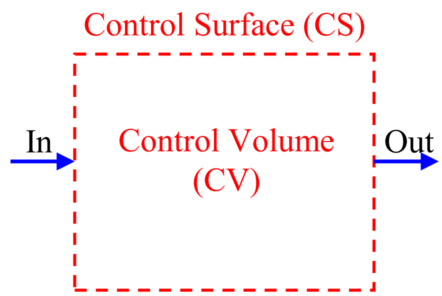
Today, we will:
- Begin Chapter 5 – Conservation of mass and energy for control volumes
- Do some example problems, conservation of mass
- If time, begin to discuss conservation of energy

III. Conservation Laws and the Control Volume (Integral) Technique (Chapter 5)

A. Introduction

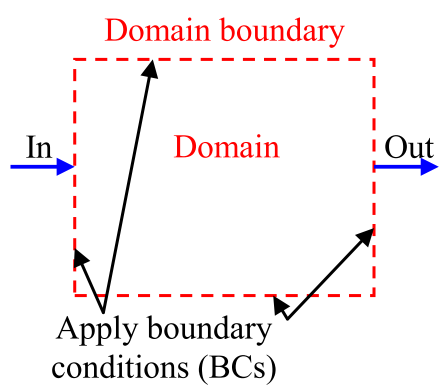
1. Overview – Techniques for solving fluid flow problems

a. Control volume analysis (Ch. 5, 6, 8)



b. Dimensional analysis and experiment (Ch. 7)

c. Differential analysis (Ch. 9, 10, 15)



B. Conservation of Mass

1. Equations and definitions

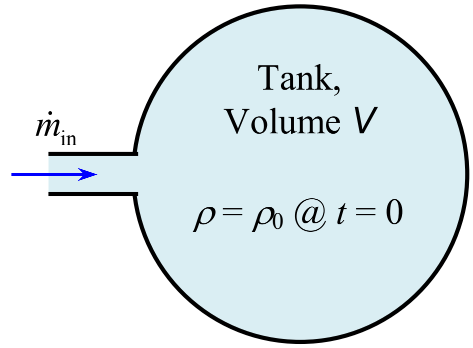
From previous lecture…**the conservation of mass equation for a fixed control volume:**

$$\frac{d}{dt}\int_{\mathrm{CV}} \rho d\mathsf{V} + \int_{\mathrm{CS}} \rho\left(\vec{V}\cdot\vec{n}\right) dA = 0 \tag{1}$$

2. Examples

Example: Unsteady conservation of mass (flow into a tank)

Given: Air is pumped into a rigid tank of volume V. The mass flow rate of the air entering the tank is constant, $\dot{m}_{\text{in}}$. We assume that the process is slow enough that the air in the tank remains at the same temperature (isothermal conditions).



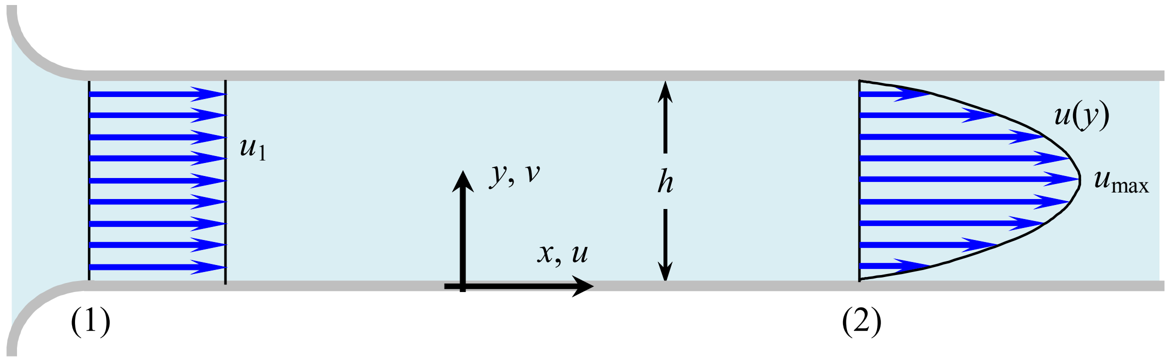
To do: Generate an equation for density ρ in the tank as a function of time.

Solution:

Example: Velocity profiles in 2-D channel flow

Given: Consider steady, incompressible, two-dimensional flow of a liquid between two very long parallel plates as sketched. At the inlet (1) there is a nice bell mouth, and the velocity is nearly uniform (except for a very thin boundary layer, not shown).

- At (1), $u = u_1$ = constant, $v = 0$, and $w = 0$.
- At (2), the flow is fully developed, and $u = ay(h - y)$, $v = 0$, and $w = 0$, where a is a constant.



To do: Generate expressions for constant a and speed u_{max} in terms of the given variables.

Solution:

C. Conservation of Energy

1. Equations and definitions

From previous lecture…**the conservation of energy equation for a fixed control volume:**

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}} = \frac{d}{dt}\int_{CV} e\rho d\mathsf{V} + \int_{CS} e\rho\left(\vec{V}\cdot\vec{n}\right)dA \qquad (2)$$

But we know from thermodynamics that specific energy $e = u + \frac{V^2}{2} + gz$. Thus, (2) becomes

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}} = \frac{d}{dt}\int_{CV} e\rho d\mathsf{V} + \int_{CS}\left(u + \frac{V^2}{2} + gz\right)\rho\left(\vec{V}\cdot\vec{n}\right)dA \qquad (3)$$