Today, we will:

- Discuss the Moody chart and the Colebrook equation for friction factor f
- Discuss major vs. minor losses in pipe flows, and do some example problems

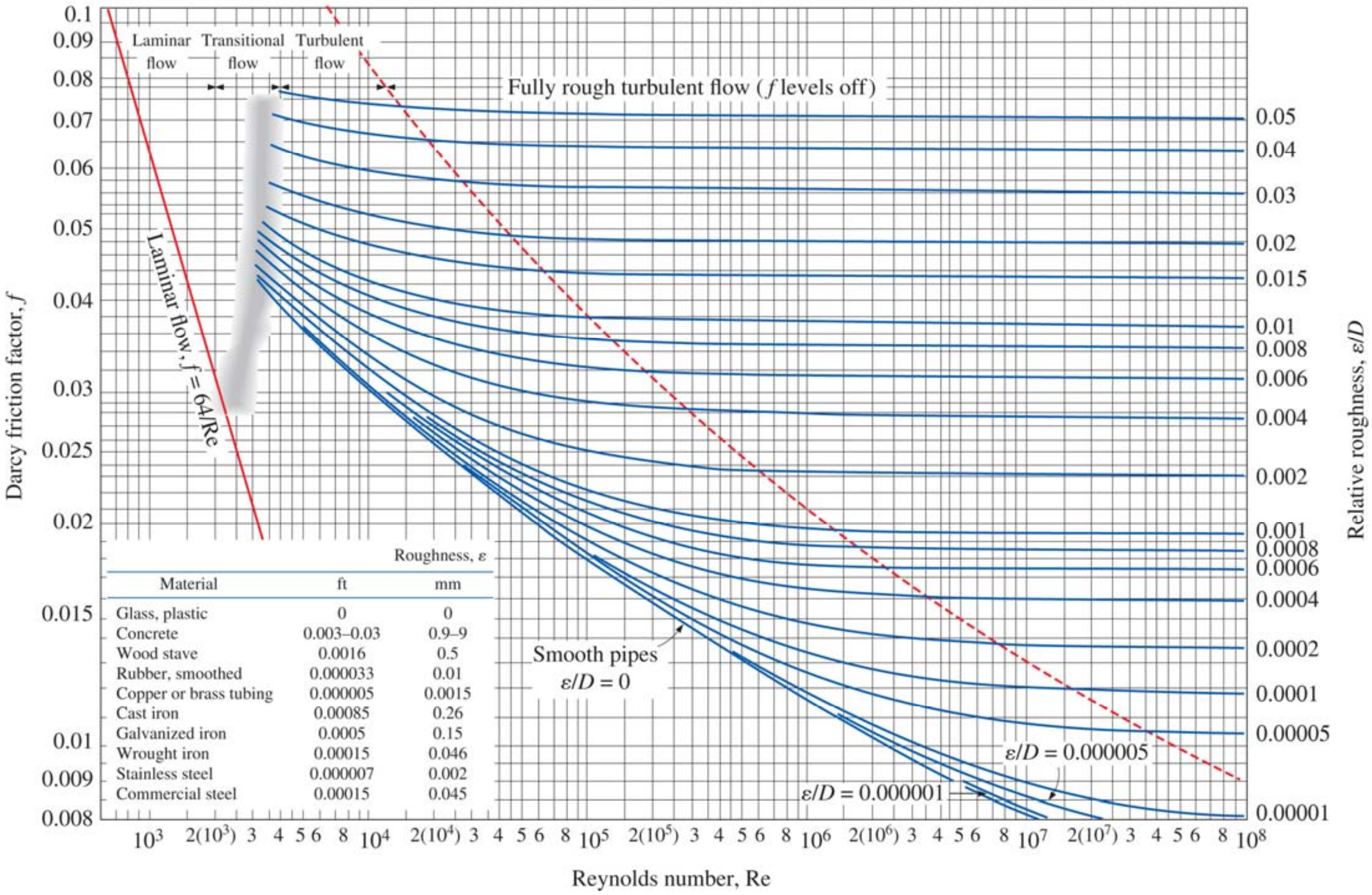
Consider steady, fully developed, incompressible pipe flow. Last time, we concluded that Darcy friction factor f is a function of (ρ, V, μ, D, ε). Dimensional analysis yields:

$$f = \text{fnc}\left(\text{Re}, \frac{\varepsilon}{D}\right) \quad \text{where} \quad f = \frac{8\tau_w}{\rho V^2} \quad \text{and} \quad \text{Re} = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$$

We also used control volume analysis to show that

$$h_L = \frac{4\tau_w}{\rho g}\frac{L}{D}$$

The Moody Chart (From Appendix, Figure A-12, in the textbook)



6. Empirical Equation for Fully Developed Pipe Flow

There are empirical equations available to use in place of the Moody chart. The most useful one (in fact, the equation with which the turbulent portion of the Moody chart is drawn) is:

The Colebrook equation

$$\frac{1}{\sqrt{f}} \approx -2.0\log_{10}\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

Note: This is $\log_{10}$, not the natural log, ln.

