Today, we will:

- Discuss dimensional analysis of turbines
- Do an example problem – dimensional analysis with turbines
- Discuss piping networks – how to deal with pipes in series or in parallel

b. Dimensionless parameters in turbine performance

We perform exactly the same dimensional analysis for turbines as we did for pumps. Result:

Dimensionless Parameters: $C_Q = \dfrac{\dot{V}}{\omega D^3}$ $\quad C_H = \dfrac{gH}{\omega^2 D^2}$ $\quad C_P = \dfrac{bhp}{\rho \omega^3 D^5}$

Capacity coefficient — Head coefficient — Power coefficient

Example: Scaling up a hydroturbine

Given: An existing hydroturbine (A): Fluid is water at 20°C, D_A = 1.95 m, $\dot{n}_A = 120$ rpm, bhp_A = 220 MW, and $\dot{V}_A = 335\ \mathrm{m^3/s}$ at $H_A = 72.4$ m. We are designing a new turbine (B) that is geometrically similar, still uses water at 20°C, and $\dot{n}_B = 120$ rpm, but $H_B = 97.4$ m. [Dam B has a higher gross head available than Dam A.]

To do: (a) Calculate D_B and $\dot{V}_B$ for operation of turbine B at a homologous point.
(b) Calculate bhp_B and estimate the turbine efficiency of both turbines.

Solution:

(a) At homologous points, the two turbines are dynamically similar. Apply the affinity laws:

$$C_{H,A} = \frac{gH_A}{\omega_A{}^2 D_A{}^2} = C_{H,B} = \frac{gH_B}{\omega_B{}^2 D_B{}^2} \quad \rightarrow \quad \text{solve for } D_B = D_A\left(\frac{\omega_A}{\omega_B}\right)\sqrt{\frac{H_B}{H_A}} = D_A\left(\frac{\dot{n}_A}{\dot{n}_B}\right)\sqrt{\frac{H_B}{H_A}}$$

Plug in numbers:

Similarly, $$C_{Q,A} = \frac{\dot{V}_A}{\omega_A D_A{}^3} = C_{Q,B} = \frac{\dot{V}_B}{\omega_B D_B{}^3} \quad \rightarrow \quad \dot{V}_B = \dot{V}_A\left(\frac{\omega_B}{\omega_A}\right)\left(\frac{D_B}{D_A}\right)^3 = \dot{V}_A\left(\frac{\dot{n}_B}{\dot{n}_A}\right)\left(\frac{D_B}{D_A}\right)^3$$

Plug in numbers:

(b) Similarly, $C_{P,A} = \dfrac{bhp_A}{\rho_A \omega_A{}^3 D_A{}^5} = C_{P,B} = \dfrac{bhp_B}{\rho_B \omega_B{}^3 D_B{}^5} \quad \rightarrow \quad bhp_B = bhp_A \left(\dfrac{\rho_B}{\rho_A}\right)\left(\dfrac{\dot{n}_B}{\dot{n}_A}\right)^3 \left(\dfrac{D_B}{D_A}\right)^5$

Plug in numbers:

Finally, the efficiency is calculated for each turbine:

