Today [last day of class ☺], we will:

- Finish our discussion about shock waves; converging-diverging nozzles
- Do some example problems

☆ Office Hours Tues. 1-5 – Prof. Settles (in his office)

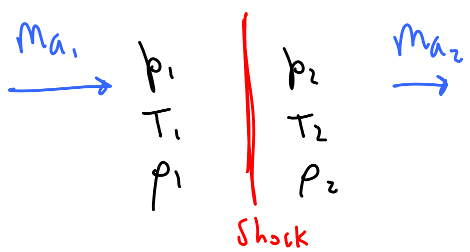
Normal Shock:

1) Normal shock can go only from supersonic to subsonic
$Ma_1 > 1$ $Ma_2 < 1$

Δs goes ↑ (not isentropic) across a shock

(a normal shock from subsonic to supersonic would violate 2nd law)

2) Shock causes compression of the gas → $P_2 > P_1$, $T_2 > T_1$, $\rho_2 > \rho_1$



3) But, total pressure goes down $P_{0_2} < P_{0_1}$

total temp remains the same $T_{0_2} = T_{0_1}$

4) "Weak shocks" → nearly isentropic & travel at speed of sound.

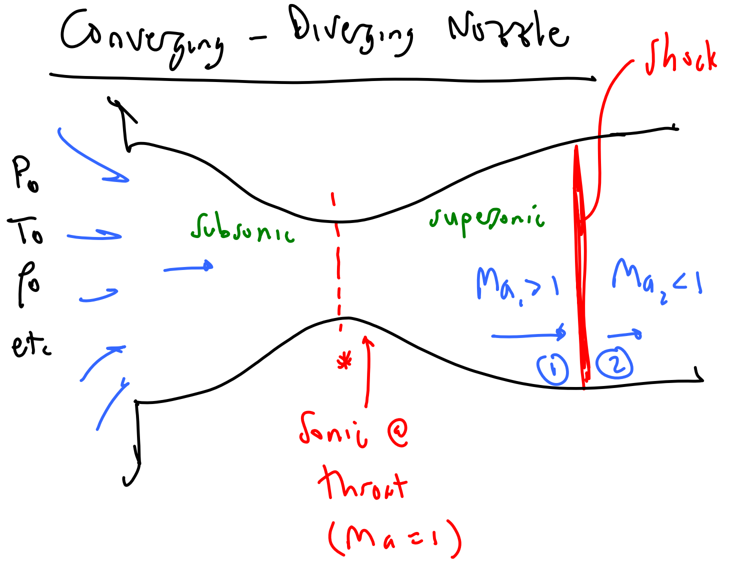
E.g. sound waves are weak shocks — Ma close to 1 (e.g. 1.001)

"Strong shocks" → not isentropic & travel faster than sound (Ma ≈ 2 or 3)

The stronger the shock, the faster it travels.

