Today, we will:

- Discuss various kinds of flow patterns and flow visualization techniques
- Discuss the motion and deformation of fluid particles
- Discuss linear strain, shear strain, and the strain rate tensor

B. Flow Patterns and Flow Visualization (Section 4-2)

1. Streamlines, pathlines, streaklines, and timelines

a. Streamline: A streamline is a curve everywhere parallel to the local velocity field.

b. Pathline: A pathline is the path traveled by a marked fluid particle over some time period.

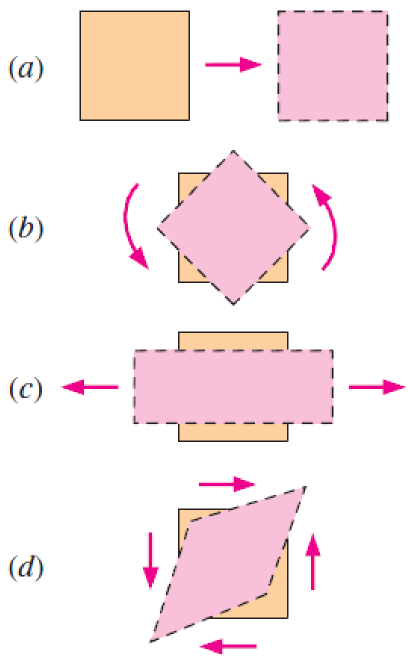
c. Streakline: A streakline is the locus of fluid particles introduced at a point.

d. Timeline: A timeline is a *set* of adjacent fluid particles that were marked at the same time.

2. Other flow visualization techniques (Section 4.2) – read on your own.
3. Fluid flow plots (Sec. 4.3) – read on your own (self explanatory).
   Examples: profile plots, vector plots, contour plots.

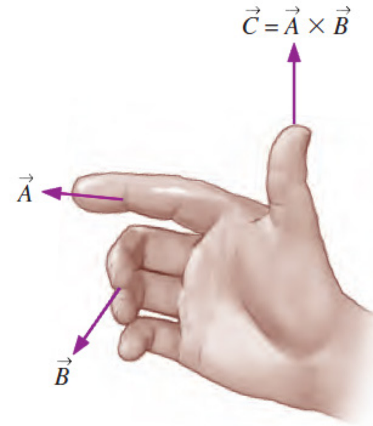
C. Other Kinematic Descriptions (Section 4-4)

1. Motion and deformation of fluid particles. There are four fundamental types of fluid element motion or deformation:



Rate of rotation: $\vec{\omega} = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$

Vorticity and Rotationality (Section 4-5)



The **vorticity vector** is defined as the **curl of the velocity vector**, using the **right-hand rule**.

Greek letter zeta → $\vec{\zeta} = \vec{\nabla} \times \vec{V}$

It turns out that **vorticity is equal to twice the angular velocity of a fluid particle**,

$$\vec{\zeta} = 2\vec{\omega}$$

Thus, ***vorticity is a measure of rotation of a fluid particle***.

if $\vec{\zeta} = 0$, the flow is irrotational

if $\vec{\zeta} \neq 0$, the flow is rotational

Vorticity vector in Cartesian coordinates:

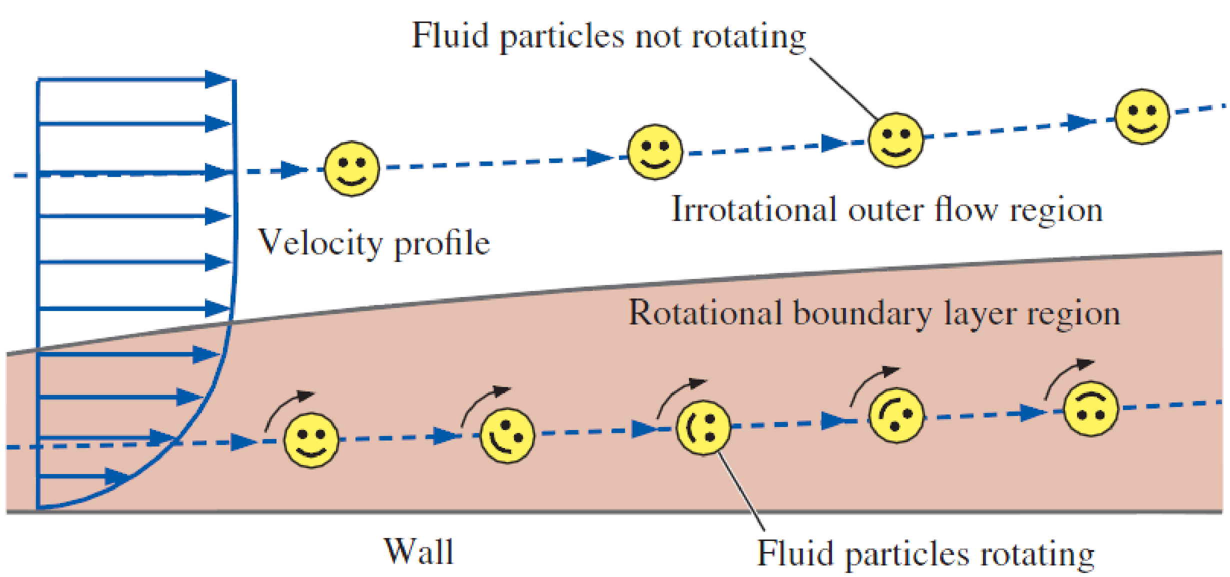
$$\vec{\zeta} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k} \qquad (4\text{–}30)$$

Vorticity vector in cylindrical coordinates:

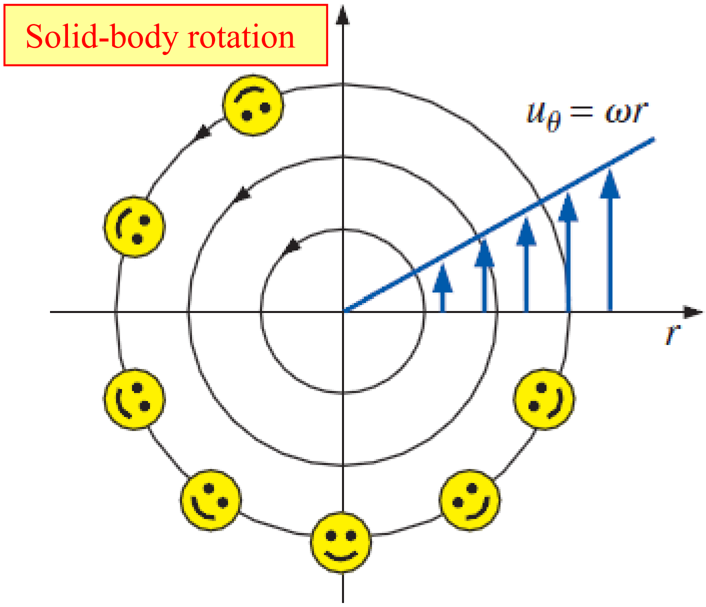
$$\vec{\zeta} = \left(\frac{1}{r}\frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z}\right)\vec{e}_r + \left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right)\vec{e}_\theta + \frac{1}{r}\left(\frac{\partial (r u_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta}\right)\vec{e}_z \qquad (4\text{–}32)$$

Examples:

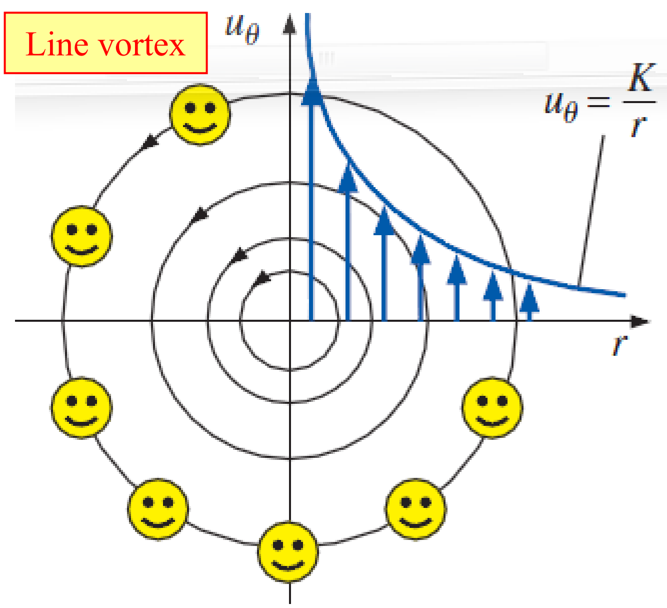
1. Inside a **boundary layer**, where viscous forces are important, the flow in this region is *rotational* ($\vec{\zeta} \neq 0$). However, outside the boundary layer, where viscous forces are not important, the flow in this region is *irrotational* ($\vec{\zeta} = 0$).



2. A **solid-body rotation** (rigid-body rotation) flow is *rotational* ($\vec{\zeta} \neq 0$). In fact, since vorticity is equal to twice the angular velocity, $\vec{\zeta} = 2\vec{\omega}$ *everywhere* in the flow field. Fluid particles rotate as they revolve around the center of the flow. This is analogous to a merry-go-round or a roundabout.



3. A **line vortex** flow, however, is *irrotational* ($\vec{\zeta} = 0$), and fluid particles do not rotate, even though they revolve around the center of the flow. This is analogous to a Ferris wheel.



See text for details and calculations.

Example: Vorticity and irrotationality

Given: A two-dimensional velocity field in the *x*-*y* plane: $\vec{V} = (u, v) = 2xy\vec{i} - y^2\vec{j}$. ($w = 0$)

To do: Is this flow rotational or irrotational?

Solution:

The rate of rotation is

$$\vec{\omega} = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$$ and vorticity = $\vec{\zeta} = 2\vec{\omega}$.

Example: Vorticity and irrotationality

Given: A two-dimensional velocity field in the *x*-*y* plane: $\vec{V} = (u, v) = 3x\vec{i} - 3y\vec{j}$. ($w = 0$)

To do: Calculate (**a**) the rate of translation and (**b**) the rate of rotation.

Solution:

(**a**) The rate of translation is simply the velocity vector,

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

(**b**) The rate of rotation is

$$\vec{\omega} = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$$ and vorticity = $\vec{\zeta} = 2\vec{\omega}$.

Example: Strain rates (Continuation of previous example problem)

Given: A two-dimensional velocity field in the *x*-*y* plane: $\vec{V} = (u, v) = 3x\vec{i} - 3y\vec{j}$ ($w = 0$). In a previous example, we had already calculated (**a**) the rate of translation and (**b**) the rate of rotation.

To do: Calculate (**a**) rate of translation, (**b**) rate of rotation, (**c**) linear strain rate, (**d**) the shear strain rate, and (**e**) the strain rate tensor.

Solution: We did Parts (**a**) and (**b**) already. Recall,

(**a**) The rate of translation is simply the velocity vector, $\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$. Here, the rate of translation = $\vec{V} = 3x\vec{i} - 3y\vec{j}$.

(**b**) The rate of rotation is $\vec{\omega} = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$. Here, the rate of rotation is $\vec{\omega} = 0$, and the vorticity = $\vec{\zeta} = 2\vec{\omega} = 0$. This flow is *irrotational*.

(**c**) The three components of linear strain rate are

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}, \quad \varepsilon_{yy} = \frac{\partial v}{\partial y}, \quad \varepsilon_{zz} = \frac{\partial w}{\partial z}$$

(**d**) The three components of shear strain rate are

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \quad \varepsilon_{zx} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right), \quad \varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right),$$