M E 320 Professor John M. Cimbala Lecture 42

Today, we will:

- Introduce the concept of ***choking*** in compressible duct flows
- Introduce shock waves, particularly *normal* shock waves

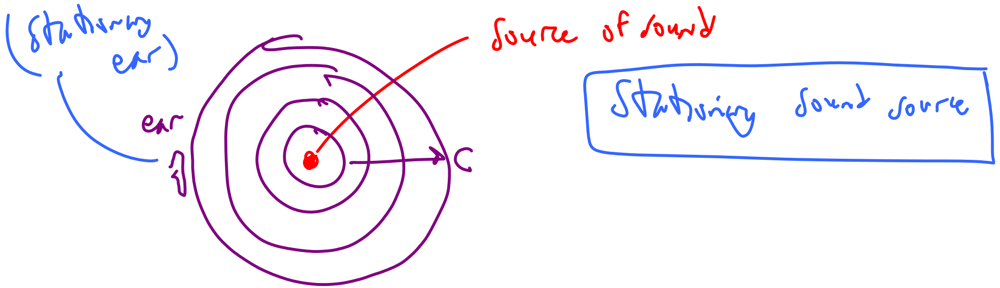
X. Introduction to Compressible Flow (Chapter 12, continued)

A. Introduction

1. Definitions and review (stagnation properties)
2. One-dimensional isentropic adiabatic flow in ducts (converging-diverging nozzles)
3. Choking

Disturbances (pressure waves) travel @ the speed of sound c

No flow:

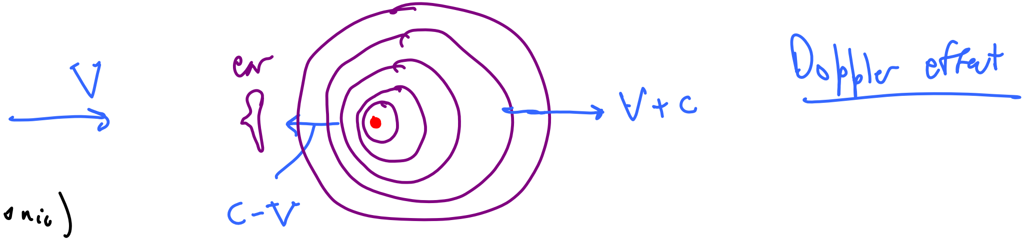


With flow:

a) $V < c$

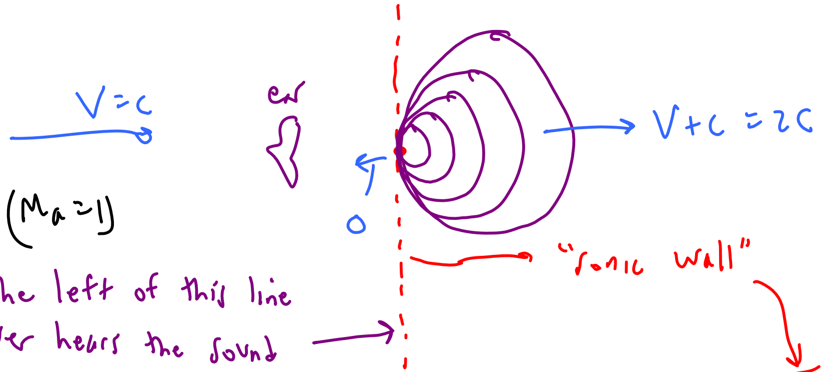
$(Ma < 1)$

(subsonic)



