Today, we will:

- Discuss the ***turbulent*** boundary layer on a flat plate, and compare with laminar flow
- Talk about boundary layers with ***pressure gradients***

F. The Boundary Layer Approximation (continued)

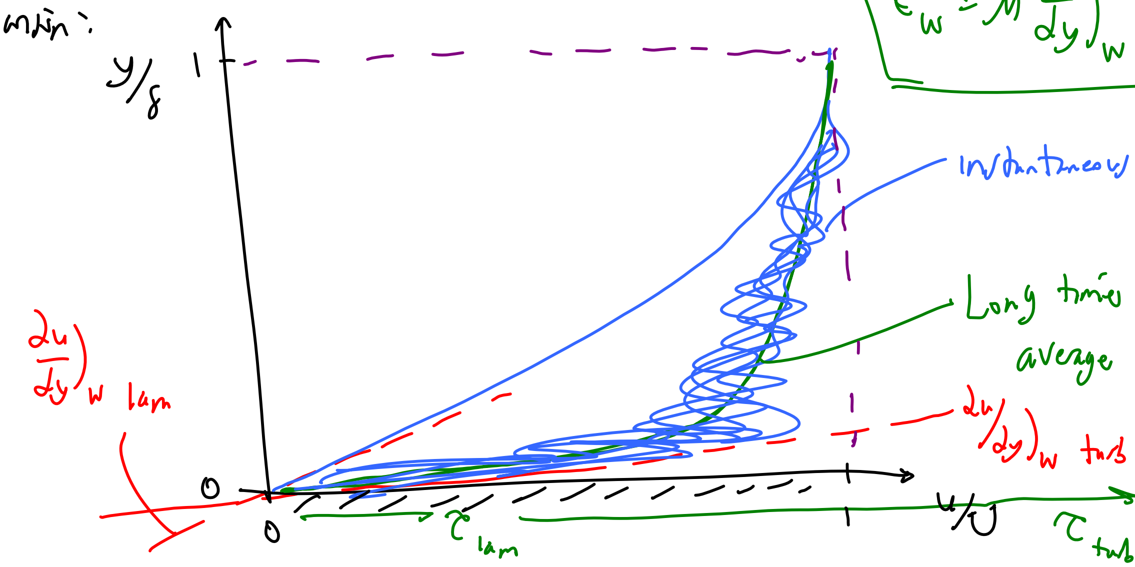
6. Turbulent BL on a Flat Plate

- We talked about laminar BLs
- Turbulent BLs are more difficult because there are no solutions

At high enough Re (Re_x), the BL will go turbulent.

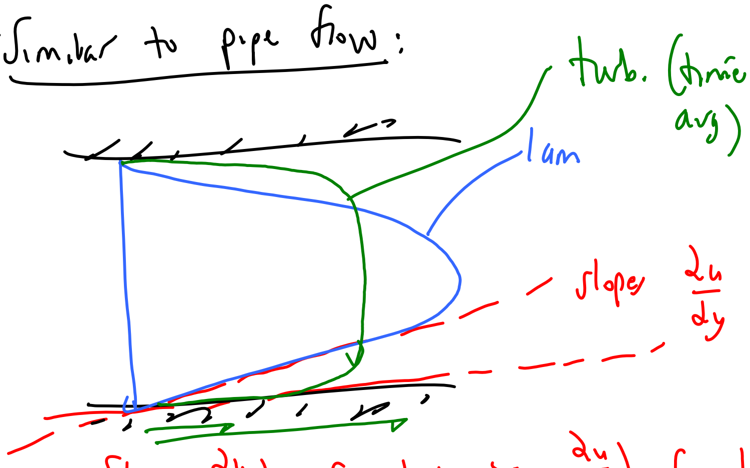
$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right)_w$$

Comparison:



