Today, we will:

- Do a BL example, boundary layer on a flat plate aligned with the flow

Review: The Boundary Layer Equations

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\nu\frac{\partial^2 u}{\partial y^2}$$

Review: The Boundary Layer Procedure

Step 1: Calculate $U(x)$ (outer flow).

↓

Step 2: Assume a thin boundary layer.

↓

Step 3: Solve boundary layer equations.

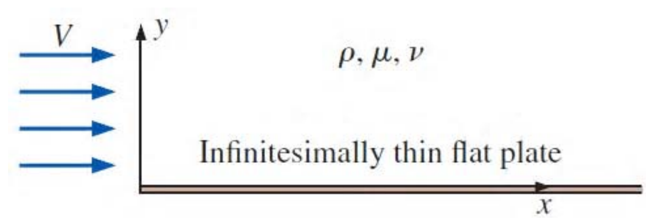
↓

Step 4: Calculate quantities of interest.

↓

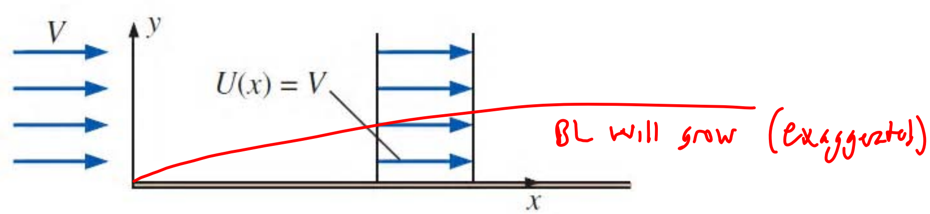
Step 5: Verify that boundary layer is thin.

Example: The Laminar Flat Plate Boundary Layer



We go through the steps of the boundary layer procedure:

- **Step 1**: The outer flow is ***U*(*x*) = *U* = *V* = constant**. In other words, the outer flow is simply a uniform stream of constant velocity.
- **Step 2**: A very thin boundary layer is assumed (so thin that it does not affect the outer flow). In other words, the outer flow does not even know that the boundary layer is there.



- **Step 3**: The boundary layer equations must be solved; they reduce to

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}$$

$\frac{\partial P}{\partial y}=0$

P does not change through the BL

There are four required boundary conditions,

NO SLIP →

$u=0$ at $y=0$ $u=U$ as $y\to\infty$

$v=0$ at $y=0$ $u=U$ for all y at $x=0$

This equation set was first solved by **P. R. H. Blasius** in 1908 – numerically, but *by hand*!

NOTE: For this problem, $\frac{\partial P}{\partial x}=0$ & $\frac{\partial P}{\partial y}=0$ → ∴ $P=$ constant everywhere

Blasius introduced a **similarity variable** η that combines independent variables *x* and *y* into one nondimensional independent variable,

Similarity variable

$$\eta = y\sqrt{\frac{U}{\nu x}}$$

(Nondimensional)

and he solved for a nondimensionalized form of the *x*-component of velocity,

$$f' = \frac{u}{U} = \text{function of } \eta$$

The similarity solution is f' as a function of η.

The key here is that ***one single similarity velocity profile holds for any x-location along the flat plate***. In other words, the velocity profile shape is the same ("similar") at any location, but it is merely *stretched vertically* as the boundary layer grows down the plate. This is illustrated in Fig. 10-98 in the text.

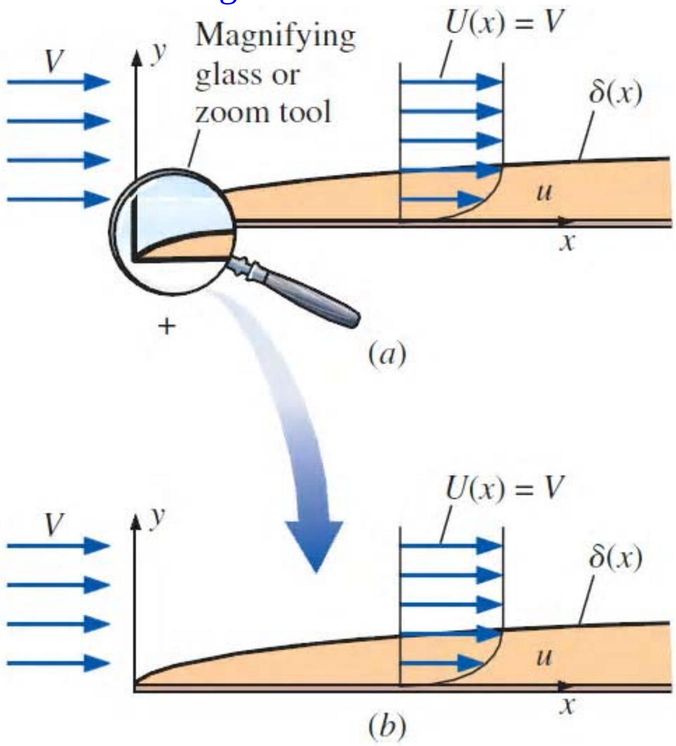
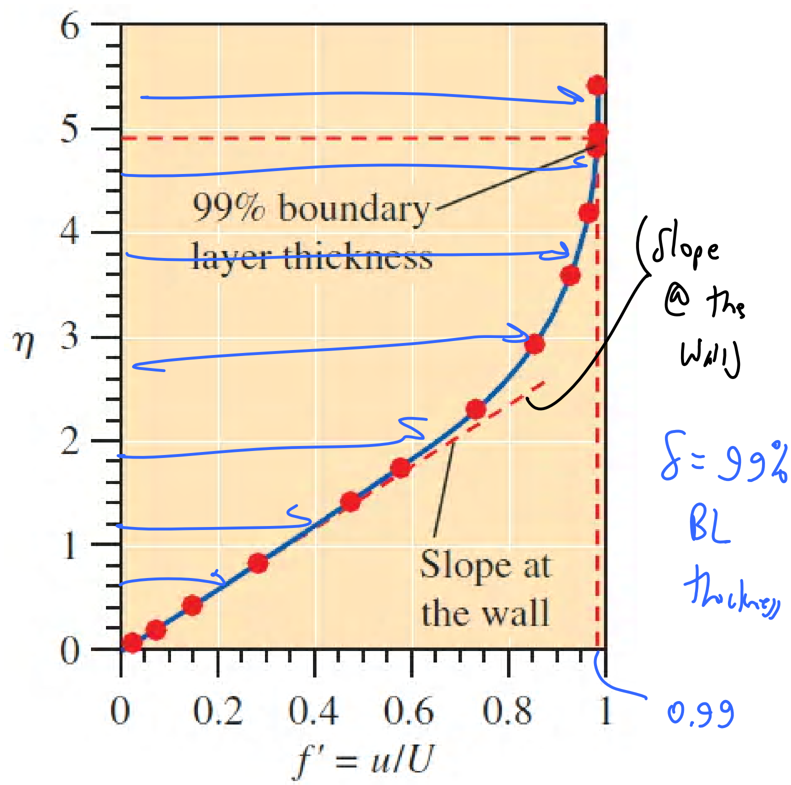


FIGURE 10–98
A useful result of the similarity assumption is that the flow looks the same (is *similar*) regardless of how far we zoom in or out; (*a*) view from a distance, as a person might see, (*b*) close-up view, as an ant might see.

The similarity solution itself is tabulated in Table 10-3, and is plotted in Fig. 10-99.

[See text for details]

FIGURE 10–99
The Blasius profile in similarity variables for the boundary layer growing on a semi-infinite flat plate. Experimental data (circles) are at $Re_x = 3.64 \times 10^5$.
From Panton (1996).



This one velocity profile, plotted in nondimensional form as above, applies at *any x*- location in the boundary layer.

Step 4: Calculate quantities of interest

a. δ → δ = y location where $u/U = 0.99$

$$\boxed{\frac{\delta}{x} = \frac{4.91}{\sqrt{Re_x}}} \qquad \boxed{Re_x = \frac{\rho U x}{\mu} = \frac{Ux}{\nu}}$$