b) $V = c$

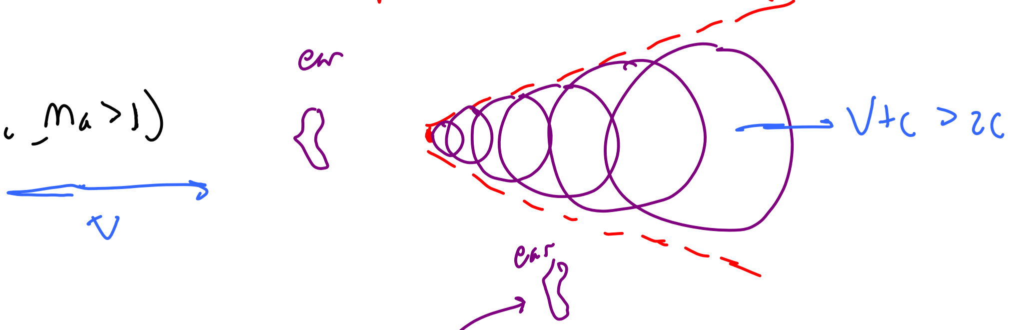
(sonic) $(Ma = 1)$



Any ear to the left of this line never hears the sound

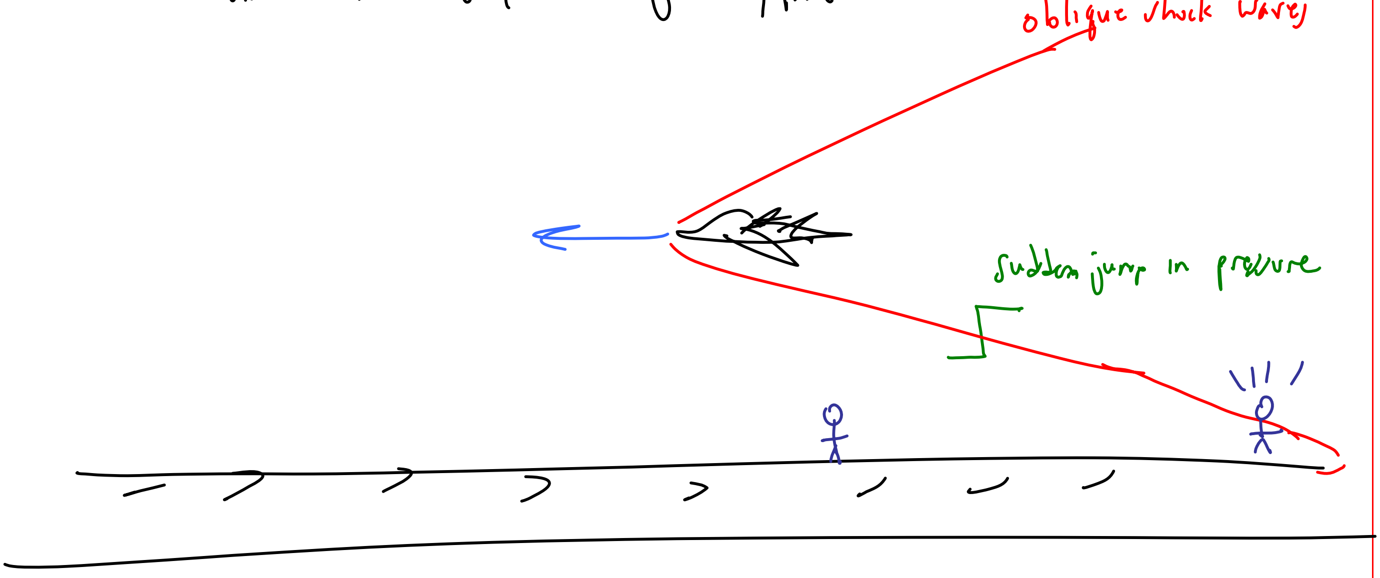
c) $V > c$

(supersonic $Ma > 1$)

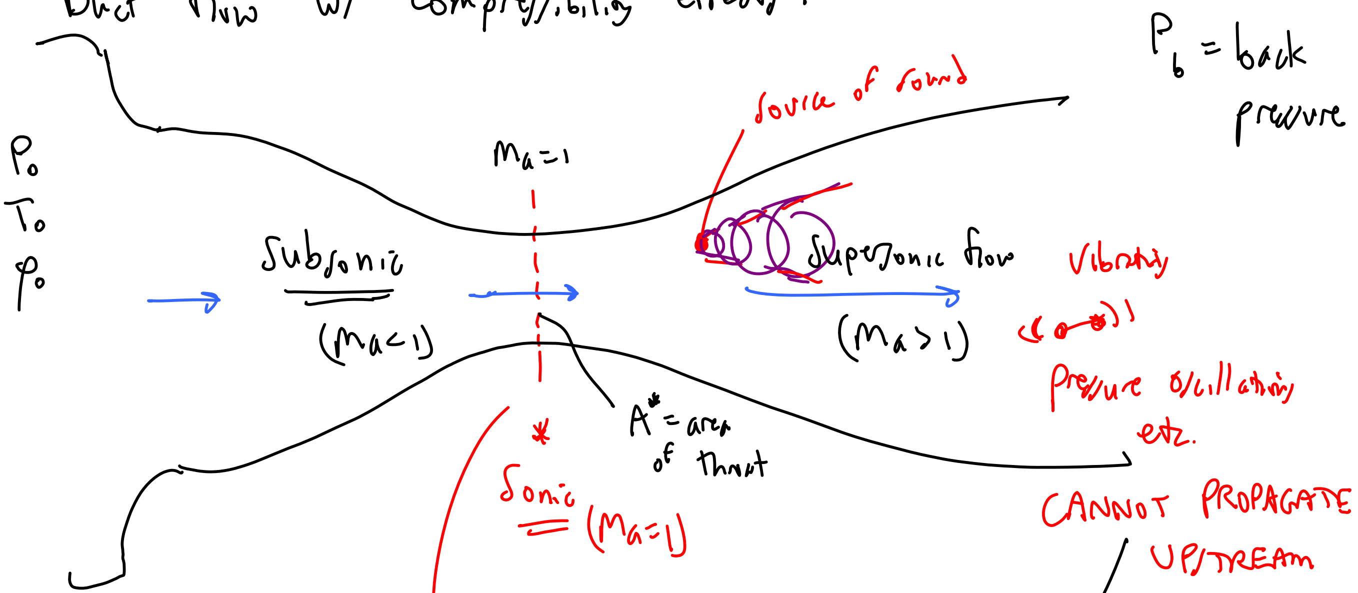


An ear even downstream (off-axis) of the source, as shown, does not hear anything

At an airshow w/ a supersonic jet plane



Duct flow w/ compressibility effects:



SONIC BARRIER

We call this **choking** ☆

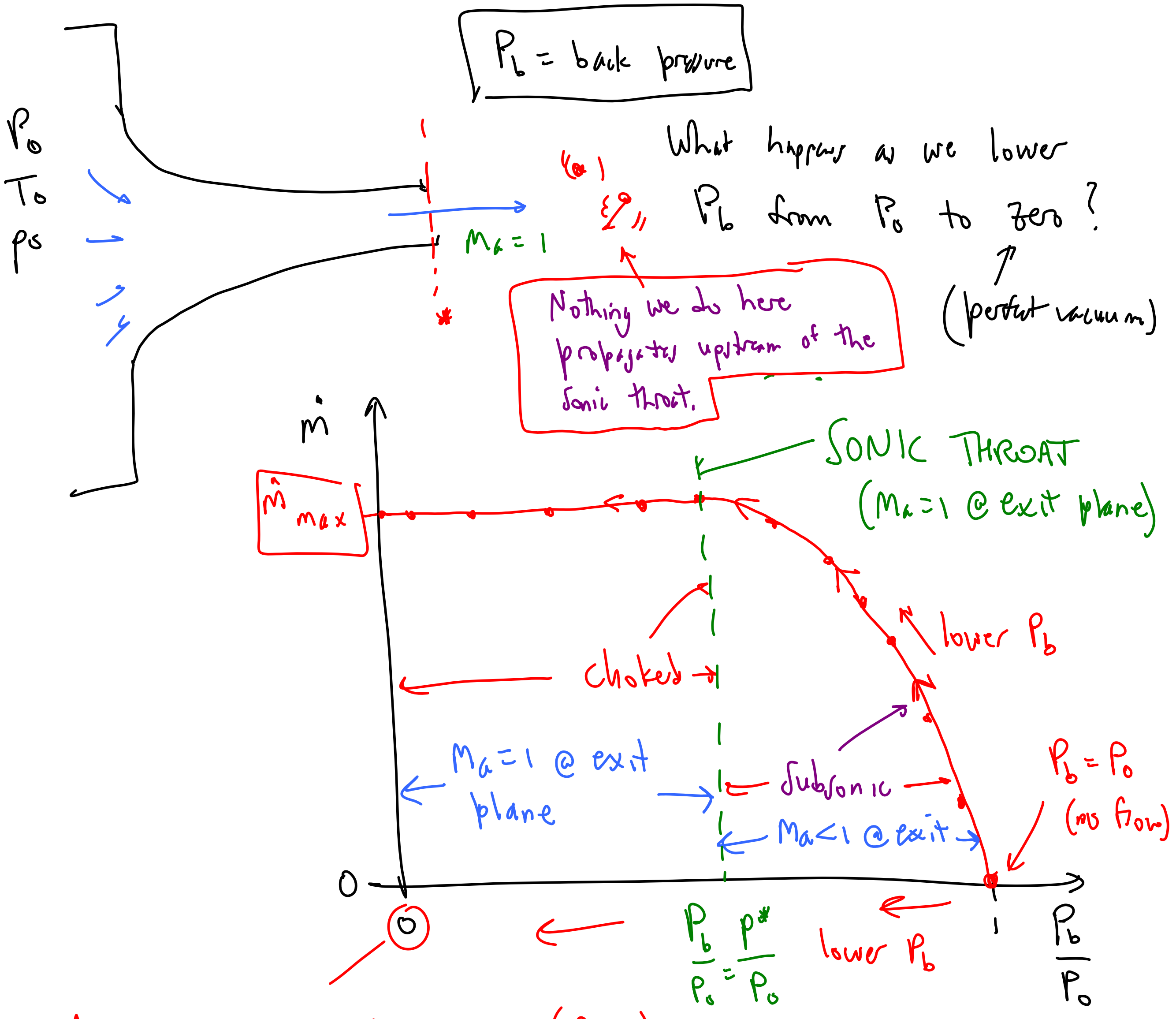
See text for derivation → Maximum mass flow rate $\dot{m}_{max}$ occurs when the flow is choked.

$$\boxed{\dot{m}_{max} = \frac{0.6847\, P_0 A^*}{\sqrt{RT_0}}}$$ ☆

for air $(k = 1.4)$

→ Does not depend on any downstream conditions, as long as back pressure is low enough.

Consider a converging duct only:



What happens as we lower P_b from P_o to zero?

(perfect vacuum)

★ even @ a perfect vacuum ($P_b = 0$), $\dot{m}$ is still $\dot{m}_{max}$.

For air $\frac{p^*}{P_o} = 0.528$

Example → bike tire @ 50 psig   atm pressure = 14.7 psi

$$\frac{P}{P_o} = \frac{14.7}{50 + 14.7} = 0.23 < 0.528$$

∴ Flow out of a hole in a bike tire is sonic (@ speed of sound)

$$\frac{T_o}{T} = 1 + \frac{k-1}{2} Ma^2 \rightarrow @\ Ma = 1,\ \frac{T_o}{T} = 1.2 \rightarrow @\ \overset{T_o =}{300\,K},\ T = \frac{300}{1.2} = 250\,K = \boxed{-23^\circ C = T}$$