Unfortunately, the Colebrook equation is ***implicit*** in *f* (since *f* appears on both sides of the equation), and the equation must be solved by *iteration*. An approximation to the Colebrook equation was created by Haaland, accurate to ±2% compared to the Colebrook equation, and can be used as a quick estimate or as a "first guess" to begin a Colebrook equation iteration:

The Haaland equation

$$\frac{1}{\sqrt{f}} \approx -1.8\log_{10}\left[\frac{6.9}{\mathrm{Re}}+\left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]$$

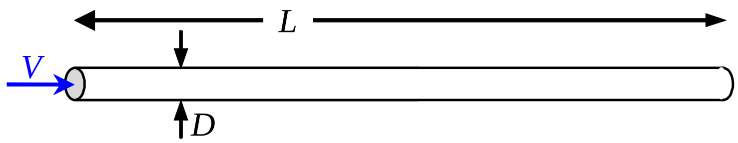
Also $\log_{10}$, not ln.

Finally, there are some other approximations in the textbook, e.g., those of Swamee and Jain. **In this course, we will always use the Colebrook equation or the Moody chart.**

7. Examples

Example: Hydrodynamic entrance length

Given: Water at 10.0 ^{o}C flows at a steady volume flow rate of 0.0100 m^3/s through a pipe of diameter 5.00 cm. The pipe is 100 m long, and the flow is fully developed through the entire section of pipe.



To do:

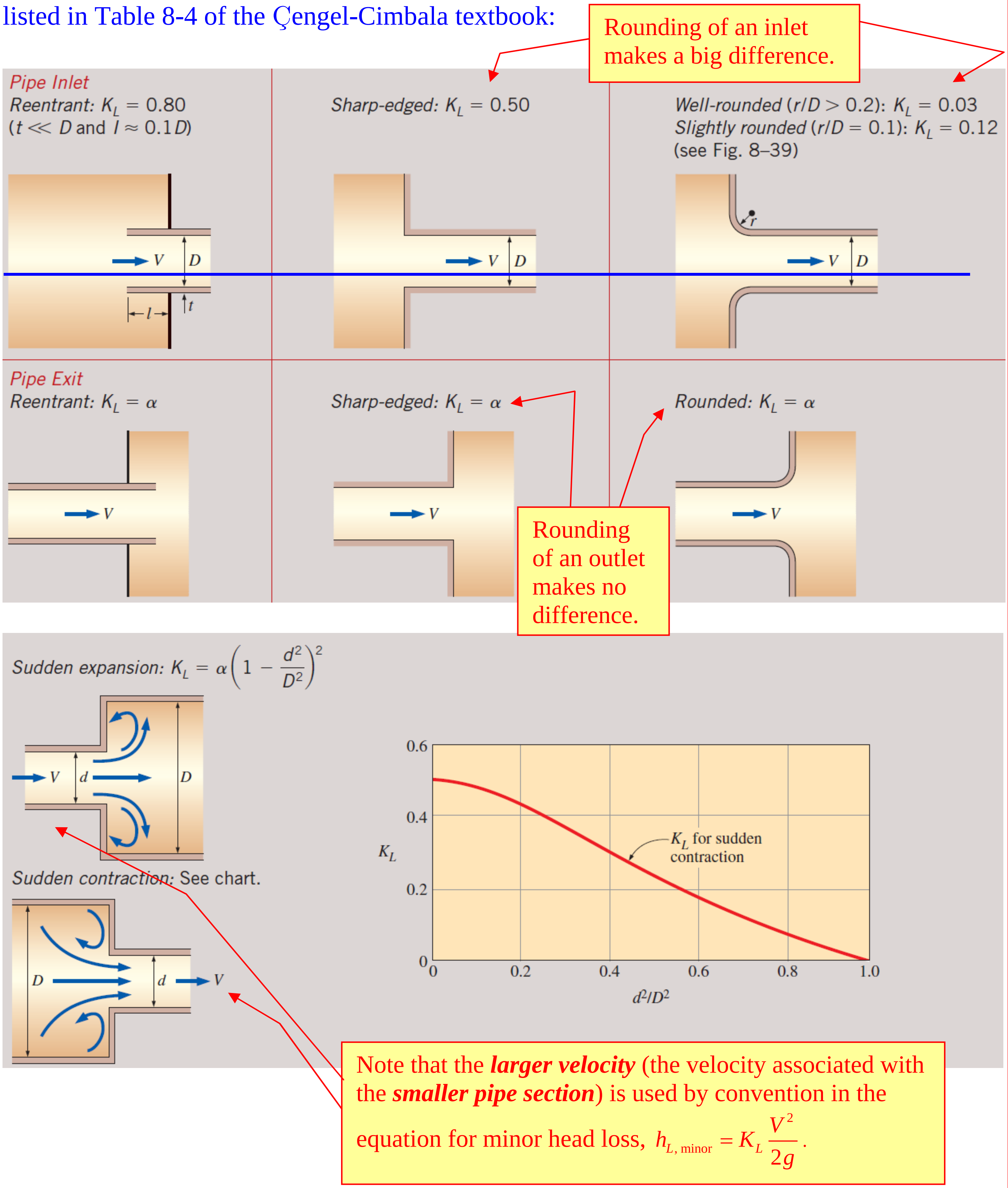
(a) Calculate the pressure drop if roughness height ε = 0.00050 cm.

