Today, we will:

- Discuss the kinetic energy correction factor
- Derive the "head" form of the energy equation
- Discuss pumps and turbines and their efficiencies
- Do an example problem – energy equation with pumps and turbines

C. Conservation of Energy (continued)

3. The kinetic energy correction factor

From previous lecture…the Steady-State Steady-Flow (**SSSF**) conservation of energy equation for a fixed control volume (no shear work term and no "other" work terms) for fixed known inlets and outlets:

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{net shaft in}} = \sum_{\text{outlets}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{\text{inlets}} \dot{m}\left(h + \frac{V^2}{2} + gz\right)$$

The summation terms on the right are actually *approximations* of the *exact* integral form of these terms. For steady flow the exact form is

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}} = \int_{CS}\left(u + \frac{V^2}{2} + gz\right)\rho\left(\vec{V}\cdot\vec{n}\right)dA$$

Corrected SSSF conservation of energy equation for a fixed control volume:

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{net shaft in}} = \sum_{\text{outlets}} \dot{m}\left(h + \alpha\frac{V^2}{2} + gz\right) - \sum_{\text{inlets}} \dot{m}\left(h + \alpha\frac{V^2}{2} + gz\right)$$

C. Conservation of Energy (continued)

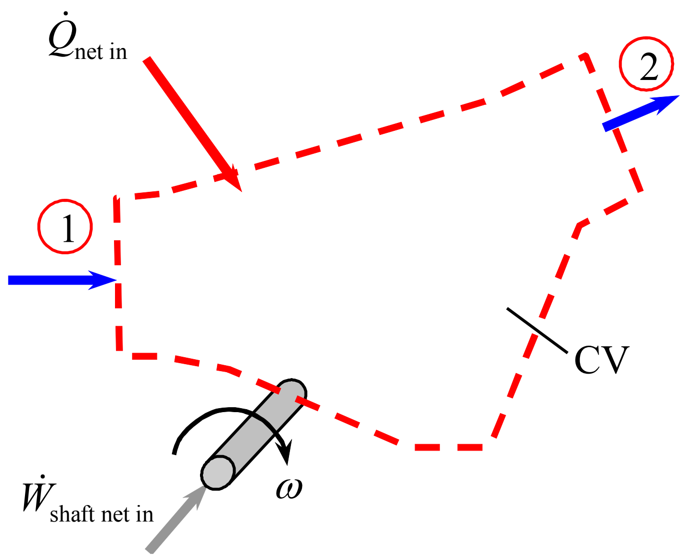
4. The "head" form of the energy equation

Start with the SSSF conservation of energy equation for a fixed control volume:

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{net shaft in}} = \sum_{\text{outlets}} \dot{m}\left(h + \alpha\frac{V^2}{2} + gz\right) - \sum_{\text{inlets}} \dot{m}\left(h + \alpha\frac{V^2}{2} + gz\right) \quad (1)$$

Assumptions and approximations:

1. steady (we already removed the unsteady term in Eq. 1)
2. only one inlet (get rid of the sigma for inlets in Eq. 1 – call the inlet **1**)
3. only one outlet (get rid of the sigma for outlets in Eq. 1 – call the inlet **2**)



Equation (1) becomes

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{net shaft in}} = \dot{m}\left(h + \alpha\frac{V^2}{2} + gz\right)_{2\text{ (outlet)}} - \dot{m}\left(h + \alpha\frac{V^2}{2} + gz\right)_{1\text{ (inlet)}}$$

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{net shaft in}} = \dot{m}\left(u + \frac{P}{\rho} + \alpha\frac{V^2}{2} + gz\right)_2 - \dot{m}\left(u + \frac{P}{\rho} + \alpha\frac{V^2}{2} + gz\right)_1$$

Now divide each term by $\dot{m}g$ and rearrange,

$$\frac{\dot{W}_{\text{net shaft in}}}{\dot{m}g} + \left(\frac{P}{\rho g} + \alpha\frac{V^2}{2g} + z\right)_1 = \left(\frac{P}{\rho g} + \alpha\frac{V^2}{2g} + z\right)_2 + \frac{u_2 - u_1}{g} - \frac{\dot{Q}_{\text{net in}}}{\dot{m}g}$$

The "**head form**" of the energy equation:

$$\frac{\dot{W}_{\text{net shaft in}}}{\dot{m}g} + \left(\frac{P}{\rho g} + \alpha\frac{V^2}{2g} + z\right)_1 = \left(\frac{P}{\rho g} + \alpha\frac{V^2}{2g} + z\right)_2 + h_L \quad (2)$$

Finally, here is the head form of the energy equation in its most useful form:

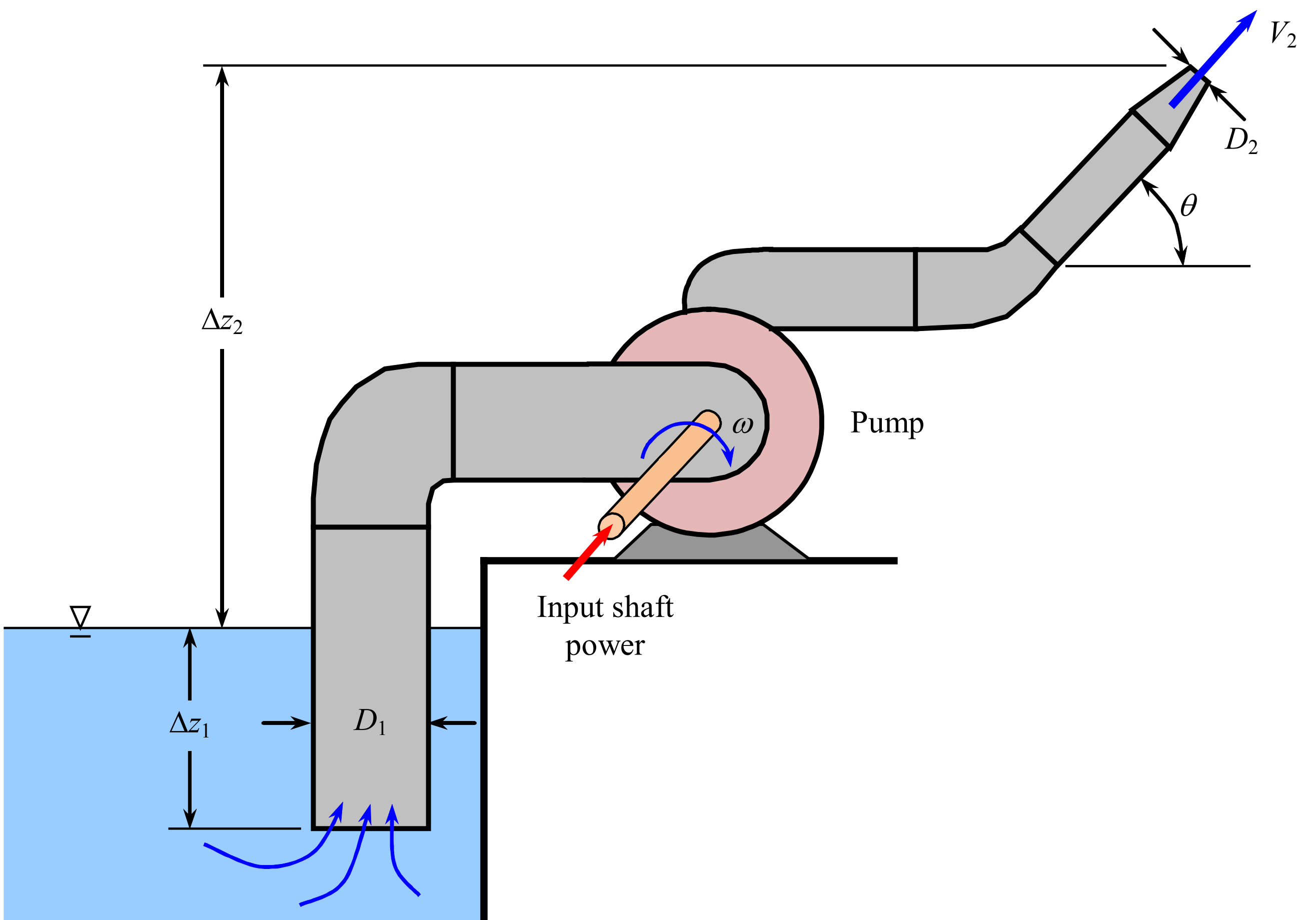
$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^{\,2}}{2g} + z_1 + \sum h_{\text{pump},u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^{\,2}}{2g} + z_2 + \sum h_{\text{turbine},e} + h_L \qquad (3)$$

The head form of the energy equation for *one* pump and *one* turbine:

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump},u}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine},e}+h_L$$

Example – Fire-fighting water pump

Given: A self-priming pump is used to draw water from a lake and shoot it through a nozzle, as sketched. The diameter of the pump inlet is D_1 = 12.0 cm. The diameter of the nozzle outlet is D_2 = 2.54 cm, and the average velocity at the nozzle outlet is V_2 = 65.8 m/s. The pump efficiency is 80%. The vertical distances are Δz_1 = 1.00 m and Δz_2 = 2.00 m. The irreversible head losses in the piping system (not counting inefficiencies associated with the pump itself) are estimated as h_L = 4.50 m of equivalent water column height. *Note*: Later on, in Chapter 8, you will learn how to calculate the irreversible head losses associated with piping systems on your own. For now, they are given.



(**a**) **To do**: Calculate the volume flow rate of the water in m³/hr and gallons per minute (gpm).

Solution: At the outlet, $\dot{V} = V_{2,\text{ avg}} A_2 = V_2 \frac{\pi D_2^{\;2}}{4} = \left(65.8 \frac{\text{m}}{\text{s}}\right) \frac{\pi \left(0.0254 \text{ m}\right)^2}{4} = 0.033341 \frac{\text{m}^3}{\text{s}}$, where we have dropped the subscript "avg" for convenience. We convert to the required units as follows:

$\dot{V} = 0.033341 \frac{\text{m}^3}{\text{s}} \left(\frac{3600 \text{ s}}{\text{hr}}\right) = \mathbf{120.} \frac{\mathbf{m^3}}{\mathbf{hr}}$ and $\dot{V} = 0.033341 \frac{\text{m}^3}{\text{s}} \left(\frac{15{,}850 \text{ gpm}}{\text{m}^3/\text{hr}}\right) = \mathbf{528.\ gpm}$, where both answers are given to three significant digits of precision.

(**b**) **To do**: Calculate the power delivered by the pump to the water, i.e. calculate the ***water horsepower*** $\dot{W}_{\text{water horsepower}}$ in units of kW.

(**c**) **To do**: Calculate the required shaft power to the pump, i.e. calculate the ***brake horsepower*** bhp in units of kW.

Solutions for parts (b) and (c) to be completed in class.

First, pick a control volume (always the first step!).
Now apply the head form of the energy equation:

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump},u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine},e} + h_L$$