B. SHOCK WAVES (Sec 12-4)

Defn → a strong pressure wave

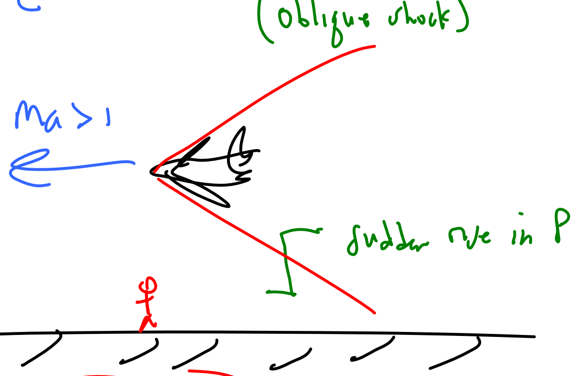
[a sound wave is a very weak shock wave)

SHOCK WAVES → travels @ speed greater than c

sound wave → travels @ c

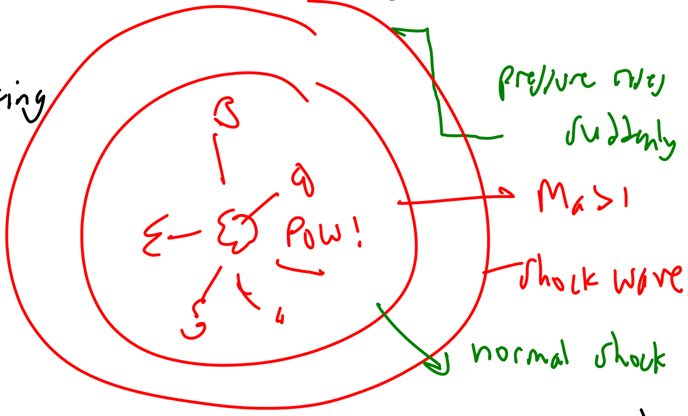
examples:

1) sonic boom

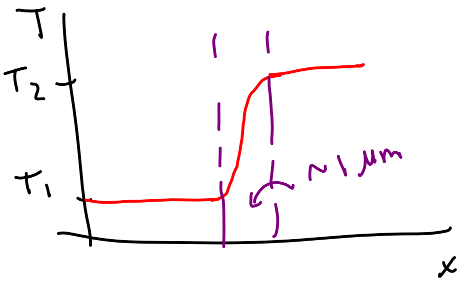


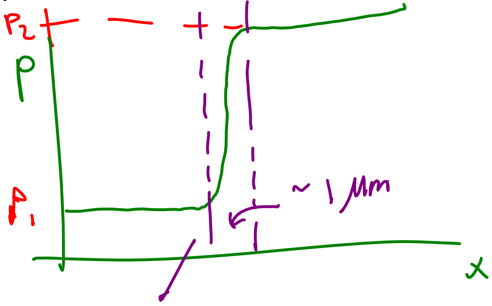
2) whip (or towel) cracking

3) bomb blast



Shock wave thickness is ~ 1 μm (order of magnitude – very thin!)

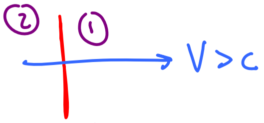




approx as a sudden jump in pressure.

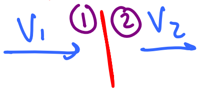
Two ways to look @ shock waves (NORMAL SHOCK WAVES)

a) moving shock



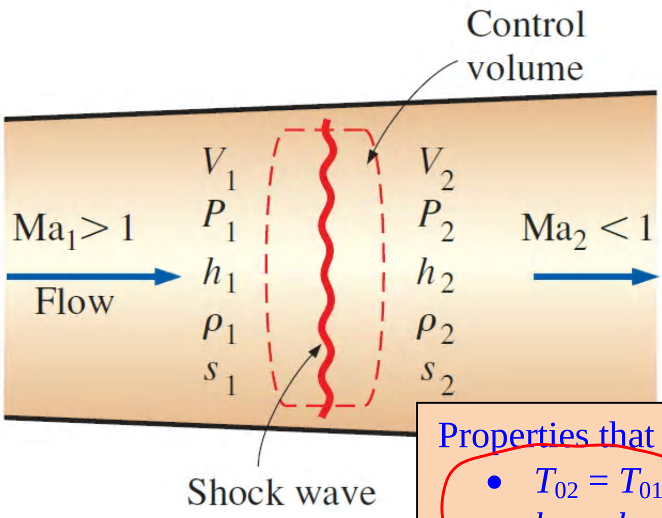
V=0 (like in a blast wave / explosion)

b) stationary shock



(like in a wind tunnel)

Consider a *stationary* normal shock wave (as in a supersonic wind tunnel)



Properties that ***increase*** across the shock:

- $P_2 > P_1$
- $T_2 > T_1$, thus:
  - $c_2 > c_1$
  - $h_2 > h_1$
- $\rho_2 > \rho_1$
- $s_2 > s_1$
- $A_2^* > A_1^*$

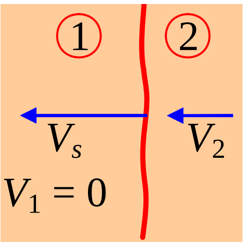
Properties that ***decrease*** across the shock:

- $Ma_2 < Ma_1$
- $P_{02} < P_{01}$
- ρ_{02} ~~$>$~~ $<$ ρ_{02}
- $V_2 < V_1$

Properties that ***stay the same*** across the shock:

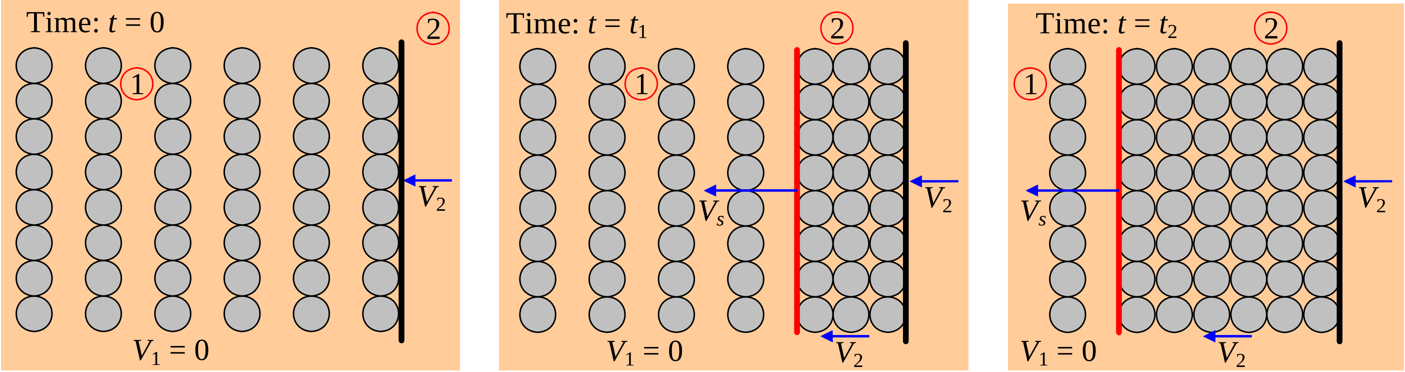
- $T_{02} = T_{01}$
- $h_{02} = h_{01}$

Consider instead a *moving* normal shock wave (as in a blast wave from an explosion)



- The shock is moving into quiescent air (region 1)
- In this frame of reference we define $Ma_1 = V_s/c_1$
- The shock wave travels into region 1 at supersonic speed ($Ma_1 > 1$)
- The air behind the shock (region 2) follows at a slower speed

The "dime analogy" (model the moving shock as rows of dimes that pile up when pushed by a rod or "piston" as sketched; three sequential times):



Comments:

- The vertical red line is analogous to a shock wave: $V_1 = 0$, $V_s > V_2$, $\rho_2 > \rho_1$ (there is sudden increase in density, and the "wave front" of dimes moves faster than the piston).
- The dimes in region 1 don't "know" anything is happening until the shock hits them.
- Similarly in a shock wave in air, the air in region 1 does not "know" anything is happening until the shock wave hits it.