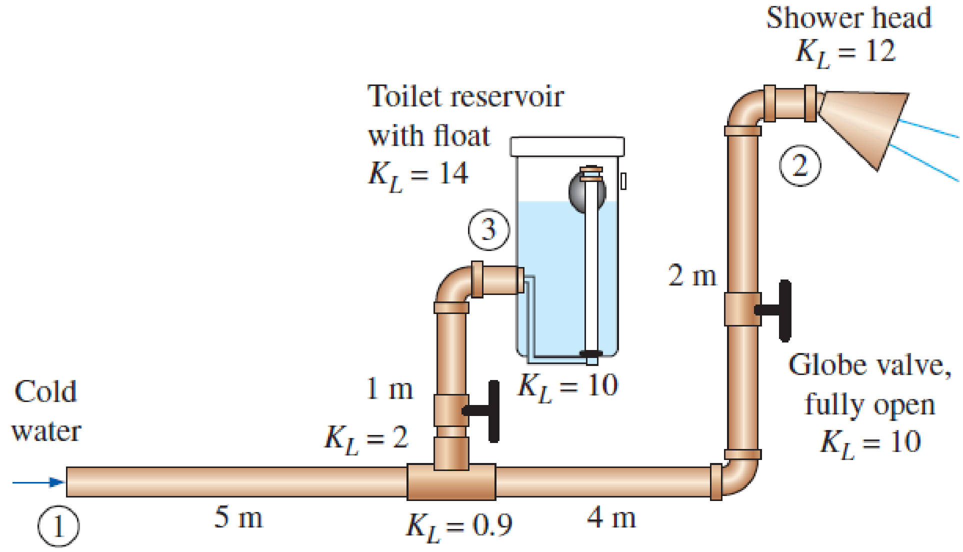
$$\eta_{\text{turbine},A} = \frac{bhp_A}{\rho_A g H_A \dot{V}_A} = \frac{220{,}000{,}000\ \text{W}}{\left(1000\,\dfrac{\text{kg}}{\text{m}^3}\right)\left(9.81\,\dfrac{\text{m}}{\text{s}^2}\right)72.4\ \text{m}\left(335\,\dfrac{\text{m}^3}{\text{s}}\right)}\left(\frac{\text{N}\cdot\text{m}}{\text{W}\cdot\text{s}}\right)\left(\frac{\text{kg}\cdot\text{m}}{\text{s}^2\cdot\text{N}}\right) =$$

$$\eta_{\text{turbine},B} = \frac{bhp_B}{\rho_B g H_B \dot{V}_B} = \frac{461{,}820{,}979\ \text{W}}{\left(1000\,\dfrac{\text{kg}}{\text{m}^3}\right)\left(9.81\,\dfrac{\text{m}}{\text{s}^2}\right)97.4\ \text{m}\left(522.728\,\dfrac{\text{m}^3}{\text{s}}\right)}\left(\frac{\text{N}\cdot\text{m}}{\text{W}\cdot\text{s}}\right)\left(\frac{\text{kg}\cdot\text{m}}{\text{s}^2\cdot\text{N}}\right) =$$

4. Examples

Example: Taking a Shower and Flushing a Toilet (E.g. 8-9, Çengel and Cimbala)

Given: This is a very practical everyday example of a parallel piping network! You are taking a shower. The piping is 1.50-cm coper pipes with threaded connectors as sketched. The gage pressure at the inlet of the system is 200 kPa, and the shower is on. The hot water is from a separate supply – only the cold water system is shown here.



To do:

(a) Calculate the volume flow rate $\dot{V}_2$ through the shower head when there is no water flowing through the toilet.

(b) Calculate the volume flow rate $\dot{V}_2$ through the shower head when someone flushes the toilet, and water flows into the toilet reservoir.

Solution: (copied from the textbook)

SOLUTION The cold-water plumbing system of a bathroom is given. The flow rate through the shower and the effect of flushing the toilet on the flow rate are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The flow is turbulent and fully developed. **3** The reservoir is open to the atmosphere. **4** The velocity heads are negligible.

This is a simplifying assumption that may or may not be valid. We should check the validity later.

We consider only the cold water line. The hot water line is separate, and is not connected to the toilet, so the volume flow rate of hot water through the shower remains constant. The cold water, however, is affected by flushing the toilet.

Properties The properties of water at 20°C are $\rho = 998\ \text{kg/m}^3$, $\mu = 1.002 \times 10^{-3}$ kg/m·s, and $\nu = \mu/\rho = 1.004 \times 10^{-6}\ \text{m}^2/\text{s}$. The roughness of copper pipes is $\varepsilon = 1.5 \times 10^{-6}$ m.

Analysis This is a problem of the second type since it involves the determination of the flow rate for a specified pipe diameter and pressure drop. The solution involves an iterative approach since the flow rate (and thus the flow velocity) is not known.

Part (a) is not a parallel system since no flow through the toilet.