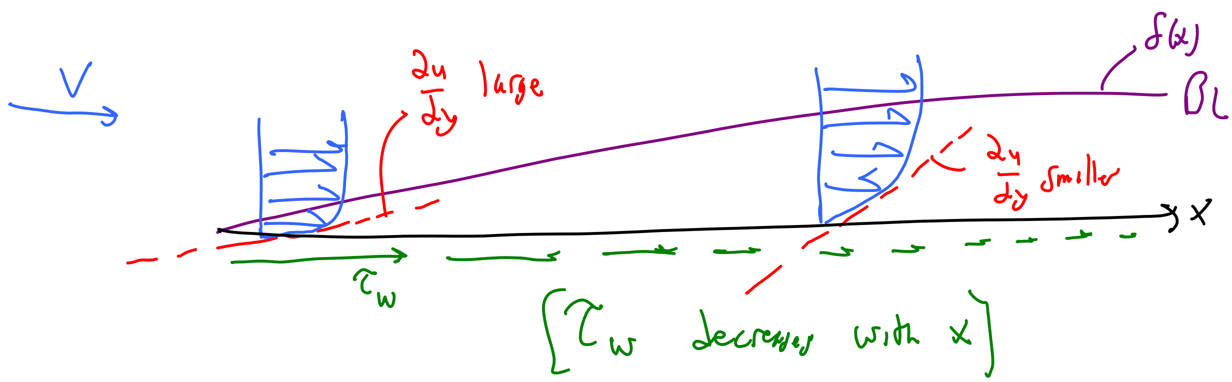
δ = BL thickness

b. τ_w = shear stress @ the wall $\quad \boxed{\tau_w = \mu \left.\frac{du}{dy}\right)_{y=0}}$

See text → $\boxed{C_{f,x} = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.664}{\sqrt{Re_x}}}$

Local skin friction coefficient

$$\boxed{\tau_w = \frac{1}{2}\rho U^2 \left(0.664/\sqrt{Re_x}\right)}$$



c. Total skin friction drag (on one side (top))

$$F_{D_{\text{friction}}} = b\int_0^x \tau_w \, dx = b\int_0^x \frac{\frac{1}{2}\rho U^2 (0.664)}{\sqrt{Re_x}}\, dx$$

b = width into page

algebra

Define a nondimensional $F_{D_{\text{friction}}}$ → let $\boxed{F_{D_{\text{friction}}} = \frac{1}{2}\rho U^2 A C_f}$ ← $b \cdot x$

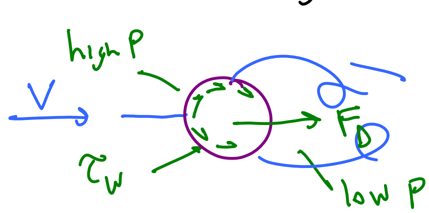
$\boxed{C_f = \text{average or overall skin friction coeff.}}$ (over the whole plate)

Nondimensionally $\boxed{C_f = \frac{1}{x}\int_0^x C_{f,x}\, dx} = \boxed{C_f = \frac{1.33}{\sqrt{Re_x}}}$

Intro to Ch. 11: The aerodynamic drag on a body is composed of

- Skin friction drag (due to τ_w)
- Pressure drag (due to pressure variation along the body)

e.g. cylinder:



- skin friction drag
- pressure drag

$$F_D = F_{D_{\text{friction}}} + F_{D_{\text{pressure}}}$$

$$\boxed{C_D = \text{drag coeff.} \equiv \frac{F_D}{\frac{1}{2}\rho V^2 A}}$$

characteristic area, usually
1) frontal area (looking at the body from far upstream)
2 planform area (looking from the top)

☆ More appropriate for our flat plate problem

Here, flat plate BL → we have NO pressure drag

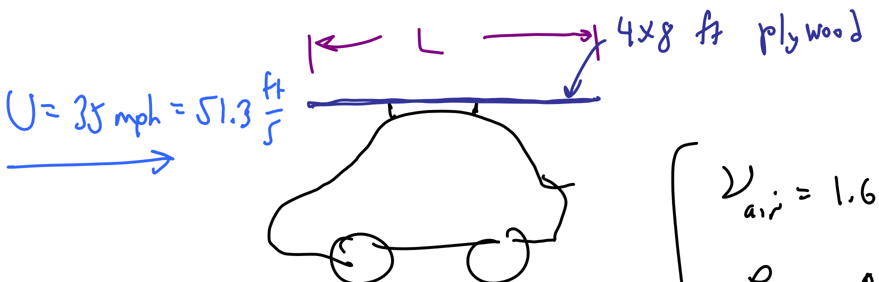
P = const everywhere

We have only skin friction drag

$\therefore\ C_D = C_f$ for a flat plate

[if there are 2 sides of the plate (top & bottom), $C_D = 2C_f$]

Example:



$$\left[\ \nu_{air} = 1.632\times10^{-4}\ \frac{ft^2}{s} \qquad \rho_{air} = 0.07518\ {lbm}/{ft^3}\ \right]$$

Calculate δ & F_D on the plate @ $x = L = 8$ ft

Soln: @ $x = L$, $$Re_L = \frac{\rho U L}{\mu} = \frac{UL}{\nu} = \frac{(51.3\ ft/s)(8\ ft)}{1.632\times10^{-4}\ ft^2/s} = 2.515\times10^6$$

[A flat plate BL is typ. turbulent for $Re_x \gtrsim 3\times10^6$]

Assuming a laminar BL (Blasius soln),

@ $x=L$, $\delta = \frac{4.91}{\sqrt{Re_L}} \cdot L = 0.297$ inches $\approx$ 0.30 inches

Compare to 8 ft

If δ small compared to L $\delta \ll L$

Step 5 → verify $\delta \ll L$ ✓

$F_D = \left(\frac{1}{2}\rho U^2 C_f A\right)(2)$ — 2 sides (top & bottom)

$C_f = \frac{1.33}{\sqrt{Re_L}}$

use planform area
$A = 4 \times 8\ ft^2$

$F_D = 0.165$ lbf → $\boxed{0.17 \text{ lbf} = F_D}$

This is small ← WHY IS F_D SO SMALL?