Turb. BL is more "full" than a laminar BL under the same conditions

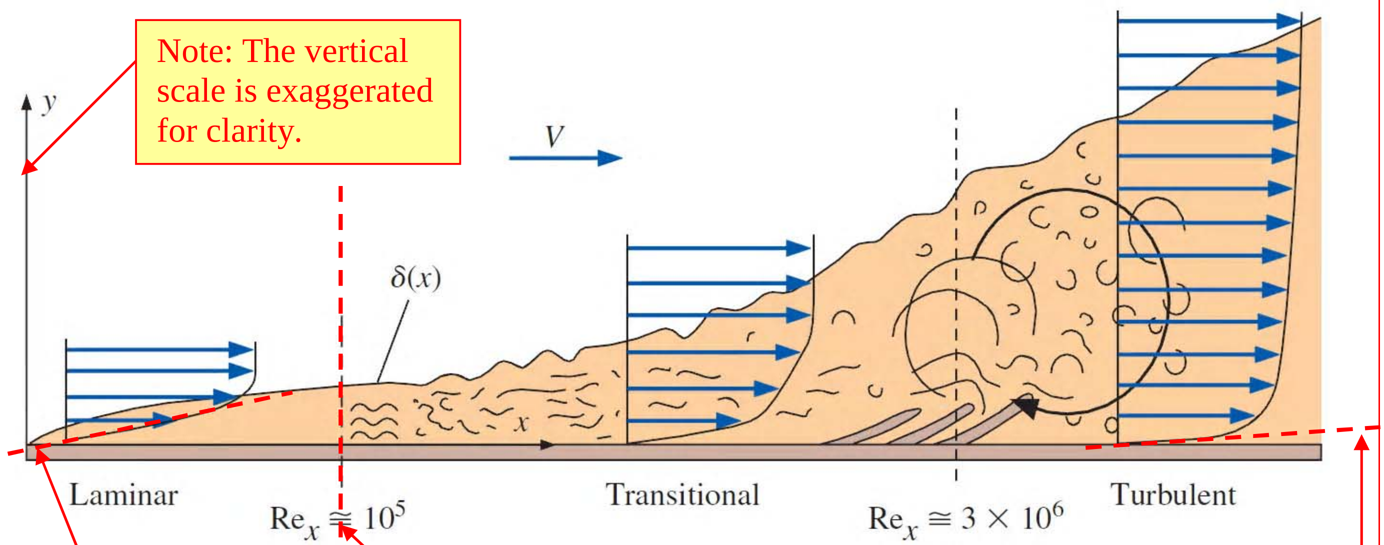
- Similar to pipe flow:



Slope $\left.\frac{\partial u}{\partial y}\right)_w$ for turb. >> $\left.\frac{\partial u}{\partial y}\right)_w$ for laminar

Turb. BL has larger shear stress than a laminar BL under the same conditions (same Re_x)

The Turbulent Flat Plate Boundary Layer (Section 10-6, Çengel and Cimbala)



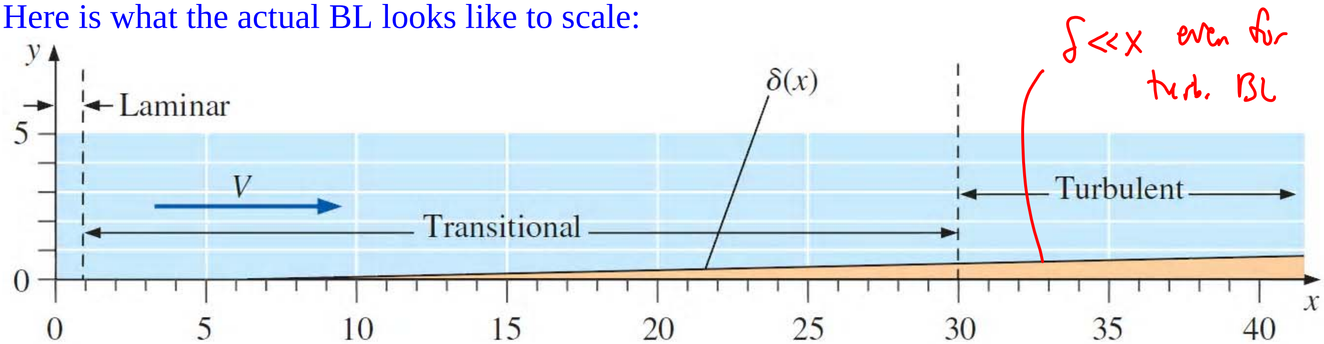
$\tau_w = \mu\partial u/\partial y$ at $y = 0$

$\mathrm{Re}_{x,\,cr} = 5 \times 10^5$

We define the engineering critical Reynolds number:

- $\mathrm{Re}_x < \mathrm{Re}_{x,\,cr}$ BL most likely laminar
- $\mathrm{Re}_x > \mathrm{Re}_{x,\,cr}$ BL most likely turbulent

Here is what the actual BL looks like to scale:



The turbulent flat plate boundary layer velocity profile:

The time-averaged turbulent flat plate (zero pressure gradient) boundary layer velocity profile is much *fuller* than the laminar flat plate boundary layer profile, and therefore has a larger slope $\partial u/\partial y$ at the wall, leading to greater skin friction drag along the wall.

