Today, we will:

- Start Chapter 12 – Compressible Flow
- Discuss isentropic compressible flow, converging-diverging ducts

X. INTRODUCTION TO COMPRESSIBLE FLOW (Chapter 12)

A. Introduction

- Everything so far has been incompressible ($\rho \approx$ constant)
  - liquids
  - gases @ low Mach #'s
- Compressibility effects cause some new phenomena.

1. Definitions & Review

a. Mach # $\boxed{Ma = \frac{V}{c}}$ ☆ most important parameter

If $Ma \lesssim 0.3$ incompressible approx. is reasonable ($\approx 5\%$ error)

In air @ room temp. $c \approx 1100$ ft/s $\approx 750 \frac{mi}{hr} \approx 335$ m/s

@ 100 mph → $Ma \approx 0.133$

@ 600 mph → $Ma \approx 0.8$

Flow regimes:

$Ma \lesssim 0.3$ incompressible

$0.3 \lesssim Ma < 1.0$ subsonic

$Ma > 1.0$ supersonic

Ma near 1.0 transonic

$Ma = 1.0 \Rightarrow$ sonic

$Ma \gtrsim 3$ → hypersonic

b. Thermodynamic relationships: Ideal gases

Use absolute temp!

$\boxed{P = \rho R T}$ $\boxed{k = \frac{c_p}{c_v}}$ (some books use γ instead of k)

$\boxed{R = c_p - c_v}$ $\boxed{R = \text{specific gas constant} = \frac{R_u}{M}}$ $\boxed{c = \sqrt{kRT}}$

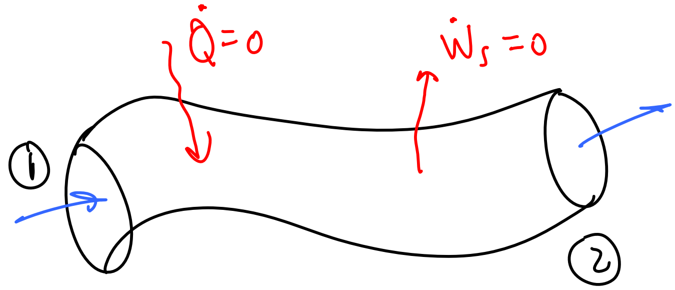
For isentropic flow (neglect friction & other losses)

From ① to ②

$$\boxed{\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} \qquad \frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{k-1}}}$$

Energy eq.

duct



For isentropic, adiabatic, no work (neglecting gravity)

$$\underbrace{h_1 + \frac{1}{2}V_1^2}_{h_{0_1}} = \underbrace{h_2 + \frac{1}{2}V_2^2}_{h_{0_2}} \qquad \boxed{h_{0_1} = h_{0_2}} \ ☆$$

h_{0_1} stagnation enthalpy

• Stagnation temperature ☆ $\boxed{T_0 \equiv \text{The temp. you get when you bring the flow to rest isentropically}}$

Ideal gas → $h = C_p T + \text{const}$ → $h_0 = C_p T_0 + \text{const}$

in a duct like this, $\boxed{T_0 = \text{constant}}$ $(T_{0_2} = T_{0_1})$

energy eq. $\quad h_0 = h + \frac{1}{2}V^2 = \text{const}$

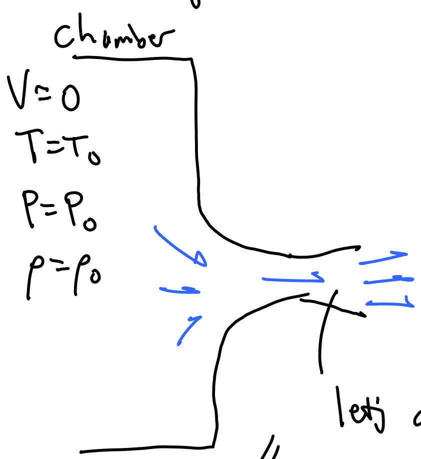
$$C_p T_0 = C_p T + \frac{1}{2}V^2 = \text{const} \qquad \left[C_p = \frac{kR}{k-1},\ c = \sqrt{kRT}\right]$$

$\therefore$

$$☆\ \boxed{\frac{T_0}{T} = 1 + \frac{k-1}{2}(M_a)^2} \qquad (3)$$

2. 1-D Isentropic Adiabatic Flow in Ducts

a. Eqs.



P_b = back pressure (in the downstream volume)

See Table A-13

let's calculate T, P, ρ, Ma, etc. in the duct

ISENTROPIC RELATIONSHIPS for ideal gas (adiabatic)

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}(Ma)^2$$

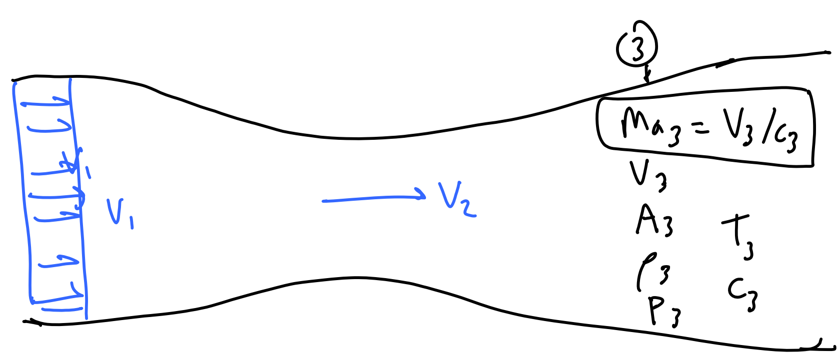
$$\frac{c_0}{c} = \left(\frac{T_0}{T}\right)^{\frac{1}{2}} \qquad \frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\frac{k}{k-1}} \qquad \frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{k-1}}$$

For a given Ma, we can calculate all these values
& k [$k = 1.4$ for air]

How do we calc. Ma?