Converging - Diverging Nozzle

P_b = back pressure

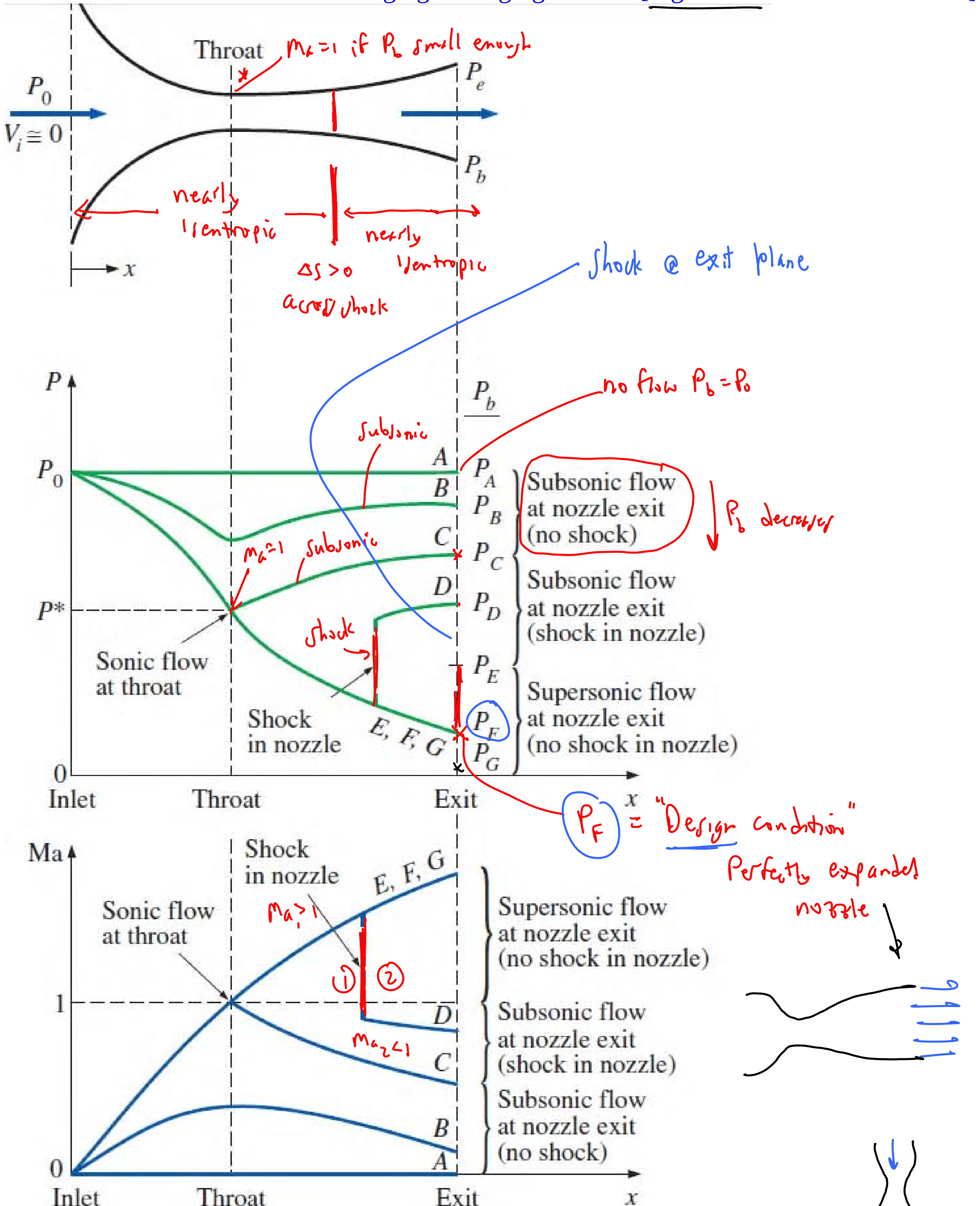


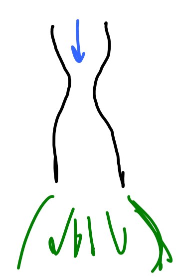
Let's look at a whole family of cases, with P_b varying from $P_b = P_0$ to $P_b = 0$

See next page → we run a "thought experiment," varying P_b to see what happens

↓

Effect of Back Pressure in a Converging-Diverging Nozzle: [Fig. 12-22 in Ed 2 of textbook]





OVEREXPANDED → when $P_F < P_b < P_E$ → get shock diamonds

UNDEREXPANDED → when $P_b < P_F$ (eg. P_G) → get flare out

Overexpanded nozzles:

Example – High speed jet aircraft

Given: The SR-71 travels at Ma = 3.2 at 24 km altitude (80,000 ft).

To do: Calculate its air speed.