There are three common empirical relationships for the turbulent flat plate boundary layer velocity profile:

- **The log law**:

The log law:
$$\frac{u}{u_*} = \frac{1}{\kappa}\ln\frac{yu_*}{\nu} + B \qquad \textbf{(10–83)}$$

where

Friction velocity:
$$u_* = \sqrt{\frac{\tau_w}{\rho}} \qquad \textbf{(10–84)}$$

- **Spalding's law of the wall**:

$$\frac{yu_*}{\nu} = \frac{u}{u_*} + e^{-\kappa B}\left[e^{\kappa(u/u_*)} - 1 - \kappa(u/u_*) - \frac{[\kappa(u/u_*)]^2}{2} - \frac{[\kappa\,(u/u_*)]^3}{6}\right] \quad \textbf{(10–85)}$$

- **The one-seventh-power law**:

$$\boxed{\frac{u}{U} \cong \left(\frac{y}{\delta}\right)^{1/7}} \text{ for } y \le \delta, \quad \rightarrow \quad \frac{u}{U} \cong 1 \quad \text{for } y > \delta \qquad \textbf{(10–82)}$$

Quantities of interest for the turbulent flat plate boundary layer:

Just as we did for the laminar (Blasius) flat plate boundary layer, we can use these expressions for the velocity profile to estimate quantities of interest, such as the 99% boundary layer thickness δ, the displacement thickness δ^*, the local skin friction coefficient $C_{f,x}$, etc. These are summarized in Table 10-4 in the text.

Column (b) expressions are generally preferred for engineering analysis.

TABLE 10–4

Summary of expressions for laminar and turbulent boundary layers on a smooth flat plate aligned parallel to a uniform stream*

Property	Laminar	(a) Turbulent(†)	(b) Turbulent(‡)
Boundary layer thickness	$\frac{\delta}{x} = \frac{4.91}{\sqrt{\mathrm{Re}_x}}$	$\frac{\delta}{x} \cong \frac{0.16}{(\mathrm{Re}_x)^{1/7}}$	$\frac{\delta}{x} \cong \frac{0.38}{(\mathrm{Re}_x)^{1/5}}$
Displacement thickness	$\frac{\delta^*}{x} = \frac{1.72}{\sqrt{\mathrm{Re}_x}}$	$\frac{\delta^*}{x} \cong \frac{0.020}{(\mathrm{Re}_x)^{1/7}}$	$\frac{\delta^*}{x} \cong \frac{0.048}{(\mathrm{Re}_x)^{1/5}}$
Momentum thickness	$\frac{\theta}{x} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$	$\frac{\theta}{x} \cong \frac{0.016}{(\mathrm{Re}_x)^{1/7}}$	$\frac{\theta}{x} \cong \frac{0.037}{(\mathrm{Re}_x)^{1/5}}$
Local skin friction coefficient	$C_{f,x} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$	$C_{f,x} \cong \frac{0.027}{(\mathrm{Re}_x)^{1/7}}$	$C_{f,x} \cong \frac{0.059}{(\mathrm{Re}_x)^{1/5}}$

Note that $C_{f,x}$ is the *local* skin friction coefficient, applied at only *one* value of x.

To these we add the integrated ***average skin friction coefficients*** for *one side* of a flat plate of length L, noting that C_f applies to the entire plate from $x = 0$ to $x = L$ (see Chapter 11):

Laminar: $$C_f = \frac{1.33}{\mathrm{Re}_L^{1/2}} \qquad \mathrm{Re}_L \lesssim 5 \times 10^5 \qquad \textbf{(11–19)}$$

Turbulent: $$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} \qquad 5 \times 10^5 \lesssim \mathrm{Re}_L \lesssim 10^7 \qquad \textbf{(11–20)}$$

C_f, δ, δ^*, etc. are all larger for turb. BL than for a laminar BL

For cases in which the laminar portion of the plate is taken into consideration, we use:

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L} \qquad 5 \times 10^5 \lesssim \mathrm{Re}_L \lesssim 10^7 \tag{11–22}$$

Turbulent flat plate boundary layers with wall roughness:

Finally, all of the above are for *smooth* flat plates. However, if the plate is *rough*, the average skin friction coefficient C_f increases with roughness ε. This is similar to the situation in pipe flows, in which Darcy friction factor f increases with pipe wall roughness.

