Vorticity and Rotationality (Section 4-5)

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The **vorticity vector** is defined as the **curl of the velocity vector**,

Greek letter zeta →

$$\vec{\zeta} = \vec{\nabla} \times \vec{V}$$

It turns out that **vorticity is equal to twice the angular velocity of a fluid particle**,

$$\vec{\zeta} = 2\vec{\omega}$$

Thus, ***vorticity is a measure of rotation of a fluid particle***.

if $\vec{\zeta} = 0$, the flow is irrotational

if $\vec{\zeta} \neq 0$, the flow is rotational

Vorticity vector in Cartesian coordinates:

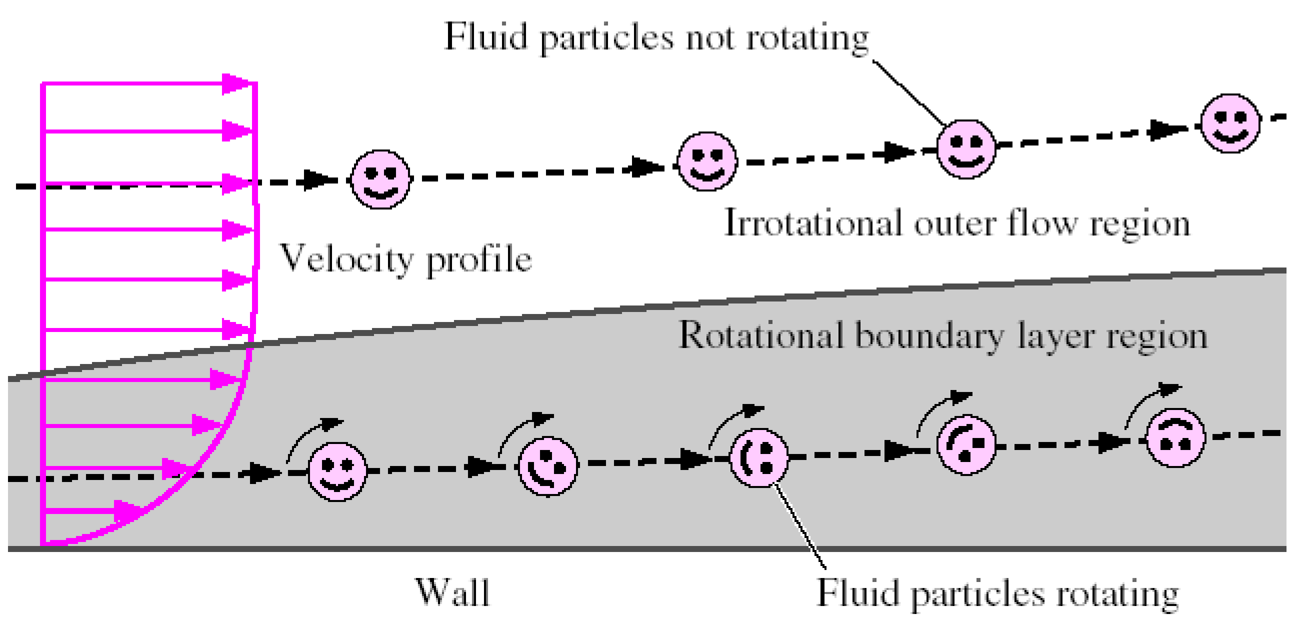
$$\vec{\zeta} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k} \qquad \textbf{(4–30)}$$

Vorticity vector in cylindrical coordinates:

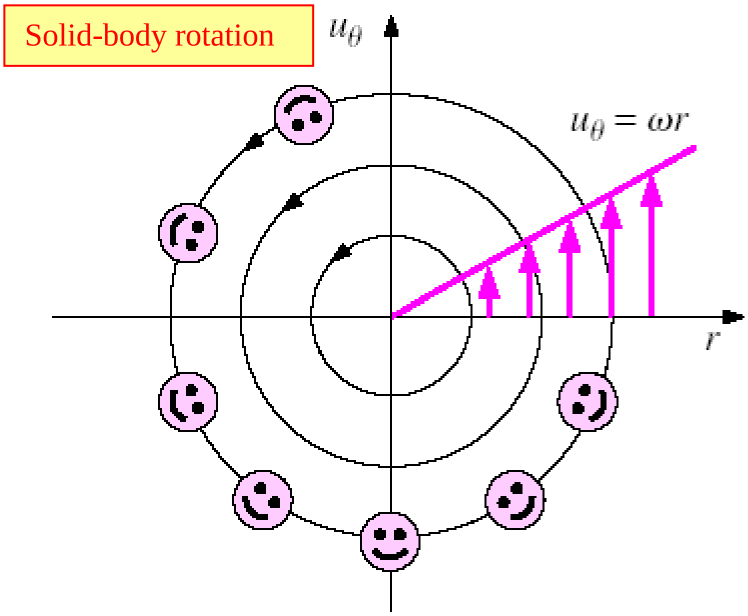
$$\vec{\zeta} = \left(\frac{1}{r}\frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z}\right)\vec{e}_r + \left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right)\vec{e}_\theta + \frac{1}{r}\left(\frac{\partial (r u_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta}\right)\vec{e}_z \qquad \textbf{(4–32)}$$

Examples:

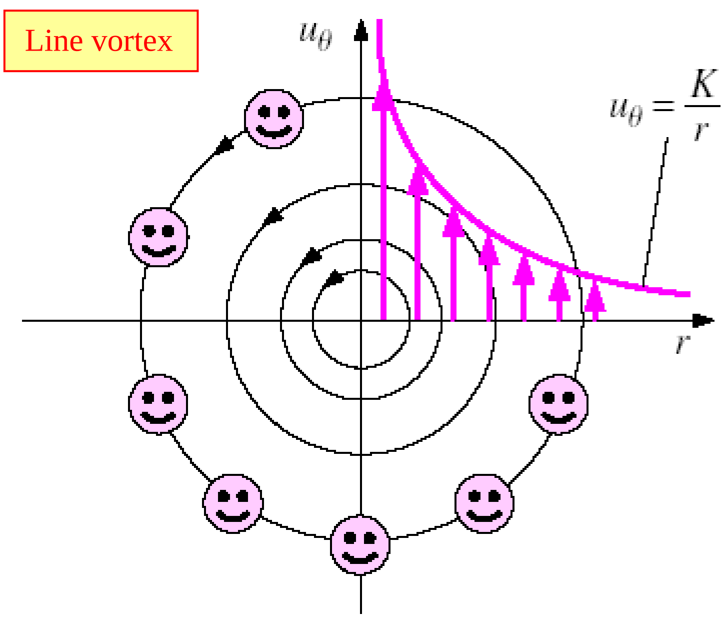
1. Inside a **boundary layer**, where viscous forces are important, the flow in this region is *rotational* ($\vec{\zeta} \neq 0$). However, outside the boundary layer, where viscous forces are not important, the flow in this region is *irrotational* ($\vec{\zeta} = 0$).



2. A **solid-body rotation** (rigid-body rotation) flow is *rotational* ($\vec{\zeta} \neq 0$). In fact, since vorticity is equal to twice the angular velocity, $\vec{\zeta} = 2\vec{\omega}$ *everywhere* in the flow field. Fluid particles rotate as they revolve around the center of the flow. This is analogous to a merry-go-round or a roundabout.



3. A **line vortex** flow, however, is *irrotational* ($\vec{\zeta} = 0$), and fluid particles do not rotate, even though they revolve around the center of the flow. This is analogous to a Ferris wheel.



See text for details and calculations.