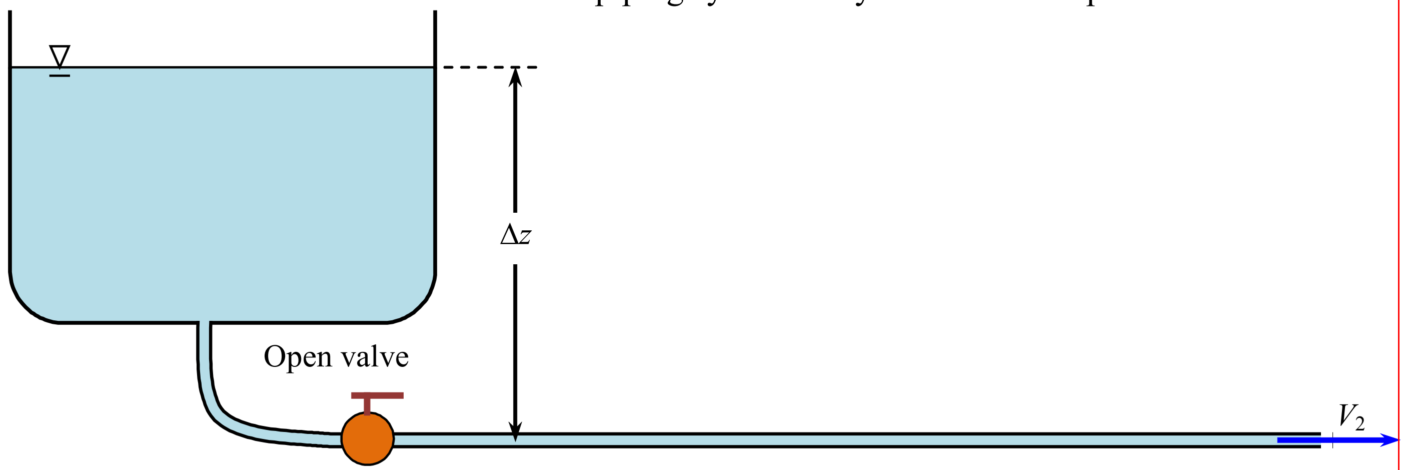
Today, we will:

- Do another example problem – head form of the energy equation
- Discuss grade lines – energy grade line and hydraulic grade line
- Derive and discuss the Bernoulli equation

5. Examples (continued)

Example – Water draining from a tank

Given: Water drains by gravity from a tank exposed to atmospheric pressure. The vertical distance from the pipe outlet to the surface of the water in the tank is Δz = 0.500 m. The irreversible head losses in the piping system (due to friction in the pipe, losses through the valve, elbow, etc.) are estimated as h_L = 0.400 m of equivalent water column height. *Note*: You will learn how to calculate the irreversible head losses associated with piping systems on your own in Chapter 8.



To do: Calculate the average velocity at the outlet, V_2.

Solution:

From previous lecture…use the head form of the conservation of energy equation:

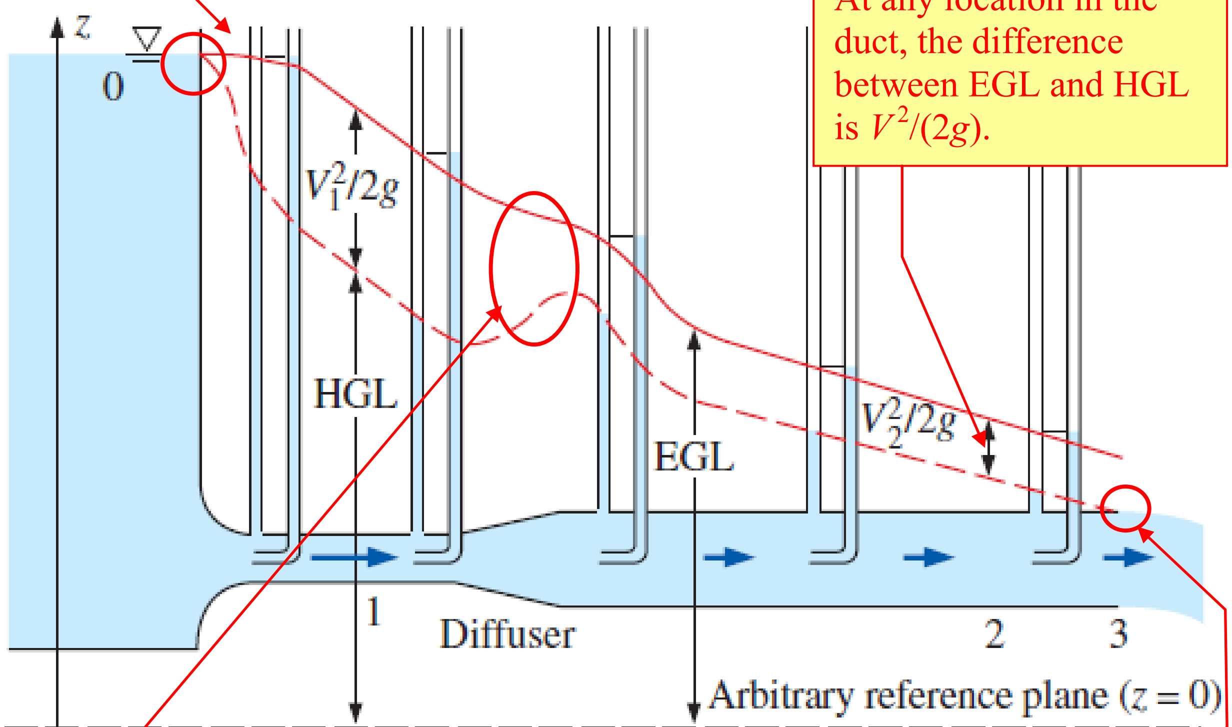
$$\left(\frac{P_1}{\rho_1 g}+\alpha_1\frac{V_1^2}{2g}+z_1\right)+\sum h_{\text{pump,u}}=\left(\frac{P_2}{\rho_2 g}+\alpha_2\frac{V_2^2}{2g}+z_2\right)+\sum h_{\text{turbine, e}}+h_L$$

Example of Grade Lines in a Fluid Flow

At point 0, HGL = EGL inside the tank, since the fluid is at rest ($V = 0$). Neither EGL or HGL can rise above this value unless work is added to the flow (e.g., with a pump).

FIGURE 5–35
The *hydraulic grade line* (HGL) and the *energy grade line* (EGL) for free discharge from a reservoir through a horizontal pipe with a diffuser.

At any location in the duct, the difference between EGL and HGL is $V^2/(2g)$.

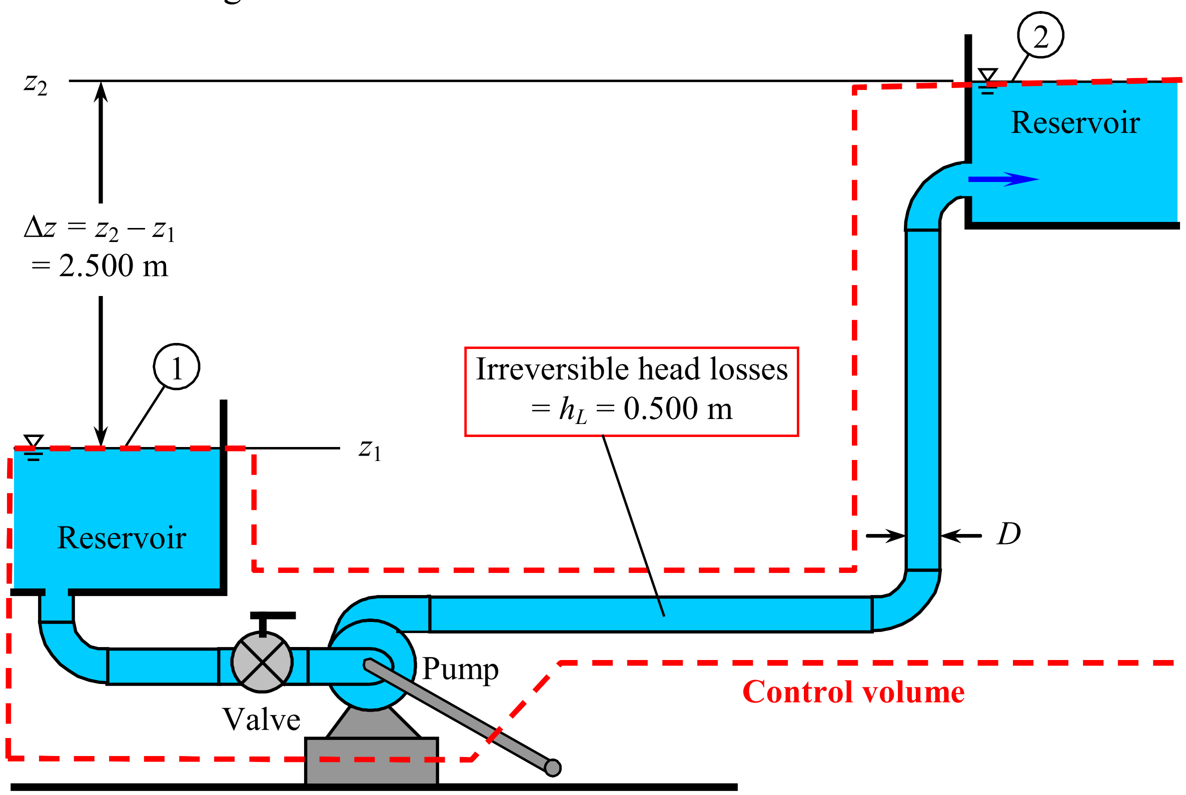


EGL continually falls due to irreversible losses, but HGL can rise or fall. Overall, however, HGL also must fall. In fact, HGL can *never* rise above EGL.

Since the jet exits at atmospheric pressure at the outlet of the pipe, $P_3 = P_{atm}$, and HGL is equal to the height of the free surface of the liquid.

Example – Pumping water from one reservoir to another

Given: Water is pumped from one reservoir to another. Both reservoirs are exposed to atmospheric pressure. $\Delta z = 2.50$ m. The irreversible head losses in the piping system (due to friction in the pipe, losses through the valve, elbows, etc.) are estimated as $h_L = 0.50$ m of equivalent water column height.



To do: Calculate $h_{\text{pump,u}}$, the useful pump head supplied to the water in meters of water.

Solution: First we pick a control volume wisely. The CV shown above is a *wise* CV.
Now apply our workhorse equation, the head form of the conservation of energy equation:

$$\left(\frac{P_1}{\rho_1 g} + \alpha_1 \frac{V_1^2}{2g} + z_1\right) + \sum h_{\text{pump,u}} = \left(\frac{P_2}{\rho_2 g} + \alpha_2 \frac{V_2^2}{2g} + z_2\right) + \sum h_{\text{turbine, e}} + h_L$$

D. The Bernoulli Equation

1. Derivation

Begin with the head form of the conservation of energy equation, but apply it ***along a streamline***:

$$\left(\frac{P_1}{\rho_1 g}+\frac{V_1^2}{2g}+z_1\right)+h_{\text{pump,u}}=\left(\frac{P_2}{\rho_2 g}+\frac{V_2^2}{2g}+z_2\right)+h_{\text{turbine, e}}+h_L$$