Like a "moody chart" for B.L's

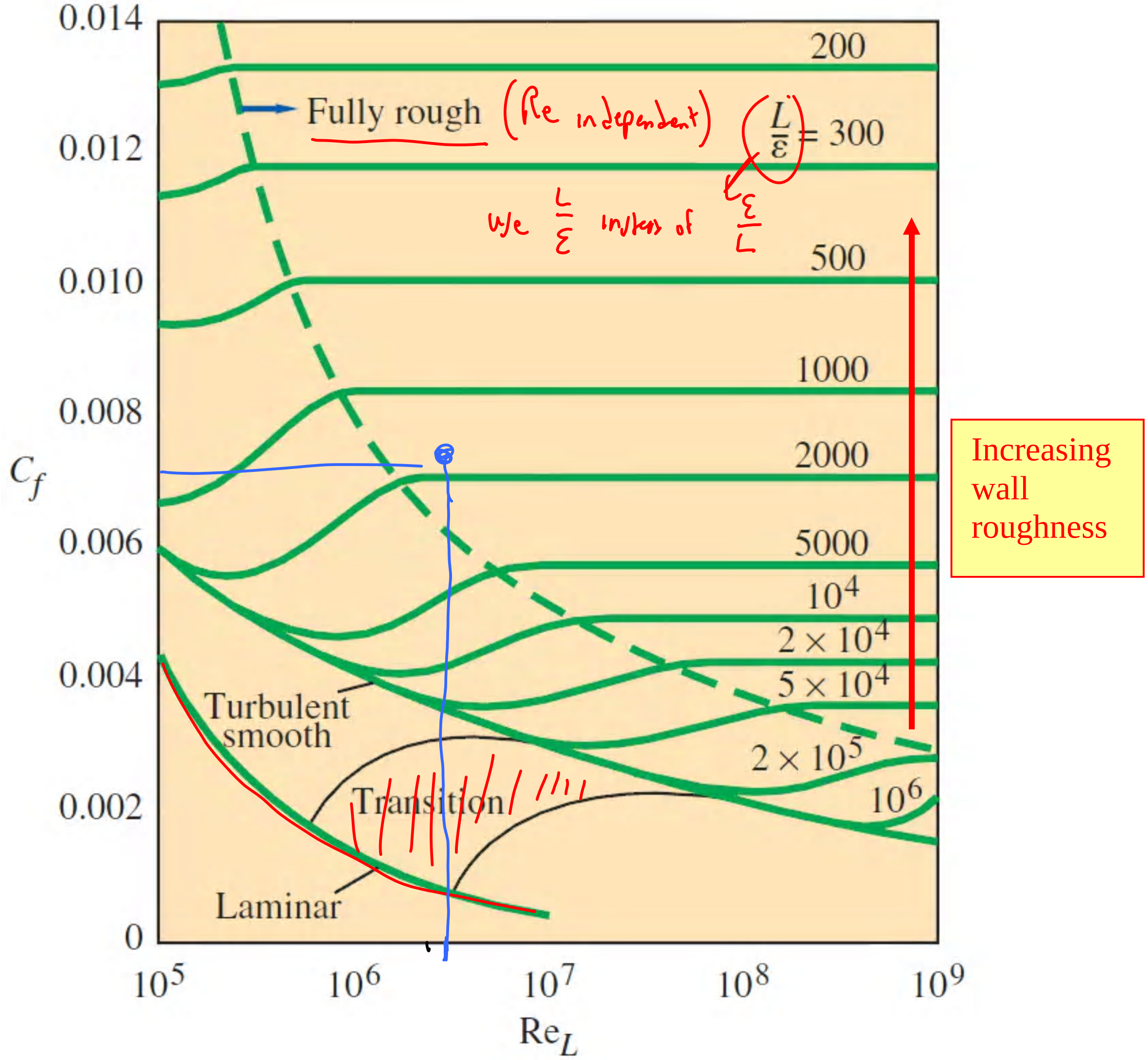


FIGURE 11–31

Friction coefficient for parallel flow over smooth and rough flat plates.

