Today, we will:

- Discuss applications of the Bernoulli equation
- Derive and discuss the linear momentum equation for a control volume (Chapter 6)
- Discuss the momentum flux correction factor, β
- Discuss all the various forces acting on a control volume, and do some examples

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Recall, the beloved Bernoulli equation:

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2= \text{ constant along a sreamline}$$

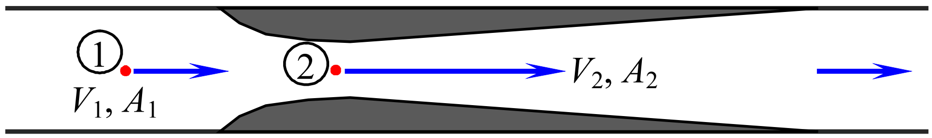
Note that the Bernoulli equation is valid *only* if *all* of the following limitations are met:

- no pumps or turbines
- steady
- incompressible
- negligible irreversible head losses (this is the limitation that usually restricts its use)

2. Applications of the Bernoulli equation

Example: Pressure drop through a Venturi tube

Given: Water flows horizontally in a round pipe with a converging-diverging section (a Venturi tube) as sketched. Cross-sectional area A_1 is four times larger than cross-sectional area A_2 (at the throat). Neglect any irreversible losses in the flow. The average speed at Section 1 is V_1 = 2.00 m/s.



To do: If $P_{1,\text{gage}}$ = 50,000 Pa, estimate $P_{2,\text{gage}}$ in units of Pa.

Solution:

E. The Linear Momentum Equation for a Control Volume (Chapter 6)

1. Equations and Definitions

Recall the RTT at the end of Chapter 4, applied to linear momentum:

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho \vec{V} dV + \int_{CS} \rho \vec{V} \left(\vec{V} \cdot \vec{n}\right) dA$$

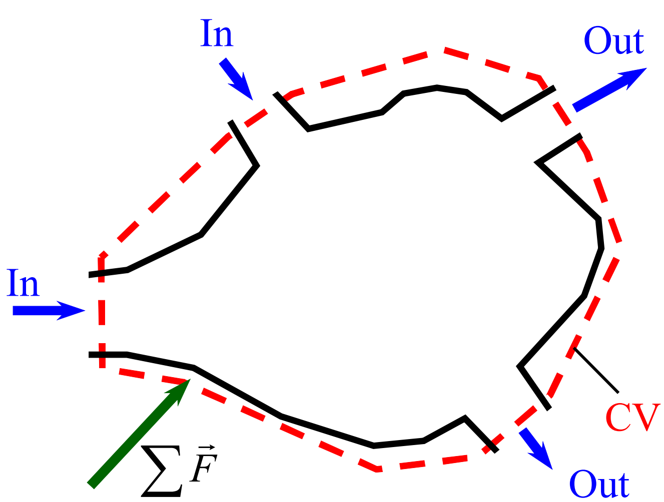
Total force (***vector***) acting on the control volume

Rate of change of linear momentum inside the control volume

Net rate of linear momentum flow out of the control volume

Use *relative* velocity $\vec{V}_r$ here if we have a moving or deforming control volume

Simplification for well-defined inlets and outlets

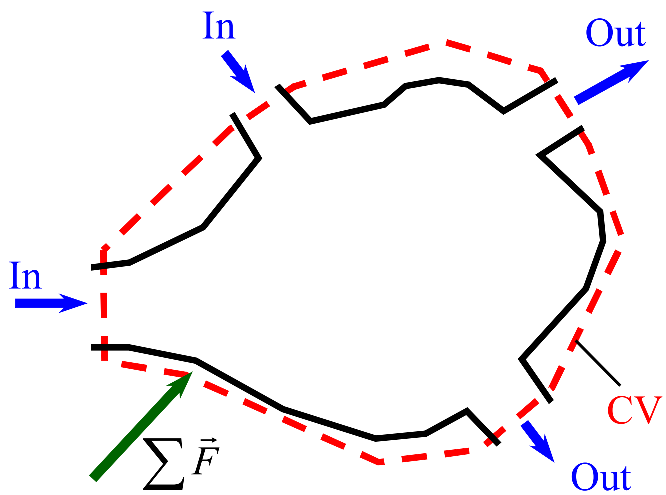


$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho \vec{V} dV + \sum_{\text{out}} \dot{m}\vec{V} - \sum_{\text{in}} \dot{m}\vec{V}$$

2. The momentum flux correction factor

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho \vec{V} dV + \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$$

3. Forces acting on a control volume



The approximate, most useful form of the linear momentum equation:

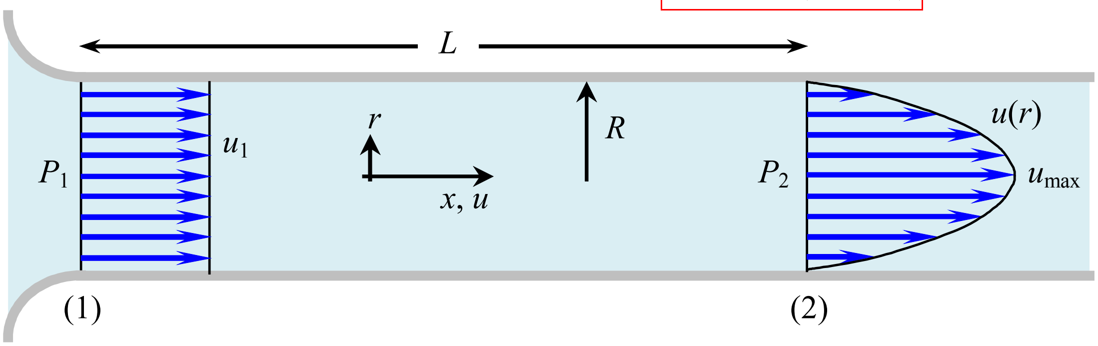
$$\sum \vec{F} = \sum \vec{F}_{\text{gravity}} + \sum \vec{F}_{\text{pressure}} + \sum \vec{F}_{\text{viscous}} + \sum \vec{F}_{\text{other}} = \frac{d}{dt}\int_{CV} \rho \vec{V} d\mathsf{V} + \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$$

4. Examples

Example: Friction force in a pipe

Given: Consider steady, laminar, incompressible, axisymmetric flow of a liquid in a pipe as sketched. At the inlet (1) there is a nice bell mouth, and the velocity is nearly uniform (except for a very thin boundary layer, not shown).

- At (1), $u = u_1$ = constant, $v = 0$, and $w = 0$. P_1 is measured.
- At (2), the flow is fully developed and parabolic: $u_2 = u_{max}\left(1 - \frac{r^2}{R^2}\right)$. P_2 is measured.



To do: Calculate the total friction force acting on the fluid by the pipe wall from 1 to 2.

Solution:

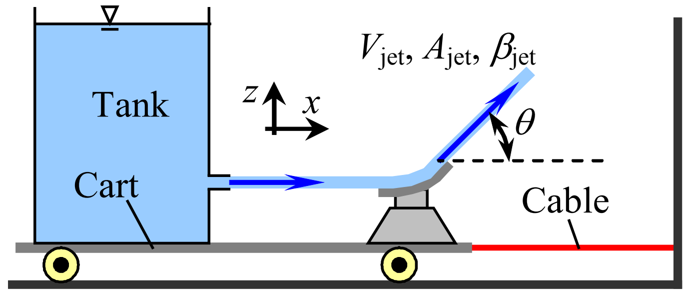
- First step:

- Now use the approximate, most useful form of the linear momentum equation,

$$\sum \vec{F} = \sum \vec{F}_{\text{gravity}} + \sum \vec{F}_{\text{pressure}} + \sum \vec{F}_{\text{viscous}} + \sum \vec{F}_{\text{other}} = \frac{d}{dt}\int_{CV} \rho \vec{V} dV + \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$$

Example: Tension in a cable

Given: A cart with frictionless wheels and a large tank shoots water at a deflector plate, turning it by angle θ as sketched. The cart tries to move to the left, but a cable prevents it from doing so. At the exit of the deflector, the water jet area A_{jet}, its average velocity V_{jet}, and its momentum flux correction factor β_{jet} are known.



To do: Calculate the tension T in the cable.

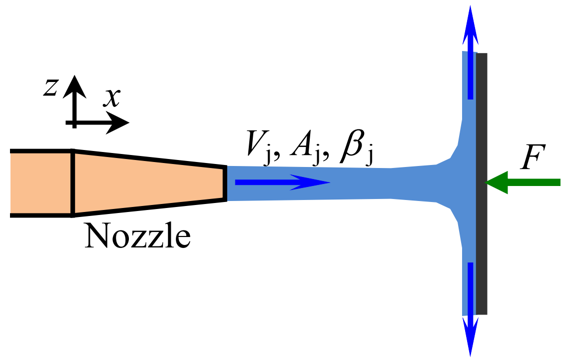
Solution:

- First step:
- Second step: Use the approximate, most useful form of the linear momentum equation,

$$\sum \vec{F} = \sum \vec{F}_{\text{gravity}} + \sum \vec{F}_{\text{pressure}} + \sum \vec{F}_{\text{viscous}} + \sum \vec{F}_{\text{other}} = \frac{d}{dt}\int_{CV} \rho \vec{V} dV + \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$$

Example: Force imparted by a water jet hitting a stationary plate

Given: A horizontal water jet of area A_j, average velocity V_j, and momentum flux correction factor β_j impinges normal to a stationary vertical flat plate.



To do: Calculate the horizontal force F required to keep the plate from moving.

Solution:

- First step:
- Second step: Use the approximate, most useful form of the linear momentum equation,

$$\sum \vec{F} = \sum \vec{F}_{\text{gravity}} + \sum \vec{F}_{\text{pressure}} + \sum \vec{F}_{\text{viscous}} + \sum \vec{F}_{\text{other}} = \frac{d}{dt}\int_{CV} \rho \vec{V} dV + \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$$