Cons. of mass

b. Isentropic duct flow with area changes



Approx:
1) 1-D (uniform)
2) steady
3) isentropic
4) adiabatic
5) ideal gas

Cons. of mass:

$$\boxed{\dot{m} = \rho V A = \text{constant}}$$

algebra (see text)

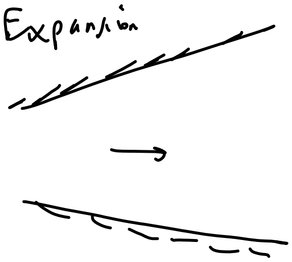
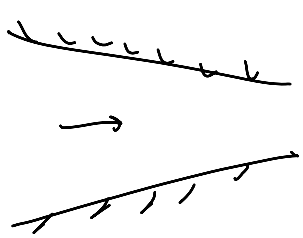
$$\boxed{\frac{dV}{V} = \frac{1}{Ma^2 - 1}\frac{dA}{A}} \quad (4)$$

- if $Ma < 1$ (subsonic) $Ma^2 - 1$ is negative $\rightarrow \frac{dV}{V} \propto -\frac{dA}{A}$

  as $A\uparrow$ (diffuser), $V\downarrow$ } same as
  as $A\downarrow$ (nozzle), $V\uparrow$ } Incomp. flow

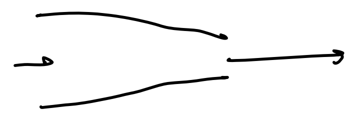
- if $Ma > 1$ (supersonic) $Ma^2 - 1$ is +ve $\rightarrow \frac{dV}{V} \propto \frac{dA}{A}$

  as $A\uparrow$ $V\uparrow$ } opposite of
  as $A\downarrow$ $V\downarrow$ } our intuition

COMPARISON	SUBSONIC	SUPERSONIC
Expansion 	(diffuser) $A\uparrow$ $Ma\downarrow$ $V\downarrow$ $\rho\uparrow$ $P\uparrow$	(nozzle) $A\uparrow$ $Ma\uparrow$ $V\uparrow$ $\rho\downarrow$ $P\downarrow$
Contraction 	**SUBSONIC** $A\downarrow$ $Ma\uparrow$ $V\uparrow$ $\rho\downarrow$ $P\downarrow$ (nozzle)	**SUPERSONIC** $A\downarrow$ $Ma\downarrow$ $V\downarrow$ $\rho\uparrow$ $P\uparrow$ (diffuser)

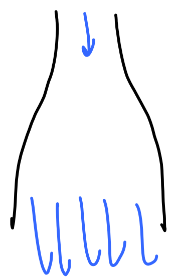
Nozzles:

Subsonic



$V\uparrow$ as $A\downarrow$

Supersonic



rocket engine

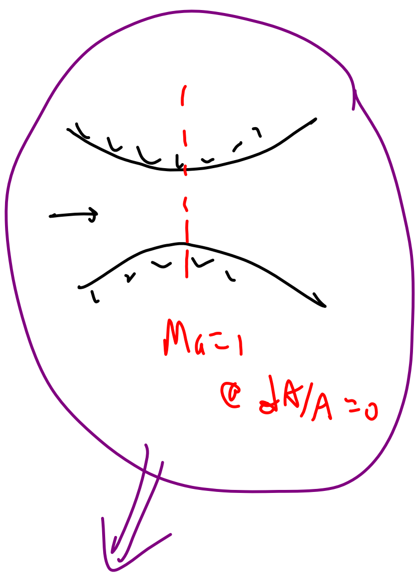
$V\uparrow$ as $A\uparrow$

What about when $Ma = 1$? (Eq 4 has a zero in denom!)

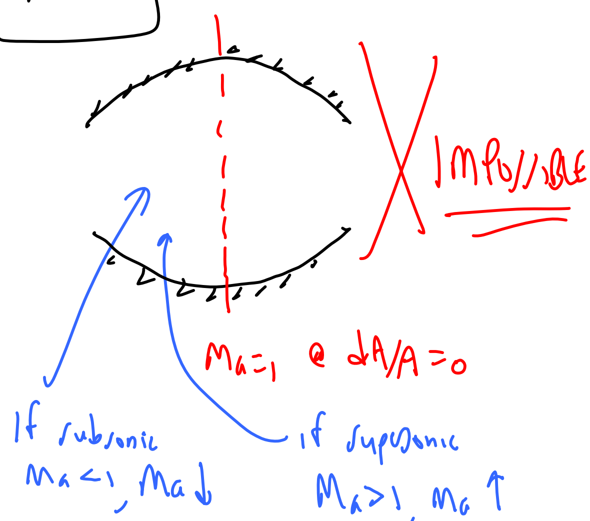
$$\text{Eq (4)} \rightarrow \quad \frac{dA}{A} = \underbrace{(Ma^2 - 1)}_{0} \frac{dV}{V} \quad (= 0 \text{ when } Ma = 1)$$

$$\rightarrow \boxed{\frac{dA}{A} = 0} \quad @\ Ma = 1 \ ☆$$

i.e. either



or



$$\boxed{\text{For 1-D isentropic adiabatic duct flow, sonic conditions } (Ma=1) \text{ can occur only at a throat (minimum area)}}$$ ☆

See eqs in text

Sonic conditions (use * for sonic)

eg $\left[\dfrac{p^*}{p_0} = 0.5283 \text{ for air} \right] \qquad \dfrac{T^*}{T_0} = 0.8333$