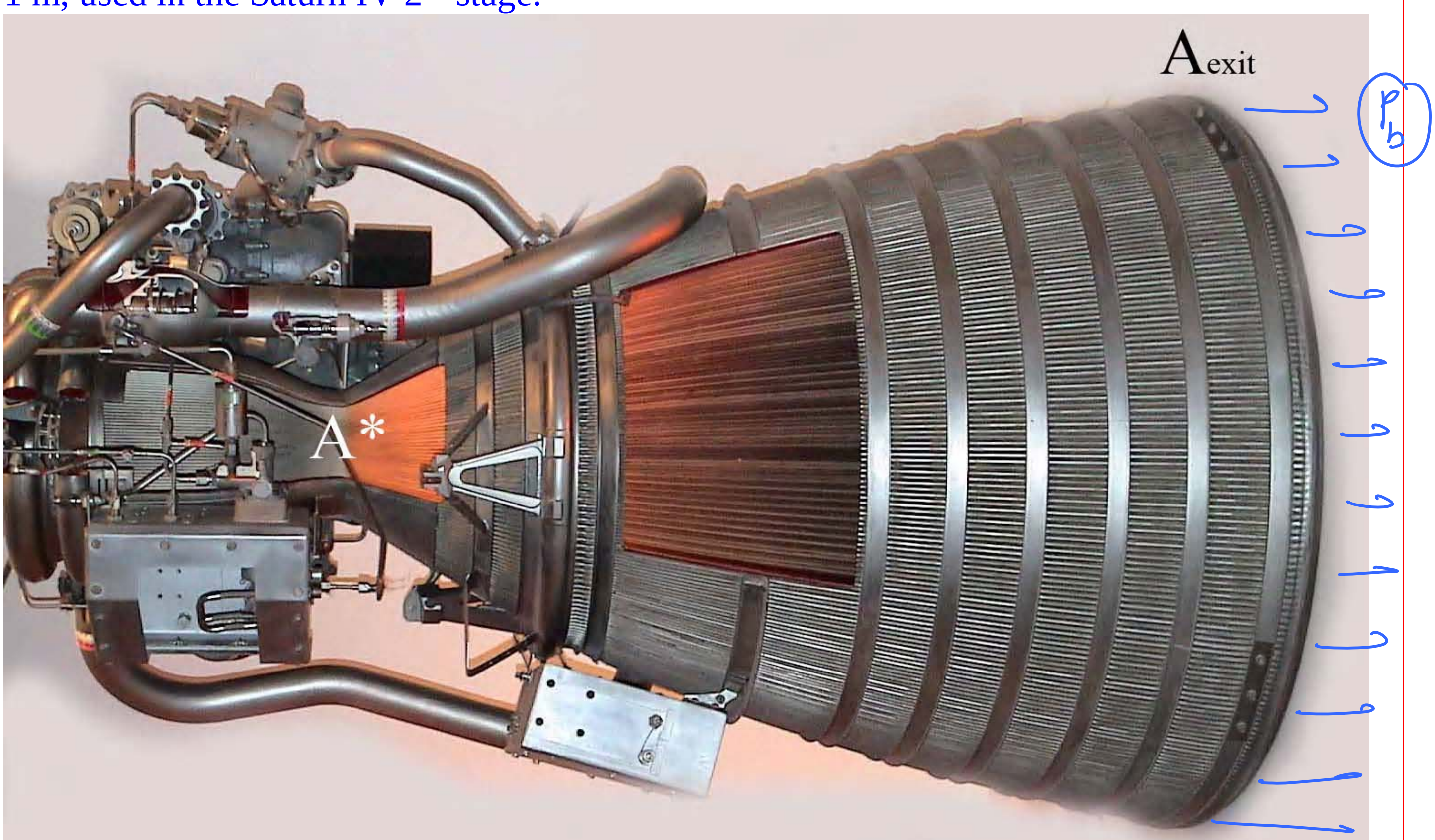
Solution:

- From Table A-11E, T at 24 km altitude is -69.7ºF = 217 K.
- Using k = 1.4 and R_{air} = 287 m^2/(s^2·K), $c = (kRT)^{1/2}$ = 295 m/s.
- Thus, V = Ma·c = 3.2(295 m/s) = 944 m/s (= 2110 mph).