Just as with pipe flows, at high enough Reynolds numbers, the boundary layer becomes "fully rough". For a ***fully rough flat plate turbulent boundary layer*** with average wall roughness height ε,

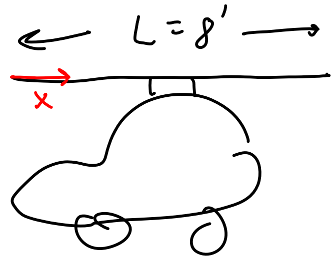
Fully rough turbulent regime:

$$C_f = \left(1.89 - 1.62 \log \frac{\varepsilon}{L}\right)^{-2.5} \tag{11–23}$$

This equation represents the flat portions of Fig. 11-31 that are labeled "Fully rough".

Example

$V = 51.3\ \text{ft/s}$



(Same prob. as last lecture except now assume the BL is turbulent)

$$@\, x = L \quad Re_x = \frac{UL}{\nu} = \underline{\underline{2.515 \times 10^6}}$$

[recall, if laminar,

$\delta = 0.297$ inches @ $x = L$

$F_D = \underline{\underline{0.17}}$ lbf (both sides)]

Here, if BL is turbulent & smooth ($\varepsilon = 0$)

$$\frac{\delta}{x} = \frac{0.38}{(Re_x)^{1/5}} \rightarrow \delta = 0.1595\ \text{ft} \rightarrow \boxed{\delta \approx 1.9 \text{ inches}}$$

turb BL is 6.5 x thicker than lam. BL @ same Re_x

is $\delta << L$? ✓ BL is still "thin"

- Calc. F_D on plate

$$F_D = 2\ \frac{1}{2} \rho V^2 C_f A$$

(top & bot)

$A = 4' \times 8'$

From Table, $C_f = \dfrac{0.074}{Re_L^{1/5}}$

numbers $\rightarrow$ $C_f = 0.003883$

$$\boxed{F_D = 0.764 \approx \underline{0.76} \text{ lbf}}$$

(4.5 x greater than laminar case)

- Real F_D may lie somewhere between lam. & turb value <u>if</u> the plate stays perfectly flat.
- Actual value of F_D will be larger than this

· Re-do if plate is rough with $\varepsilon = 0.05$ in

$$\frac{L}{\varepsilon} = \frac{96''}{0.05''} = 1920$$

@ $\frac{L}{\varepsilon} = 1920$ & $Re_L = 2.5 \times 10^6$, $C_f = \approx 0.007$

$$F_D = 1.377 \approx \boxed{1.4 \text{ lbf} = F_D}$$

($\approx$ 2x as smooth case)

---

7. BL's with pressure gradient

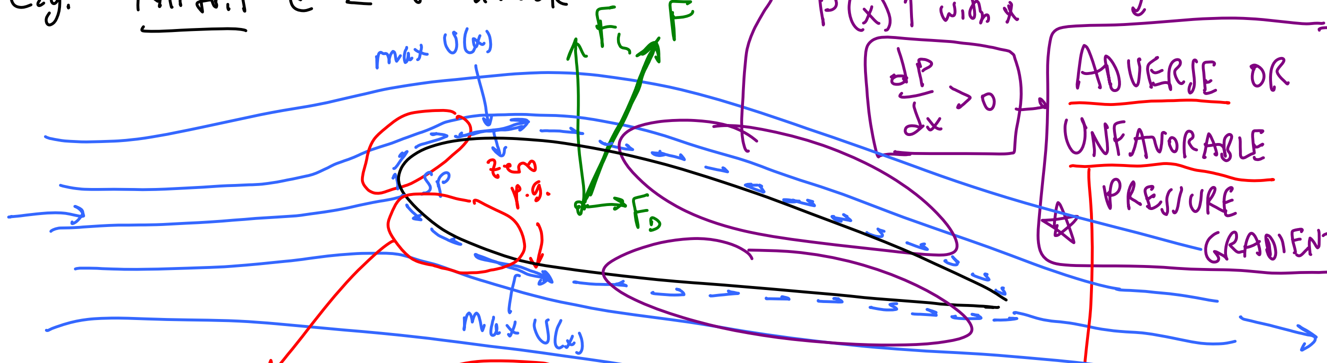
a. Definition → for a flat plate BL: $U(x) = U = V = \text{const.}$, $P(x) = P = \text{const everywhere}$