(*a*) The piping system of the shower alone involves 11 m of piping, a tee with line flow (K_L = 0.9), two standard elbows (K_L = 0.9 each), a fully open globe valve (K_L = 10), and a shower head (K_L = 12). Therefore, $\sum K_L = 0.9 + 2 \times 0.9 + 10 + 12 = 24.7$. Noting that the shower head is open to the atmosphere, and the velocity heads are negligible, the energy equation for a control volume between points 1 and 2 simplifies to

$$\frac{P_1}{\rho g} + \boxed{\alpha_1 \frac{V_1^2}{2g}} + z_1 + h_{\text{pump}, u} = \frac{P_2}{\rho g} + \boxed{\alpha_2 \frac{V_2^2}{2g}} + z_2 + h_{\text{turbine}, e} + h_L$$

We neglect the velocity heads in the energy equation. Alternatively, if $V_1 = V_2$, then these two terms cancel each other out.

$$\rightarrow \quad \frac{P_{1,\text{ gage}}}{\rho g} = (z_2 - z_1) + h_L$$

$P_2 = P_{atm}$, and therefore $P_1 - P_2 = P_{1,gage}$.

Therefore, the head loss is

$$h_L = \frac{200{,}000 \text{ N/m}^2}{(998 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} - 2 \text{ m} = 18.4 \text{ m}$$

Also,

$$h_L = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g} \quad \rightarrow \quad 18.4 = \left(f\frac{11 \text{ m}}{0.015 \text{ m}} + 24.7\right)\frac{V^2}{2(9.81 \text{ m/s}^2)}$$

since the diameter of the piping system is constant. Equations for the average velocity in the pipe, the Reynolds number, and the friction factor are

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} \quad \rightarrow \quad V = \frac{\dot{V}}{\pi(0.015 \text{ m})^2/4}$$

$$\text{Re} = \frac{VD}{\nu} \quad \rightarrow \quad \text{Re} = \frac{V(0.015 \text{ m})}{1.004 \times 10^{-6} \text{ m}^2/\text{s}}$$

Only one Re and one Colebrook equation for Part (a)

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

$$\rightarrow \quad \frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{1.5 \times 10^{-6} \text{ m}}{3.7(0.015 \text{ m})} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

This is a set of four equations with four unknowns, and solving them with an equation solver such as EES gives

Answer to part (a) – no toilet flushing

$$\dot{V} = 0.00053\ \text{m}^3/\text{s},\quad f = 0.0218,\quad V = 2.98\ \text{m/s},\quad \text{and}\quad \text{Re} = 44{,}550$$

Therefore, the flow rate of water through the shower head is $\boxed{\textbf{0.53 L/s.}}$

(*b*) When the toilet is flushed, the float moves and opens the valve. The discharged water starts to refill the reservoir, resulting in parallel flow after the tee connection. The head loss and minor loss coefficients for the shower branch were determined in (*a*) to be $h_{L,\,2} = 18.4$ m and $\sum K_{L,\,2} = 24.7$, respectively. The corresponding quantities for the reservoir branch can be determined similarly to be

$$h_{L,\,3} = \frac{200{,}000\ \text{N/m}^2}{(998\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} - 1\ \text{m} = 19.4\ \text{m}$$

$$\sum K_{L,\,3} = 2 + 10 + 0.9 + 14 = 26.9$$

The relevant equations in this case are

$$\dot{V}_1 = \dot{V}_2 + \dot{V}_3$$

$$h_{L,\,2} = f_1 \frac{5\ \text{m}}{0.015\ \text{m}} \frac{V_1^2}{2(9.81\ \text{m/s}^2)} + \left(f_2 \frac{6\ \text{m}}{0.015\ \text{m}} + 24.7\right) \frac{V_2^2}{2(9.81\ \text{m/s}^2)} = 18.4$$

$$h_{L,\,3} = f_1 \frac{5\ \text{m}}{0.015\ \text{m}} \frac{V_1^2}{2(9.81\ \text{m/s}^2)} + \left(f_3 \frac{1\ \text{m}}{0.015\ \text{m}} + 26.9\right) \frac{V_3^2}{2(9.81\ \text{m/s}^2)} = 19.4$$

