- Continue discussing the irrotational flow approximation and introduce **superposition**.
- Discuss some elementary planar irrotational flows (building block flows)
- Do some examples of superposition

~~D.~~ E. Approximation for ~~Inviscid~~ Irrotational Regions of Flow (continued)

1. ~~Definition of Inviscid Regions of Flow and the Euler Equation~~ Introduction
2. Equations of Motion for Irrotational Flow
3. 2-D Irrotational Flow (continued)
   a. Equations of motion

   Summary, equations for 2-D, steady, incompressible, irrotational flow in *x*-*y* plane:

$$\vec{\nabla} \times \vec{V} = 0 \qquad \vec{V} = \vec{\nabla}\phi \qquad \nabla^2\phi = 0 \qquad \nabla^2\psi = 0 \qquad \frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{constant everywhere}$$

Cartesian: $$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$$

Cylindrical: $$\nabla^2\phi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} = 0$$

   b. Superposition

- If a diff. eq. is linear, then we can add solutions
- Here, $\nabla^2\phi = 0$, $\nabla^2\psi = 0$ is linear, so we can add solutions

Superposition

If ϕ_1 is a solution of $\nabla^2\phi = 0$

& ϕ_2 " " " " " "

then $\phi_3 = C_1\phi_1 + C_2\phi_2$ is also a solution

(same thing holds for ψ since $\nabla^2\psi = 0$)

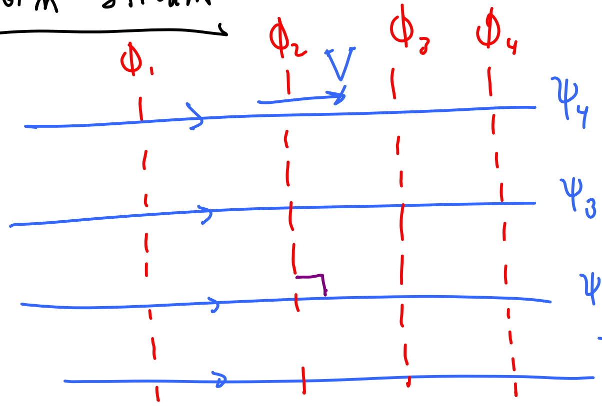
SUPERPOSITION → Adds up simple flows to generate more complicated flows

- CAUTION: Works only in irrotational regions of flow
[the full N-S eq. is not a linear eq. → ($u\frac{\partial u}{\partial y}$ is non linear)]

- CAUTION: Cannot superpose pressures (Bernoulli eq is nonlinear) (V^2)

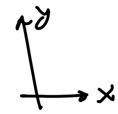
4. Elementary Planar Irrotational Flows (Building Blocks)

a. Uniform Stream



- - - - equipotential lines (const ϕ)

—— streamlines (const ψ)



☆ MUTUAL ORTHOGONALITY

ψ & ϕ intersect @ 90°

Velocity field $\vec{v}$: $u = V$, $v = 0$

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} = V \qquad v = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x} = 0$$

pick one of the above & integrate

$$\psi = Vy + f(x)$$

$$\frac{\partial \psi}{\partial x} = 0 + f'(x). \quad \text{But } \frac{\partial \psi}{\partial x} = 0$$

$$\therefore f'(x) = 0 \rightarrow f(x) = \text{const.}$$

So,

$$\boxed{\psi = Vy + \text{const}}$$

const: arbitrary constant (set = 0 for simplicity)

$$\boxed{\psi = Vy}$$

similarly – try it on your own,

$$\boxed{\phi = Vx}$$

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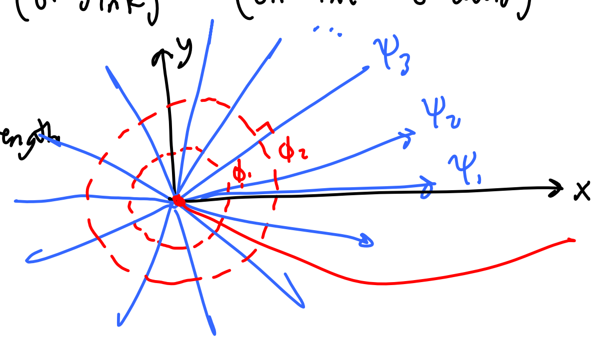
b. Line Source (or sink) (on the z axis)

Let $\frac{\dot{\forall}}{L}$ = line source strength

(L = distance into the page)