☆ Zero pressure gradient BL

In real life, most BL's have non-zero pressure gradients

i.e., $P(x) \neq$ constant P changes with x

e.g. Airfoil @ ∠ of attack



$U(x) \downarrow$ with x
$P(x) \uparrow$ with x

Positive pressure gradient

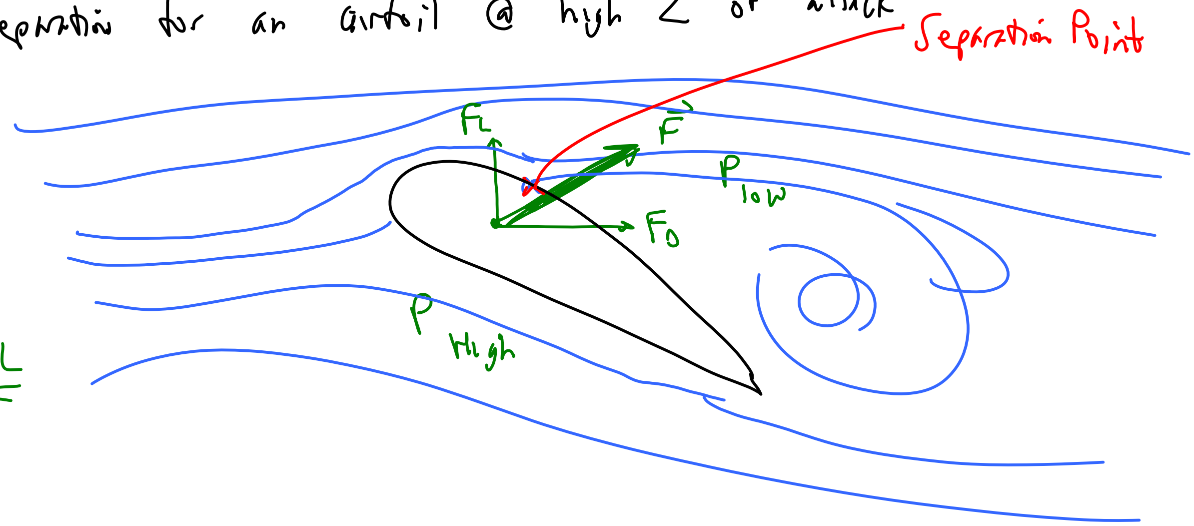
$\boxed{\frac{dP}{dx} > 0}$ → ☆ ADVERSE OR UNFAVORABLE PRESSURE GRADIENT

→ ☆ BOUNDARY LAYER MORE LIKELY TO SEPARATE

$U(x) \uparrow$ with x
∴ $P(x) \downarrow$ with x → $\boxed{\frac{dP}{dx} < 0}$ negative pressure gradient (accelerating flow)