$$V_1 = \frac{\dot{V}_1}{\pi(0.015\ \text{m})^2/4},\quad V_2 = \frac{\dot{V}_2}{\pi(0.015\ \text{m})^2/4},\quad V_3 = \frac{\dot{V}_3}{\pi(0.015\ \text{m})^2/4}$$

$$\text{Re}_1 = \frac{V_1(0.015\ \text{m})}{1.004 \times 10^{-6}\text{m}^2/\text{s}},\quad \text{Re}_2 = \frac{V_2(0.015\ \text{m})}{1.004 \times 10^{-6}\text{m}^2/\text{s}},\quad \text{Re}_3 = \frac{V_3(0.015\ \text{m})}{1.004 \times 10^{-6}\text{m}^2/\text{s}}$$

This is where EES comes in handy – solving all these simultaneous equations! (12 equations and 12 unknowns)

$$\frac{1}{\sqrt{f_1}} = -2.0 \log\left(\frac{1.5 \times 10^{-6}\ \text{m}}{3.7(0.015\ \text{m})} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right)$$

$$\frac{1}{\sqrt{f_2}} = -2.0 \log\left(\frac{1.5 \times 10^{-6}\ \text{m}}{3.7(0.015\ \text{m})} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right)$$

$$\frac{1}{\sqrt{f_3}} = -2.0 \log\left(\frac{1.5 \times 10^{-6}\ \text{m}}{3.7(0.015\ \text{m})} + \frac{2.51}{\text{Re}_3\sqrt{f_3}}\right)$$

Now we need ***three*** Colebrook equations – one for each branch!

Solving these 12 equations in 12 unknowns simultaneously using an equation solver, the flow rates are determined to be

Answer to part (b) – with toilet flushing

$$\dot{V}_1 = 0.00090 \text{ m}^3/\text{s}, \quad \boxed{\dot{V}_2 = 0.00042 \text{ m}^3/\text{s},} \quad \text{and} \quad \dot{V}_3 = 0.00048 \text{ m}^3/\text{s}$$

Therefore, the flushing of the toilet **reduces the flow rate of cold water through the shower by 21 percent** from 0.53 to 0.42 L/s, causing the shower water to suddenly get very hot (Fig. 8–53).

Discussion If the velocity heads were considered, the flow rate through the shower would be 0.43 instead of 0.42 L/s. Therefore, the assumption of negligible velocity heads is reasonable in this case. Note that a leak in a piping system would cause the same effect, and thus an unexplained drop in flow rate at an end point may signal a leak in the system.

FIGURE 8–53
Flow rate of cold water through a shower may be affected significantly by the flushing of a nearby toilet.

EES Solution – Toilet Flushing Example Problem

Part (a) EES Equation window:

```
"Example 8.9 - Toilet Flushing Problem, Part (a)"

"Contants and properties:"
g = g#
rho = 998 [kg/m^3]
mu = 1.002E-3 [kg/m-s]
D = 0.015 [m]
epsilon = 1.5e-6 [m]
P_gage_1 = 200000 [N/m^2]
L_1  = 5 [m]
L_2 = 6 [m]
SIGMAK_L_1 = 0
SIGMAK_L_2 = 0.9 + 2*0.9 + 10 + 12   "tee, two elbows, valve, and shower head"
DELTAz_1to2 = 2 [m]

"Equations to solve"
P_gage_1/(rho*g) = DELTAz_1to2 + h_L_1to2
h_L_1to2 = V^2/(2*g) * (f_1to2*(L_1+L_2)/D + SIGMAK_L_1 + SIGMAK_L_2)
A = pi*D^2/4
V_dot = V*A
Re = V*D*rho/mu
f_1to2 = MoodyChart(Re,epsilon/D)
V_dot_LPS = V_dot*CONVERT(m^3/s,L/s)
```

Part (a) EES Solution:

Unit Settings: SI K kPa kJ molar deg

$A = 0.0001767$ [m^2]	$D = 0.015$ [m]	$\Delta z_{1to2} = 2$ [m]
$\varepsilon = 0.0000015$ [m]	$f_{1to2} = 0.0217$	$g = 9.807$ [m/s^2]
$h_{L,1to2} = 18.43$ [m]	$L_1 = 5$ [m]	$L_2 = 6$ [m]
$\mu = 0.001002$ [kg/m-s]	$P_{gage,1} = 200000$ [N/m^2]	$Re = 44576$ [-]
$\rho = 998$ [kg/m^3]	$SIGMAK_{L,1} = 0$	$SIGMAK_{L,2} = 24.7$
$V = 2.984$ [m/s]	$\dot{V} = 0.0005273$ [m^3/s]	$\dot{V}_{LPS} = 0.5273$ [L/s]

No unit problems were detected.

Part (b) EES Equation Window:

```
"Example 8.9 - Toilet Flushing Problem, Part (b)"

"Contants and properties:"
g = g#
rho = 998 [kg/m^3]
mu = 1.002E-3 [kg/m-s]
D = 0.015 [m]
epsilon = 1.5e-6 [m]
P_gage_1 = 200000 [N/m^2]
L_1  = 5 [m]
L_2 = 6 [m]
L_3 = 1 [m]
SIGMAK_L_1 = 0
SIGMAK_L_2 = 0.9 + 2*0.9 + 10 + 12   "tee, two elbows, valve, and shower head"
SIGMAK_L_3 = 2 + 10 + 0.9 + 14        "tee, one elbow, valve, and toilet mechanism"
DELTAz_1to2 = 2 [m]
DELTAz_1to3 = 1 [m]

"Equations to solve"
P_gage_1/(rho*g) = DELTAz_1to2 + h_L_1to2
P_gage_1/(rho*g) = DELTAz_1to3 + h_L_1to3
h_L_1to2 = V_1^2/(2*g) * (f_1*L_1/D + SIGMAK_L_1) + V_2^2/(2*g) * (f_2*L_2/D + SIGMAK_L_2)
h_L_1to3 = V_1^2/(2*g) * (f_1*L_1/D + SIGMAK_L_1) + V_3^2/(2*g) * (f_3*L_3/D + SIGMAK_L_3)
A = pi*D^2/4
V_dot_1 = V_1*A
V_dot_2 = V_2*A
V_dot_3 = V_3*A
Re_1 = V_1*D*rho/mu
Re_2 = V_2*D*rho/mu
Re_3 = V_3*D*rho/mu
f_1 = MoodyChart(Re_1,epsilon/D)
f_2 = MoodyChart(Re_2,epsilon/D)
f_3 = MoodyChart(Re_3,epsilon/D)
V_dot_1 = V_dot_2 + V_dot_3
V_dot_2_LPS = V_dot_2*CONVERT(m^3/s,L/s)
```

Part (b) EES Solution:

$A = 0.0001767$ [m^2]	$D = 0.015$ [m]	$\Delta z_{1to2} = 2$ [m]
$\Delta z_{1to3} = 1$ [m]	$\varepsilon = 0.0000015$ [m]	$f_1 = 0.01943$
$f_2 = 0.0228$ [-]	$f_3 = 0.02212$ [-]	$g = 9.807$ [m/s^2]
$h_{L,1to2} = 18.43$ [m]	$h_{L,1to3} = 19.43$ [m]	$L_1 = 5$ [m]
$L_2 = 6$ [m]	$L_3 = 1$ [m]	$\mu = 0.001002$ [kg/m-s]
$P_{gage,1} = 200000$ [N/m^2]	$Re_1 = 76419$ [-]	$Re_2 = 35608$ [-]
$Re_3 = 40811$ [-]	$\rho = 998$ [kg/m^3]	$SIGMAK_{L,1} = 0$
$SIGMAK_{L,2} = 24.7$	$SIGMAK_{L,3} = 26.9$	$V_1 = 5.115$ [m/s]
$V_2 = 2.383$ [m/s]	$V_3 = 2.732$ [m/s]	$\dot{V}_1 = 0.0009039$ [m^3/s]
$\dot{V}_2 = 0.0004212$ [m^3/s]	$\dot{V}_{2,LPS} = 0.4212$ [L/s]	$\dot{V}_3 = 0.0004827$ [m^3/s]