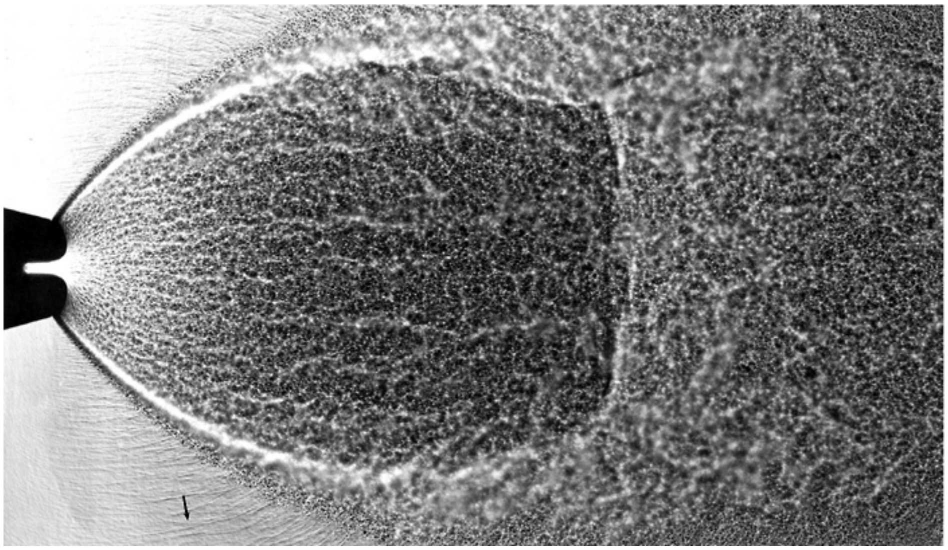
Shock Diamonds ("tiger tail"):

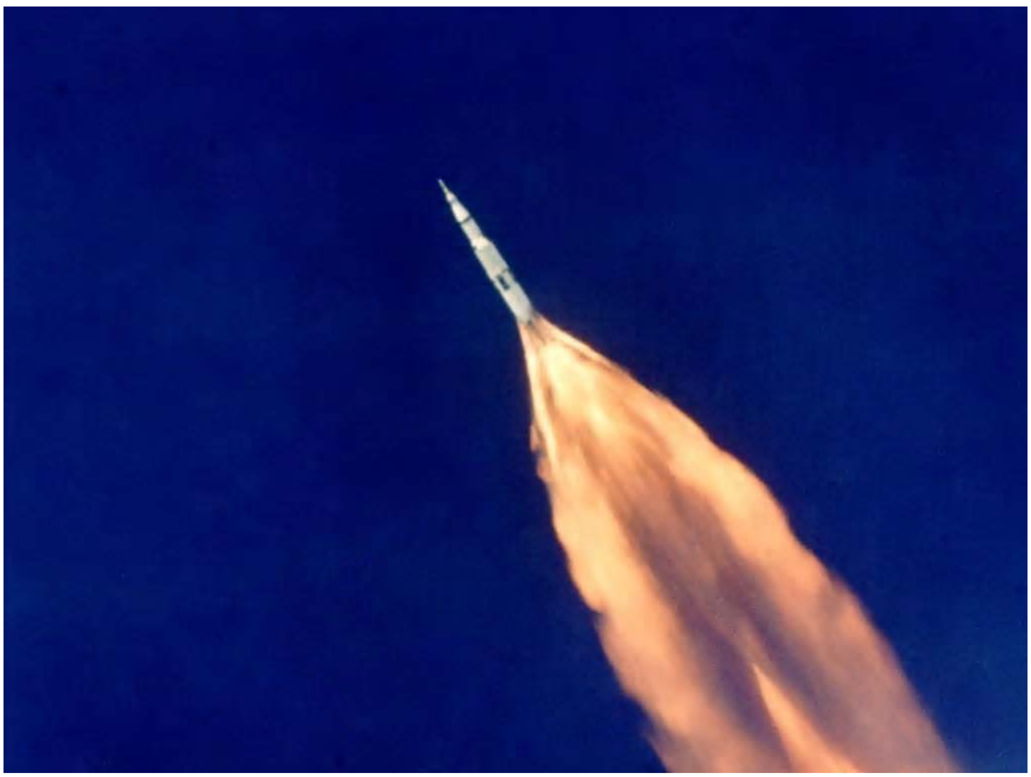
Example of a Rocket Engine:

Pratt &Whitney RL-10 rocket motor designed for a specific Ma_{exit} (photographed at the National Air & Space Museum). 1960-vintage, Ma_{exit} = 5, k = 1.33, thrust = 15,000 lbf, D_e ~ 1 m, used in the Saturn IV 2nd stage.



