Today, we will:

- Do more example problems – continuity equation (Cartesian & cylindrical coordinates)
- Discuss the stream function and its physical significance, and do some examples

Example: Continuity equation

Given: A velocity field is given by

$$u = ax + b$$
$$v = \text{ unknown}$$
$$w = 0$$

To do: Derive an expression for v so that this a valid steady, incompressible velocity field.

Solution: To be a vaild steady, incompressible velocity field, it must satisfy continuity!

$$\frac{\partial u}{\partial x} \quad + \quad \frac{\partial v}{\partial y} \quad + \quad \frac{\partial w}{\partial z} \quad = 0$$

Example: Continuity equation

Given: A flow field is 2-D in the r-θ plane, and its velocity field is given by

$$u_r = \text{ unknown}$$
$$u_\theta = c\theta$$
$$u_z = 0$$

To do: Derive an expression for u_r so that this a valid steady, incompressible velocity field.

Solution: To be a vaild steady, incompressible velocity field, it must satisfy continuity!

$$\frac{1}{r}\frac{\partial}{\partial r}(ru_r) \quad + \quad \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} \quad + \quad \frac{\partial u_z}{\partial z} \quad = 0$$

Example: Continuity equation

Given: A flow field is 2-D in the r-θ plane, and its velocity field is given by

$$u_r = -\frac{3}{r} + 2$$

$$u_\theta = 2r + a\theta$$

$$u_z = 0$$

To do: Calculate a such that this a valid steady, incompressible velocity field.

Solution: To be a vaild steady, incompressible velocity field, it must satisfy continuity!

$$\frac{1}{r}\frac{\partial}{\partial r}\left(ru_r\right) \quad + \quad \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} \quad + \quad \frac{\partial u_z}{\partial z} \quad = 0$$

Example: Stream function

Given: A flow field is 2-D in the *x-y* plane, and its stream function is given by

$$\psi\left(x,y\right)=ax^3+byx$$

To do: Calculate the velocity components and verify that this stream function represents a valid steady, incompressible velocity field.

Solution: By definition of the stream function,

$$u=\frac{\partial\psi}{\partial y} \qquad\qquad v=-\frac{\partial\psi}{\partial x}$$

To be a vaild steady, incompressible velocity field, it must satisfy continuity!

$$\frac{\partial u}{\partial x} \quad + \quad \frac{\partial v}{\partial y} \quad + \quad \frac{\partial w}{\partial z} \quad =0$$

3. Examples
[See more examples in the textbook.]

Example: Stream function and how to plot streamlines

Given: A steady, 2-D, incompressible flow is given by this velocity field:

$$u = x^2$$

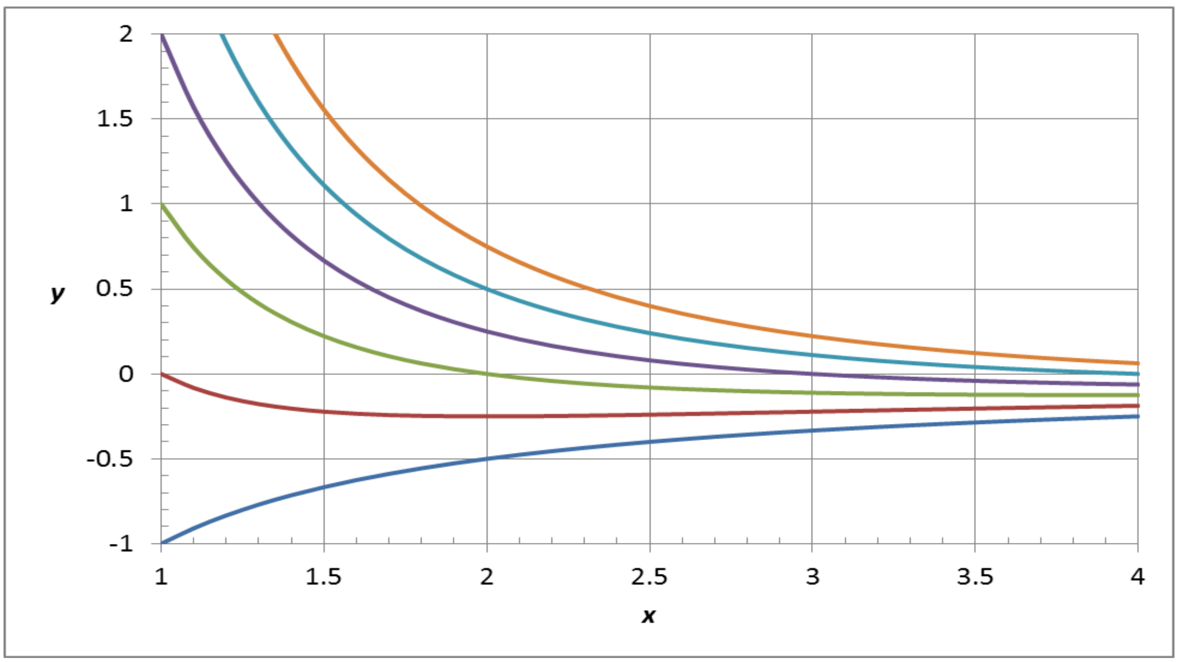
$$v = -2xy - 1$$

$$w = 0$$

To do: Generate an expression for stream function $\psi(x,y)$ and plot some streamlines.

Solution:
First, it is wise to verify that this is a vaild steady, incompressible velocity field. If not, the problem is ill-posed and we could not continue. To verify, it must satisfy continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$



Example: Streamlines and volume flow rate

Given: The 2-D flow field shown above. The width (into the page) is 0.50 m.

To do: Calculate the volume flow rate (in units of m^3/s) between streamlines $\psi = 1\ m^2/s$ and $\psi = 4\ m^2/s$.

Solution:

4. Stream function in cylindrical coordinates

c. Examples
[See more examples in the textbook.]

Example: Streamlines in Cylindrical Coordinates

Given: A flow field is steady and 2-D in the r-θ plane, and its stream function is given by

$$\psi = V_\infty r \sin\theta$$

To do:

(**a**) Derive expressions for u_r and u_θ and convert to u and v.

(**b**) Sketch the streamlines for this flow field.

Solution: