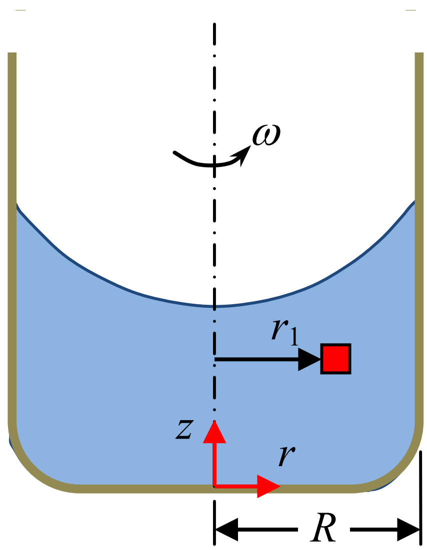
Today, we will:

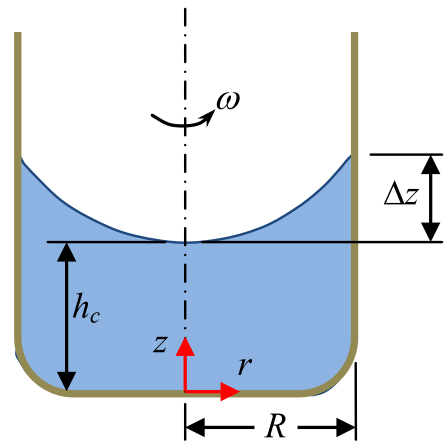
- Discuss fluids in rigid-body *rotation*
- Begin Chapter 4 – FLUID KINEMATICS
- Discuss the material acceleration and the material derivative, and show examples
- Discuss various kinds of flow patterns and flow visualization techniques

3. Rigid-body rotation

Consider a container of liquid of radius R spinning at constant angular velocity ω (angular velocity vector is straight up as shown).



Example: Rigid-body rotation



Given: A container of water spins at rotation rate $\dot{n} = 100$ rpm. The radius of the container is R = 11.05 cm. The surface height at the center is h_c = 4.62 cm.

To do: Calculate the elevation distance Δz (in units of cm) between the water surface at the center of the paraboloid and at the rim of the paraboloid.

Solution:

First, we must convert rpm to radians/s:

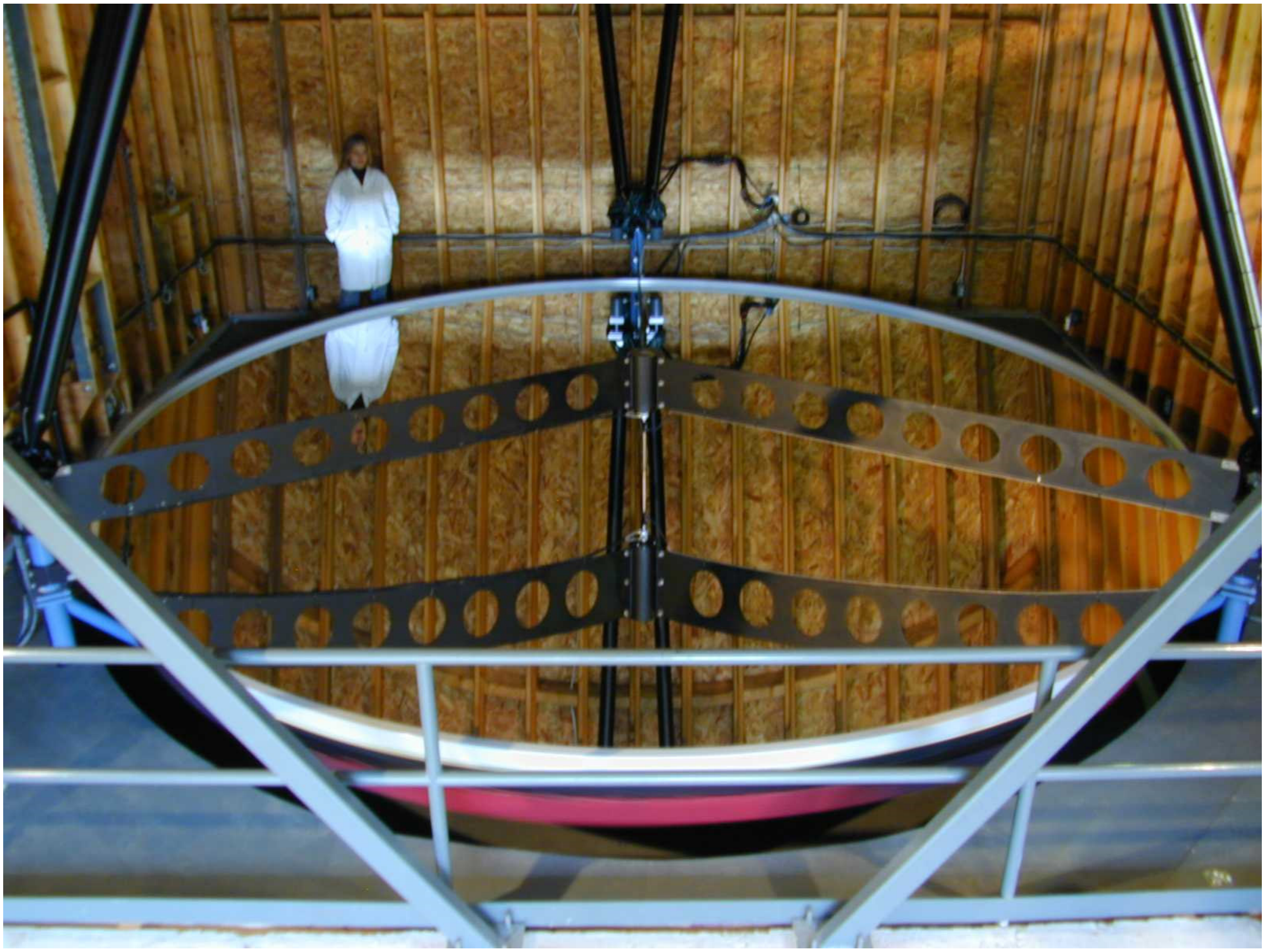
$$\omega = \left(100 \frac{\text{rot}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{\text{rot}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 10.47198 \frac{\text{rad}}{\text{s}}$$