Underexpanded Nozzles:





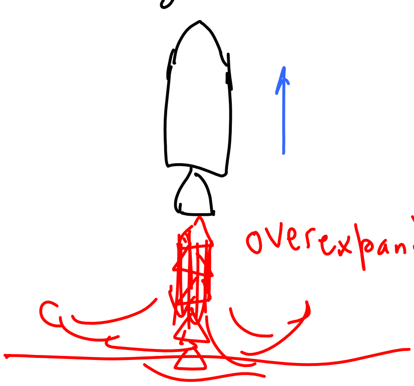
High altitude

"flare out"

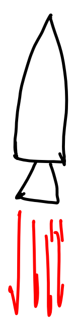
$P_b < P_f$

underexpanded

Near ground



overexpanded



@ some altitude

(typ. a few thousand feet)

$P_b = P_F$ = design conditions, perfectly expanded

Example – Normal shock at Ma = 3.0

Given: A converging/diverging nozzle accelerates air to Ma = 3.0 at the exit. The upstream stagnation properties are T_0 = 1000 K and P_0 = 1.00 MPa.

To do: If a normal shock occurs right at the exit plane, calculate the pressure, temperature, and density upstream (1) and downstream (2) of the shock.

Solution:

(1) is upstream → flow is nearly isentropic up to the shock

- From Table A-13 at Ma_1 = 3.0,
  A/A^* = 4.2346, P/P_0 = 0.0272, T/T_0 = 0.3571, and ρ/ρ_0 = 0.0760.
- From Table A-14 at Ma_1 = 3.0, → across shock @ $Ma_1 = 3.0$
  Ma_2 = 0.4752, P_2/P_1 = 10.3333, T_2/T_1 = 2.679, and ρ_2/ρ_1 = 3.8571.
- The rest of the problem to be completed in class.

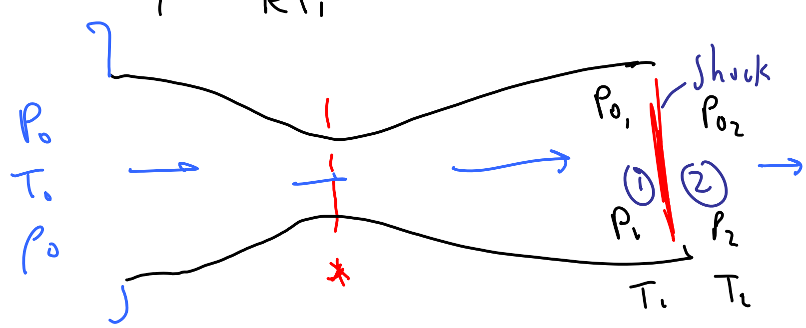
Upstream of shock:

@ (1), $P_1 = \left(\frac{P_1}{P_0}\right)P_0 = (0.272)(1.00 \text{ MPa}) = 27.2 \text{ kPa} = P_1$

$T_1 = \left(\frac{T_1}{T_0}\right)T_0 = (0.3571)(1000 \text{ K}) = 357.1 \text{ K} = 357 \text{ K} = T_1$

$\rho_1 = \text{———} = 0.265 \text{ kg/m}^3$

OR $\rho_1 = \frac{P_1}{RT_1}$ ——→ 0.265 kg/m^3



Across the normal shock:

$P_2 = \left(\frac{P_2}{P_1}\right)P_1 = (10.333)(27.2 \text{ kPa}) = 281.1 \text{ kPa} \Rightarrow P_2 = 281 \text{ kPa}$

T_2 ——— $T_2 = 957 \text{ K}$

ρ_2 ——— $\rho_2 = 1.02 \frac{\text{kg}}{\text{m}^3}$