

"Contants and properties:"
g = g#
rho = 998 [kg/m^3]
mu = 1.002E-3 [kg/m-s]
D = 0.015 [m]
epsilon = 1.5e-6 [m]
P_gage_1 = 200000 [N/m^2]
L_1  = 5 [m]
L_2 = 6 [m]
SIGMAK_L_1 = 0
SIGMAK_L_2 = 0.9 + 2*0.9 + 10 + 12   "tee, two elbows, valve, and shower head"
DELTAz_1to2 = 2 [m]

"Equations to solve"
P_gage_1/(rho*g) = DELTAz_1to2 + h_L_1to2
h_L_1to2 = V^2/(2*g) * (f_1to2*(L_1+L_2)/D + SIGMAK_L_1 + SIGMAK_L_2)
A = pi*D^2/4
V_dot = V*A
Re = V*D*rho/mu
f_1to2 = MoodyChart(Re,epsilon/D)
V_dot_LPS = V_dot*CONVERT(m^3/s,L/s)
```

Part (a) EES Solution:

Unit Settings: SI K kPa kJ molar deg

$A = 0.0001767$ [m²]	$D = 0.015$ [m]	$\Delta z_{1to2} = 2$ [m]
$\varepsilon = 0.0000015$ [m]	$f_{1to2} = 0.0217$	$g = 9.807$ [m/s²]
$h_{L,1to2} = 18.43$ [m]	$L_1 = 5$ [m]	$L_2 = 6$ [m]
$\mu = 0.001002$ [kg/m-s]	$P_{gage,1} = 200000$ [N/m²]	$Re = 44576$ [-]
$\rho = 998$ [kg/m³]	$SIGMAK_{L,1} = 0$	$SIGMAK_{L,2} = 24.7$
$V = 2.984$ [m/s]	$\dot{V} = 0.0005273$ [m³/s]	$\dot{V}_{LPS} = 0.5273$ [L/s]

No unit problems were detected.

Part (b) EES Equation Window:

```
"Example 8.9 - Toilet Flushing Problem, Part (b)"

"Contants and properties:"
g = g#
rho = 998 [kg/m^3]
mu = 1.002E-3 [kg/m-s]
D = 0.015 [m]
epsilon = 1.5e-6 [m]
P_gage_1 = 200000 [N/m^2]
L_1  = 5 [m]
L_2 = 6 [m]
L_3 = 1 [m]
SIGMAK_L_1 = 0
SIGMAK_L_2 = 0.9 + 2*0.9 + 10 + 12  "tee, two elbows, valve, and shower head"
SIGMAK_L_3 = 2 + 10 + 0.9 + 14       "tee, one elbow, valve, and toilet mechanism"
DELTAz_1to2 = 2 [m]
DELTAz_1to3 = 1 [m]

"Equations to solve"
P_gage_1/(rho*g) = DELTAz_1to2 + h_L_1to2
P_gage_1/(rho*g) = DELTAz_1to3 + h_L_1to3
h_L_1to2 = V_1^2/(2*g) * (f_1*L_1/D + SIGMAK_L_1) + V_2^2/(2*g) * (f_2*L_2/D + SIGMAK_L_2)
h_L_1to3 = V_1^2/(2*g) * (f_1*L_1/D + SIGMAK_L_1) + V_3^2/(2*g) * (f_3*L_3/D + SIGMAK_L_3)
A = pi*D^2/4
V_dot_1 = V_1*A
V_dot_2 = V_2*A
V_dot_3 = V_3*A
Re_1 = V_1*D*rho/mu
Re_2 = V_2*D*rho/mu
Re_3 = V_3*D*rho/mu
f_1 = MoodyChart(Re_1,epsilon/D)
f_2 = MoodyChart(Re_2,epsilon/D)
f_3 = MoodyChart(Re_3,epsilon/D)
V_dot_1 = V_dot_2 + V_dot_3
V_dot_2_LPS = V_dot_2*CONVERT(m^3/s,L/s)
```

Part (b) EES Solution:

$A = 0.0001767$ [m^2]	$D = 0.015$ [m]	$\Delta z_{1to2} = 2$ [m]
$\Delta z_{1to3} = 1$ [m]	$\varepsilon = 0.0000015$ [m]	$f_1 = 0.01943$
$f_2 = 0.0228$ [-]	$f_3 = 0.02212$ [-]	$g = 9.807$ [m/s^2]
$h_{L,1to2} = 18.43$ [m]	$h_{L,1to3} = 19.43$ [m]	$L_1 = 5$ [m]
$L_2 = 6$ [m]	$L_3 = 1$ [m]	$\mu = 0.001002$ [kg/m-s]
$P_{gage,1} = 200000$ [N/m^2]	$Re_1 = 76419$ [-]	$Re_2 = 35608$ [-]
$Re_3 = 40811$ [-]	$\rho = 998$ [kg/m^3]	$SIGMAK_{L,1} = 0$
$SIGMAK_{L,2} = 24.7$	$SIGMAK_{L,3} = 26.9$	$V_1 = 5.115$ [m/s]
$V_2 = 2.383$ [m/s]	$V_3 = 2.732$ [m/s]	$\dot{V}_1 = 0.0009039$ [m^3/s]
$\dot{V}_2 = 0.0004212$ [m^3/s]	$\dot{V}_{2,LPS} = 0.4212$ [L/s]	$\dot{V}_3 = 0.0004827$ [m^3/s]

NOTE: CLASS CANCELLED TODAY (10-29) DUE TO THE STORM

- Review the above example & EES solution. I will not have time to go over this in class.

[This is a fun, and practical problem, combining nearly everything that we have discussed in Ch. 8 so far]

- Also, we were going to briefly review Section 8.8 on flow measurement techniques & instrumentation. I will not have time to go over that material either — read it on your own.

- Next lecture (Wednesday), we will start in Ch. 9, Differential Analysis.

I point out that many students find this part of the course to be very difficult. I suggest you read Chapter 9 in the text very carefully.