Recall, the equation for the free surface of the liquid in rigid-body rotation,

$$z_s = \frac{\omega^2 r^2}{2g} + h_c$$

Liquid mercury mirrors. By rotating a container of mercury, a nice parabolic mirror can be generated without the need to grind or polish. Unfortunately, it can look only straight up. However, there is some discussion of creating similar mirrors in space – thrust can be used in place of gravity to produce the parabolic shape.

Example: The Large Zenith Telescope in Canada: Photo from http://www.astro.ubc.ca/LMT/lzt/index.html.



Mirrors made from spinning molten glass in a furnace, and letting it harden in the paraboloid shape.

Example: The 40-foot (12 meter) diameter spinning furnace used in casting 6.5 meter and 8.4 meter borosilicate glass "honeycomb" mirrors at the Steward Observatory Mirror Lab, University of Arizona. Image from http://uanews.org.

III. FLUID KINEMATICS

A. Descriptions of Fluid Flow – there are two ways to describe fluid flow:

1. Lagrangian description

2. Eulerian description

3. Acceleration field and material derivative

Derivation of Material Acceleration (Section 4-1)

Acceleration of a fluid particle:

$$\vec{a}_{\text{particle}} = \frac{d\vec{V}_{\text{particle}}}{dt} \quad \text{(4–6)}$$

This is a *Lagrangian* description of the acceleration of a fluid particle.

However, at any instant in time t, the velocity of the particle is the same as the local value of the velocity *field* at the location $(x_{\text{particle}}(t), y_{\text{particle}}(t), z_{\text{particle}}(t))$ of the particle, since the fluid particle moves with the fluid by definition. In other words, $\vec{V}_{\text{particle}}(t) \equiv \vec{V}(x_{\text{particle}}(t), y_{\text{particle}}(t), z_{\text{particle}}(t), t)$. To take the time derivative in Eq. 4–6, we must therefore use the *chain rule*, since the dependent variable ($\vec{V}$) is a function of *four* independent variables $(x_{\text{particle}}, y_{\text{particle}}, z_{\text{particle}},$ and $t)$,

Recall the ***chain rule***: If f is a function of two variables, t and some variable s which is itself also a function of t, then we take the total derivative of f with respect to t as follows:

$$\frac{df}{dt} = \frac{\partial f}{\partial t}\frac{dt}{dt} + \frac{\partial f}{\partial s}\frac{ds}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial s}\frac{ds}{dt}$$

Now let's apply this chain rule to the time derivative of the fluid particle's velocity:

Note that from the Lagrangian description (following a fluid particle, x_{particle} is a function of time, since the particle's location changes with time. Thus, $x_{\text{particle}} = x_{\text{particle}}(t)$. Similarly, $y_{\text{particle}} = y_{\text{particle}}(t)$ and $z_{\text{particle}} = z_{\text{particle}}(t)$.

Thus, the acceleration of a fluid particle is calculated using the chain rule as follows:

$$\vec{a}_{\text{particle}} = \frac{d\vec{V}_{\text{particle}}}{dt} = \frac{d\vec{V}}{dt} = \frac{d\vec{V}(x_{\text{particle}}, y_{\text{particle}}, z_{\text{particle}}, t)}{dt}$$

$$= \frac{\partial \vec{V}}{\partial t}\frac{dt}{dt} + \frac{\partial \vec{V}}{\partial x_{\text{particle}}}\frac{dx_{\text{particle}}}{dt} + \frac{\partial \vec{V}}{\partial y_{\text{particle}}}\frac{dy_{\text{particle}}}{dt} + \frac{\partial \vec{V}}{\partial z_{\text{particle}}}\frac{dz_{\text{particle}}}{dt} \quad \text{(4–7)}$$

$dt/dt =$	$dx_{\text{particle}}/dt =$	$dy_{\text{particle}}/dt =$	$dz_{\text{particle}}/dt =$

Or, finally,

$$\vec{a}_{\text{particle}}(x, y, z, t) = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} \quad \text{(4–8)}$$