Shock RELATIONS → See eq's in the text & Table A-14 for air

Normal Shock Equations (1 = upstream, 2 = downstream of stationary shock):

$$T_{01} = T_{02}$$

$$\mathrm{Ma}_2 = \sqrt{\frac{(k-1)\mathrm{Ma}_1^2 + 2}{2k\mathrm{Ma}_1^2 - k + 1}}$$

$$\frac{P_2}{P_1} = \frac{1 + k\mathrm{Ma}_1^2}{1 + k\mathrm{Ma}_2^2} = \frac{2k\mathrm{Ma}_1^2 - k + 1}{k + 1}$$

$$\frac{\rho_2}{\rho_1} = \frac{P_2/P_1}{T_2/T_1} = \frac{(k+1)\mathrm{Ma}_1^2}{2 + (k-1)\mathrm{Ma}_1^2} = \frac{V_1}{V_2}$$

$$\frac{T_2}{T_1} = \frac{2 + \mathrm{Ma}_1^2(k-1)}{2 + \mathrm{Ma}_2^2(k-1)}$$

$$\frac{P_{02}}{P_{01}} = \frac{\mathrm{Ma}_1}{\mathrm{Ma}_2}\left[\frac{1 + \mathrm{Ma}_2^2(k-1)/2}{1 + \mathrm{Ma}_1^2(k-1)/2}\right]^{(k+1)/[2(k-1)]}$$

$$\frac{P_{02}}{P_1} = \frac{(1 + k\mathrm{Ma}_1^2)[1 + \mathrm{Ma}_2^2(k-1)/2]^{k/(k-1)}}{1 + k\mathrm{Ma}_2^2}$$

TABLE A–14

One-dimensional normal shock functions for an ideal gas with $k = 1.4$

air

Ma_1	Ma_2	P_2/P_1	ρ_2/ρ_1	T_2/T_1	P_{02}/P_{01}	P_{02}/P_1
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.8929
1.1	0.9118	1.2450	1.1691	1.0649	0.9989	2.1328
1.2	0.8422	1.5133	1.3416	1.1280	0.9928	2.4075
1.3	0.7860	1.8050	1.5157	1.1909	0.9794	2.7136
1.4	0.7397	2.1200	1.6897	1.2547	0.9582	3.0492
1.5	0.7011	2.4583	1.8621	1.3202	0.9298	3.4133
1.6	0.6684	2.8200	2.0317	1.3880	0.8952	3.8050
1.7	0.6405	3.2050	2.1977	1.4583	0.8557	4.2238
1.8	0.6165	3.6133	2.3592	1.5316	0.8127	4.6695
1.9	0.5956	4.0450	2.5157	1.6079	0.7674	5.1418
2.0	0.5774	4.5000	2.6667	1.6875	0.7209	5.6404
2.1	0.5613	4.9783	2.8119	1.7705	0.6742	6.1654
2.2	0.5471	5.4800	2.9512	1.8569	0.6281	6.7165
2.3	0.5344	6.0050	3.0845	1.9468	0.5833	7.2937
2.4	0.5231	6.5533	3.2119	2.0403	0.5401	7.8969
2.5	0.5130	7.1250	3.3333	2.1375	0.4990	8.5261
2.6	0.5039	7.7200	3.4490	2.2383	0.4601	9.1813
2.7	0.4956	8.3383	3.5590	2.3429	0.4236	9.8624
2.8	0.4882	8.9800	3.6636	2.4512	0.3895	10.5694
2.9	0.4814	9.6450	3.7629	2.5632	0.3577	11.3022
3.0	0.4752	10.3333	3.8571	2.6790	0.3283	12.0610
4.0	0.4350	18.5000	4.5714	4.0469	0.1388	21.0681
5.0	0.4152	29.000	5.0000	5.8000	0.0617	32.6335
∞	0.3780	∞	6.0000	∞	0	∞