- Continue discussing the irrotational flow approximation and introduce **superposition**.
- Discuss some elementary planar irrotational flows (building block flows)
- Do some examples of superposition

E. The Irrotational Flow Approximation (continued)

1. Introduction
2. Equations of Motion for Irrotational Flow
3. 2-D Irrotational Flow (continued)
   a. Equations of motion

Summary, equations for 2-D, steady, incompressible, irrotational flow in the *x*-*y* plane:

$\vec{\zeta} = \vec{\nabla} \times \vec{V} = 0 \rightarrow \vec{V} = \vec{\nabla}\phi \rightarrow \nabla^2\phi = 0$; $\nabla^2\psi = 0$ & $\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant everywhere}$

Cartesian: $\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$, Cylindrical: $\nabla^2\phi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} = 0$

Cartesian: $u = \frac{\partial\psi}{\partial y}$ $v = -\frac{\partial\psi}{\partial x}$, Cylindrical planar ($r$-$\theta$ plane): $u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta}$ $u_\theta = -\frac{\partial\psi}{\partial r}$

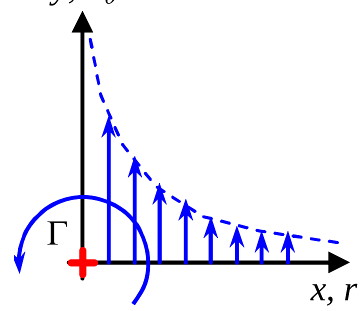
   b. Superposition

c. Line vortex

Example: Line Vortex – a third building block potential flow

Given: A line vortex around the origin of strength Γ (Γ is also called the **circulation**). The flow is steady and 2-D in the r-θ plane, and its velocity field is given by

$$u_r = 0 \qquad\qquad u_\theta = \frac{\Gamma}{2\pi r}$$



To do: Sketch streamlines and equipotential lines, and generate expressions for ψ and ϕ.

Solution:

Example: Rankine Half-Body

Given: A Rankine half-body is constructed using a horizontal freestream of velocity V = 5.0 m/s and line source at the origin of strength 2.5π m^2/s. The stream function is

$$\psi = Vr\sin\theta + \frac{1}{2\pi}\frac{\dot{V}}{L}\theta$$

To do: Generate expressions for u_r and u_θ, and calculate the distance a (the distance between the origin and the stagnation point).

Solution: