Today, we will:

- Discuss conservation of energy for a control volume
- Do an example problem – energy equation with a comresssor
- Discuss the kinetic energy correction factor

C. Conservation of Energy

1. Equations and definitions

From previous lecture…**the conservation of energy equation for a fixed control volume:**

(from the RTT) $$\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}} = \frac{d}{dt}\int_{CV} e\rho dV + \int_{CS} e\rho\left(\vec{V}\cdot\vec{n}\right)dA \qquad (2)$$

But from thermodynamics, specific total energy $e = u + \frac{V^2}{2} + gz$. Thus, (2) becomes

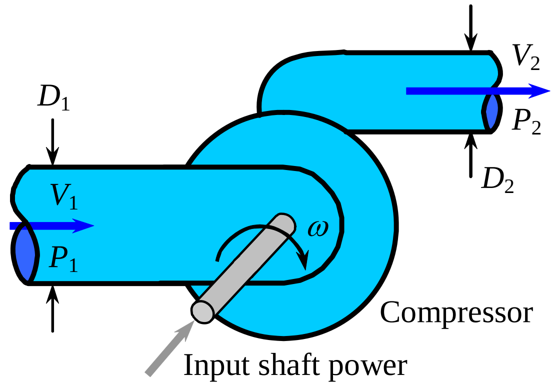
$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}} = \frac{d}{dt}\int_{CV} e\rho dV + \int_{CS}\left(u + \frac{V^2}{2} + gz\right)\rho\left(\vec{V}\cdot\vec{n}\right)dA \qquad (3)$$

Steady-state, steady-flow (SSSF) form of the 1st Law of Thermo. for a fixed CV:

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \sum_{\text{out}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{\text{in}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) \tag{3}$$

Example: Control volume energy equation applied to an air compressor

Given: A large air compressor takes in air at absolute pressure P_1 = 14.0 psia, at temperature T_1 = 80°F (539.67 R), and with mass flow rate $\dot{m}$ = 20.0 lbm/s. The diameter of the compressor inlet is D_1 = 24.5 inches. At the outlet, P_2 = 70.0 psia and T_2 = 500°F (959.67 R). The diameter of the compressor outlet is D_2 = 7.50 inches. The shaft driving the compressor supplies 3100 horsepower to the compressor.



(a) **To do**: Calculate the average velocity of the air entering the compressor.

Solution: At the inlet, $\dot{m} \approx \rho_{1,\text{avg}} V_{1,\text{avg}} A_1 = \rho_1 V_1 A_1$ where the subscripts "avg" have been dropped for convenience. Thus, $V_1 = \frac{\dot{m}}{\rho_1 A_1} = \frac{4\dot{m}}{\pi \rho_1 D_1^2} = \frac{4RT_1\dot{m}}{\pi P_1 D_1^2}$, where we have used the ideal gas law $P = \rho RT$ to calculate the density of the air. Substitution of the values yields

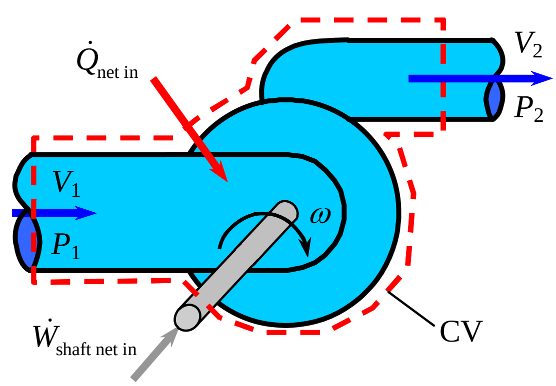
$$V_1 = \frac{4RT_1\dot{m}}{\pi P_1 D_1^2} = \frac{4\left(53.34\,\frac{\text{ft}\cdot\text{lbf}}{\text{lbm}\cdot\text{R}}\right)(539.67\text{ R})\left(20.0\,\frac{\text{lbm}}{\text{s}}\right)}{\pi\left(14.0\,\frac{\text{lbf}}{\text{in}^2}\right)(24.5\text{ in})^2} = 87.229\,\frac{\text{ft}}{\text{s}} \approx \mathbf{87.2\,\frac{ft}{s}}$$

(b) **To do**: Calculate the average velocity of the air leaving the compressor.

Solution: Similarly, using the pressure, temperature, and diameter at the compressor outlet, we get V_2 = 331.051 ft/s, or **V_2 = 331. ft/s** (to three significant digits of precision)

(c) **To do**: Calculate the net rate of heat transfer from the air compressor into the room in units of Btu/hr.

Solution: First we choose a control volume. We draw the control volume around the entire compressor, cutting through the shaft, and cutting through the inlet and outlet, as sketched. Note that we draw the net rate of heat transfer $\dot{Q}_{\text{net in}}$ *into* the control volume to keep the signs straight. We expect a negative value since the compressor will actually give off heat into the room.



Next, we apply the approximate form of the control volume energy equation,

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft net in}} = \underbrace{\frac{d}{dt}\int_{\text{CV}} e\rho\, dV}_{\text{steady}} + \sum_{\text{out}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{\text{in}} \dot{m}\left(h + \frac{V^2}{2} + gz\right)$$

and we solve for $\dot{Q}_{\text{net in}}$.

Solution to be completed in class.