- - - const ϕ lines are circles

—— const ψ lines are rays

SINGULARITY

$$u_r = \frac{\dot{\forall}/L}{2\pi r} \qquad u_\theta = 0$$

→ Do the math: (see text)

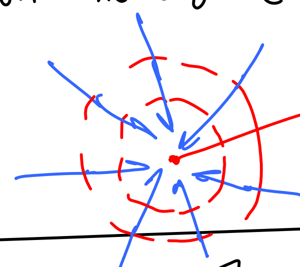
Line source @ origin,

$$\phi = \frac{\dot{\forall}/L}{2\pi} \ln r$$

$$\psi = \frac{\dot{\forall}/L}{2\pi} \theta$$

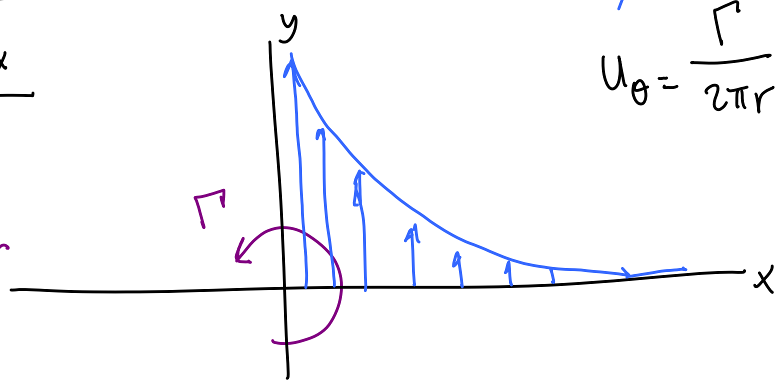
(satisfy $\nabla^2 \phi = 0$, $\nabla^2 \psi = 0$)

- See text - can do an origin shift to have a line source off the origin @ $(x=a,\ y=b)$
- Line sink = same eq's except $\dot{\forall} < 0$

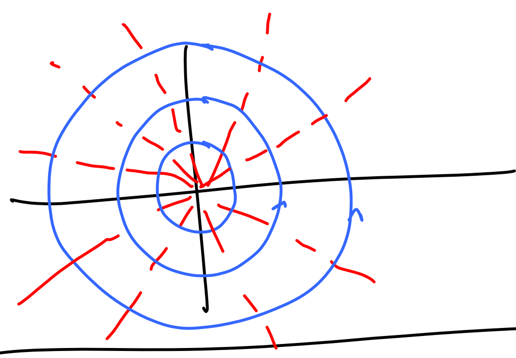


C. Line vortex

Γ = circulation = vortex strength



$$u_\theta = \frac{\Gamma}{2\pi r}, \quad u_r = 0$$



if Γ ⊕ve → counterclockwise

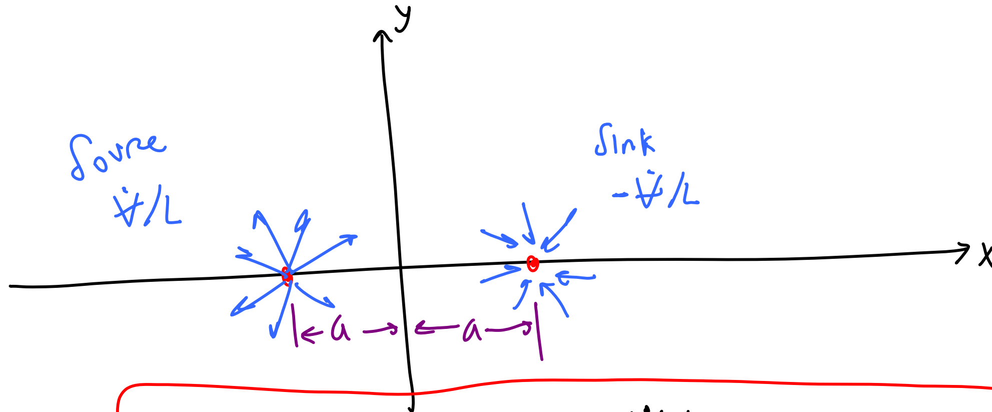
if Γ ⊖ve → clockwise

eqs:

$$\phi = \frac{\Gamma}{2\pi} \theta \qquad \psi = -\frac{\Gamma}{2\pi} \ln r$$

Again, can shift origin to $(x=a,\ y=b)$ → see text

d. <u>Doublet</u> → superposition of a source & a sink



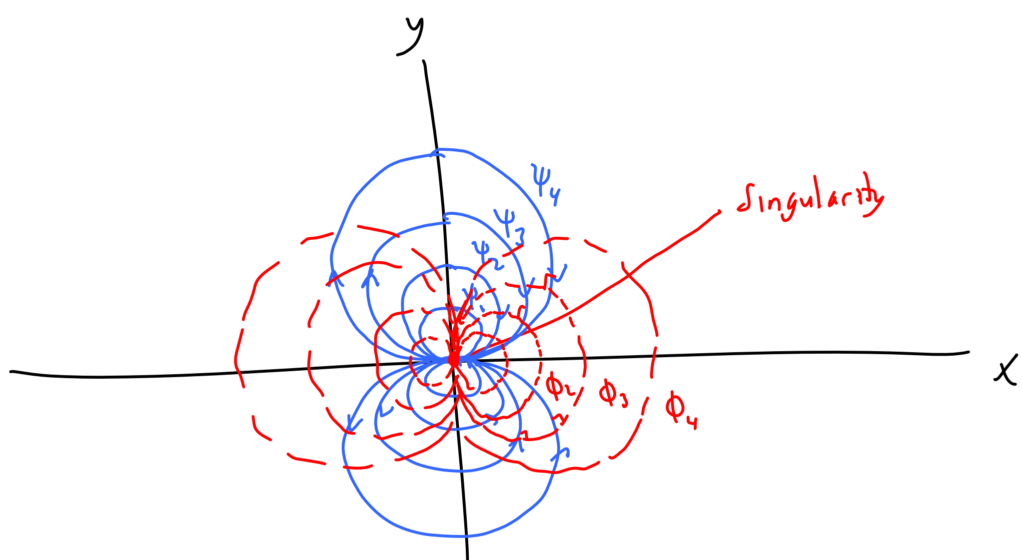
Doublet ≡ Let $a \to 0$ & let $\dot{\forall}/L \to \infty$ simultaneously such that $a \cdot \dot{\forall}/L$ = constant ✭

Result is a doublet

Define ✭ $K = \dfrac{a\,\dot{\forall}/L}{\pi}$ = doublet strength

Math → calc. ϕ & ψ:

$$\phi = K\frac{\cos\theta}{r} \qquad \psi = -K\frac{\sin\theta}{r}$$



Now we can add up some building blocks (superposition) to generate other flows of interest

5. Examples of Superposition

a. The Rankine Half-Body

Superpose
- Uniform stream V [$V = 0.50\ m/s$]
- line source @ origin $\dot{\forall}/L$ [$\dot{\forall}/L = 1.0\ m^2/s$]

Simply add ϕ or ψ to get the new soln.

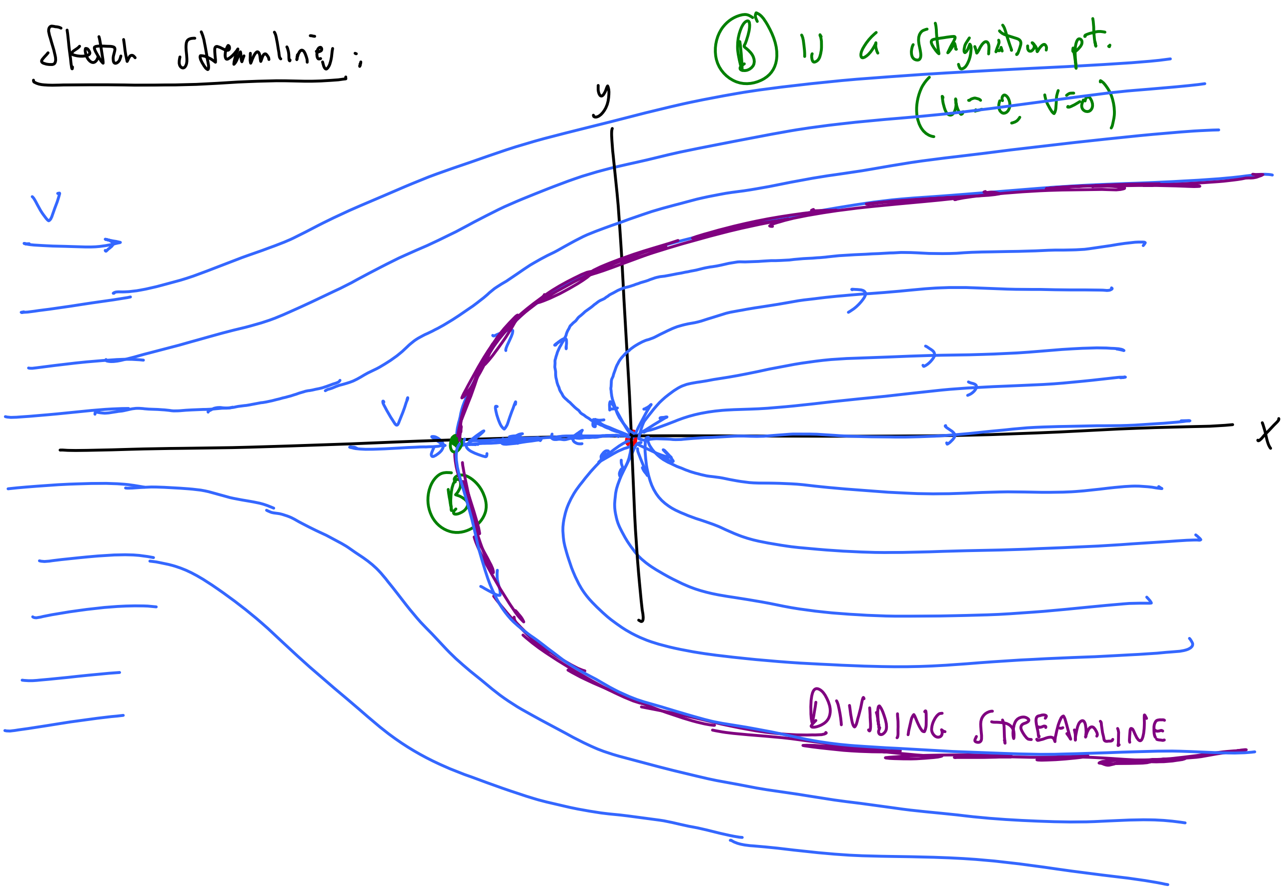
$$\psi = Vy + \frac{\dot{\forall}/L}{2\pi}\theta$$