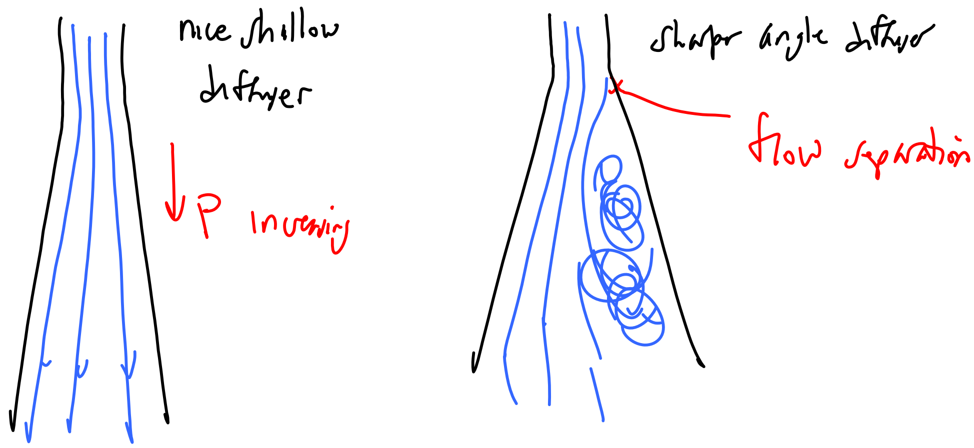
☆ FAVORABLE PRESSURE GRADIENT

BL separation for an airfoil @ high ∠ of attack

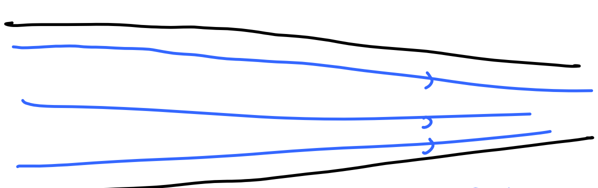


STALL

Example → a diffuser

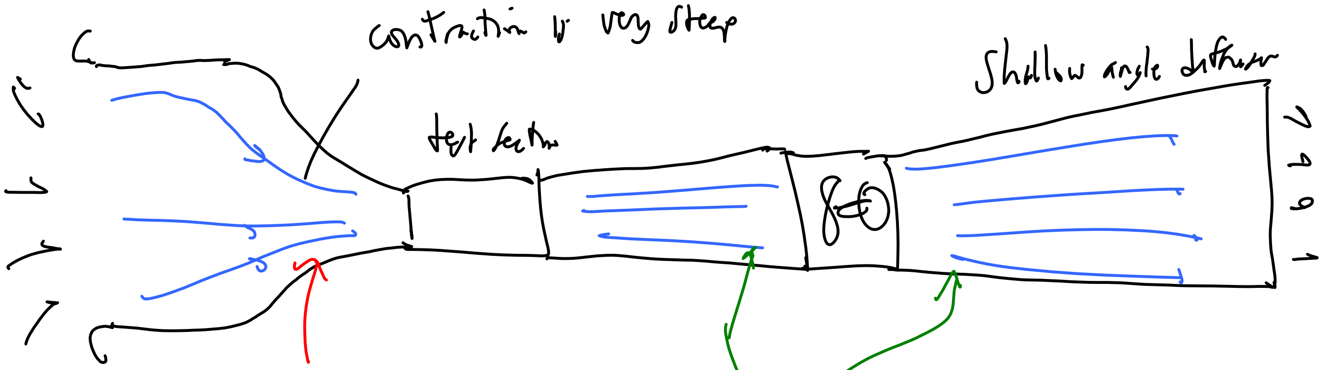


Contractions have a favorable pressure gradient (P decreasing)
so they are much less likely to separate



P(x) is decreasing

My wind tunnel in fluids lab:



favorable pressure gradient

U ↑, P ↓

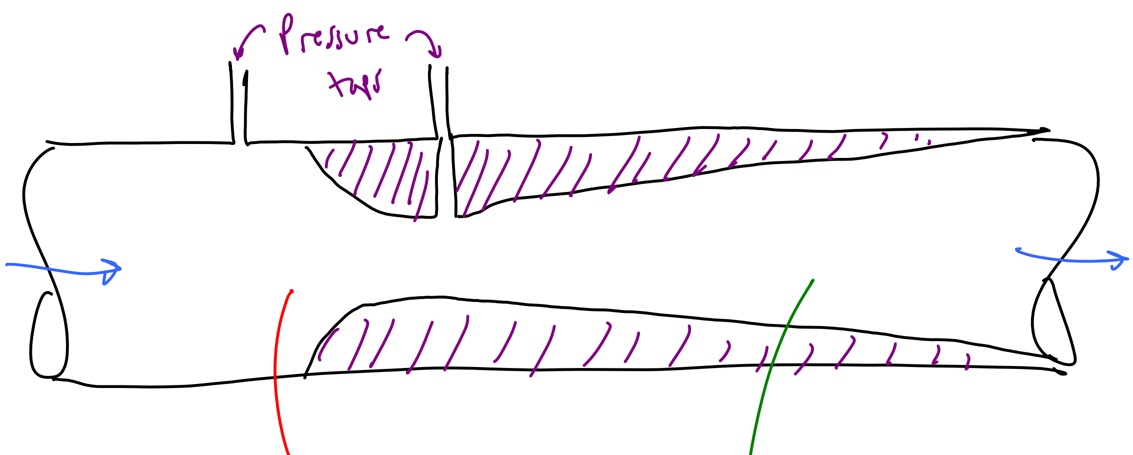
∴ can make it short (it will not separate)

Adverse pressure gradient

U ↓, P ↑

∴ must make the diffuser sections long to avoid separation

Similar for Venturi meters (Ch. 8):



Sharp contraction (favorable p.g.)

long diffuser (adverse p.g.)