(b) Calculate the pressure drop if roughness height ε = 0 (hydrodynamically smooth pipe).

Solution:

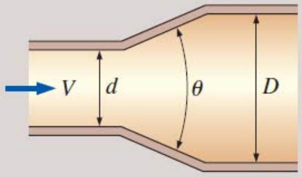
Minor Losses

Here are some sample loss coefficients for various minor loss components. More values are listed in Table 8-4 of the Çengel-Cimbala textbook:

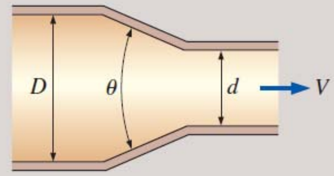


Gradual Expansion and Contraction (based on the velocity in the smaller-diameter pipe)

Expansion (for $\theta = 20°$):
$K_L = 0.30$ for $d/D = 0.2$
$K_L = 0.25$ for $d/D = 0.4$
$K_L = 0.15$ for $d/D = 0.6$
$K_L = 0.10$ for $d/D = 0.8$

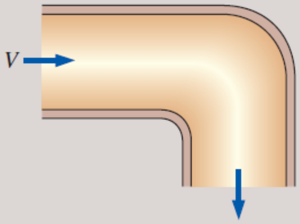


Contraction:
$K_L = 0.02$ for $\theta = 30°$
$K_L = 0.04$ for $\theta = 45°$
$K_L = 0.07$ for $\theta = 60°$

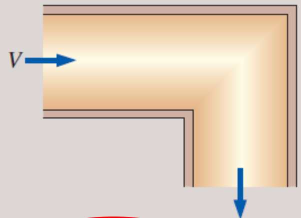


Bends and Branches

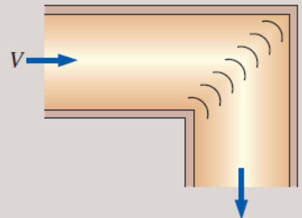
90° *smooth bend:*
Flanged: $K_L = 0.3$
Threaded: $K_L = 0.9$



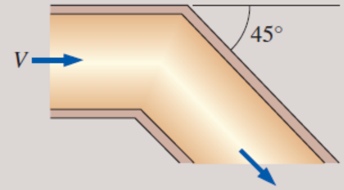
90° *miter bend* (without vanes): $K_L = 1.1$



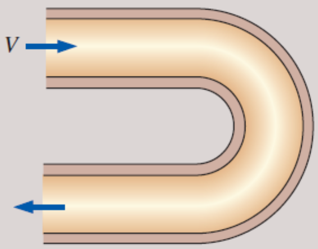
90° *miter bend* (with vanes): $K_L = 0.2$



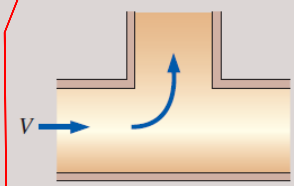
45° *threaded elbow:*
$K_L = 0.4$



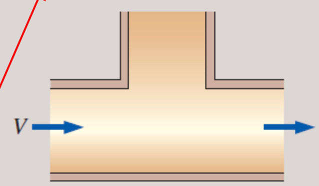
180° *return bend:*
Flanged: $K_L = 0.2$
Threaded: $K_L = 1.5$



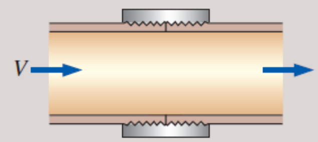
Tee (branch flow):
Flanged: $K_L = 1.0$
Threaded: $K_L = 2.0$



Tee (line flow):
Flanged: $K_L = 0.2$
Threaded: $K_L = 0.9$



Threaded union:
$K_L = 0.08$



For tees, there are *two* values of K_L, one for *branch flow* and one for *line flow*.

Valves

Globe valve, fully open: $K_L = 10$
Angle valve, fully open: $K_L = 5$
Ball valve, fully open: $K_L = 0.05$
Swing check valve: $K_L = 2$

Gate valve, fully open: $K_L = 0.2$
$\frac{1}{4}$ closed: $K_L = 0.3$
$\frac{1}{2}$ closed: $K_L = 2.1$
$\frac{3}{4}$ closed: $K_L = 17$