$\psi_{freestream} + \psi_{source}$

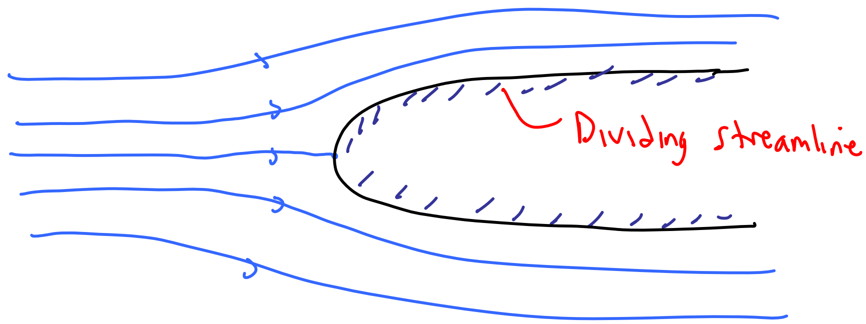
[This eq. is automatically a solution because of superposition]

What is it? What flow does this model?

Sketch streamlines:

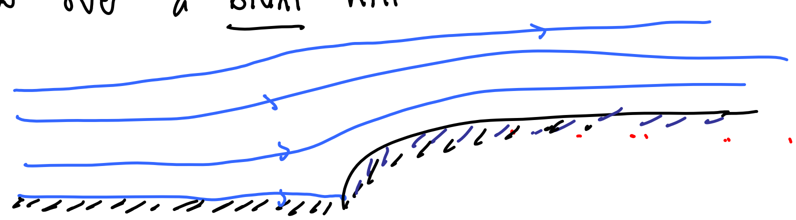


We can use this soln as an approximation of flow over a "half-body"

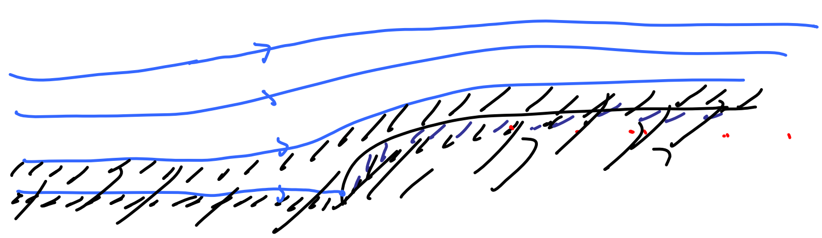


We can think of any streamline in the flow as a wall
[Here I chose the dividing streamline]

OR Flow over a blunt hill



OR Flow over a smooth hill



☆ We can pick any streamline & think of it as a wall

☆ NOTE However → the no-slip condition is not applicable to these "fake" walls → $V \neq 0$ along our "walls"