In real life → • plate will sag & not be perfectly straight & level
↳ leads to pressure drag

• The flow may separate → "

• The BL may actually be turbulent (since Re_L is big)

In real life, F_D will be much larger than our estimate

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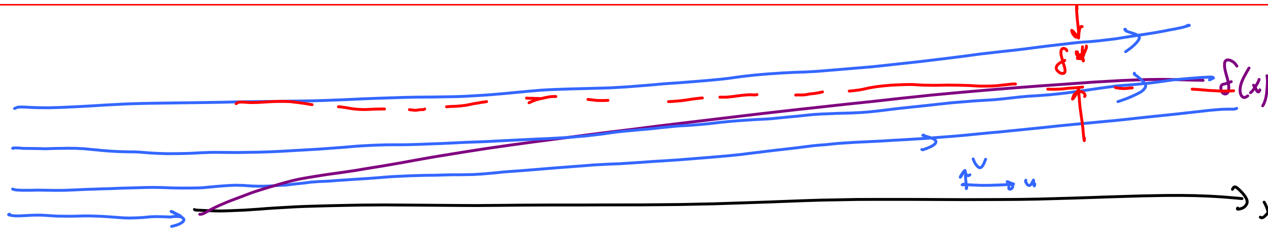
d. Displacement thickness δ^*

★ $\boxed{\delta^* = \text{distance that a streamline just outside the BL is deflected away from the wall due to the growing BL}}$



$v \ll u$, but $v \neq 0$, $v > 0$

$\delta^* = \delta^*(x)$

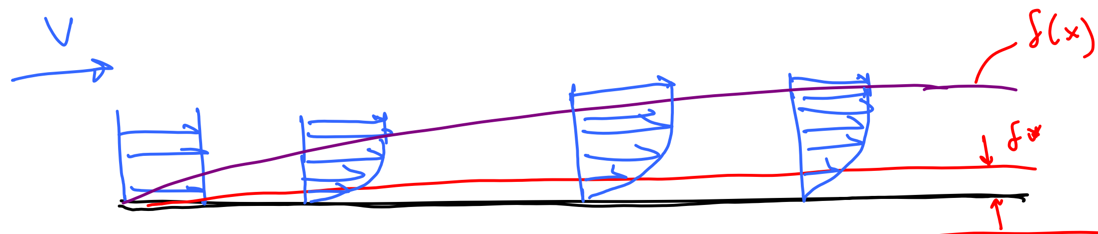
$$\boxed{\frac{\delta^*}{x} = \frac{1.72}{\sqrt{Re_x}}} \quad \left[= \frac{\delta}{3} \right]$$

AFTER CLASS.... SOME ADDITIONAL COMMENTS ABOUT δ^*:

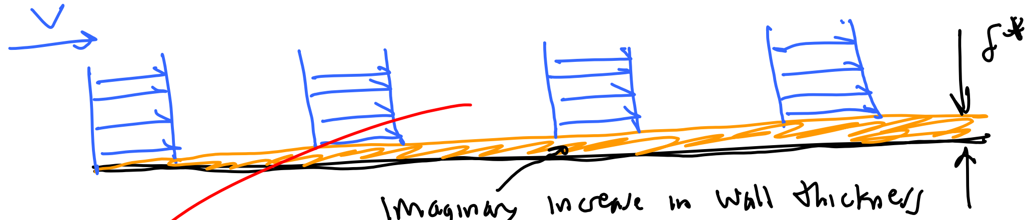
ALTERNATE DEF'N OF δ^*:

* $\boxed{\text{$\delta^*$ is the imaginary increase in wall thickness seen by the outer flow, due to the BL}}$ A

ACTUAL FLOW



IMAGINARY FLOW TAKING INTO ACCOUNT δ^*



SEE EXAMPLE 10-11 ☆

This is what the outer flow "sees" or "feels" - it feels like the flat plate is <u>thicker</u>, due to the BL.