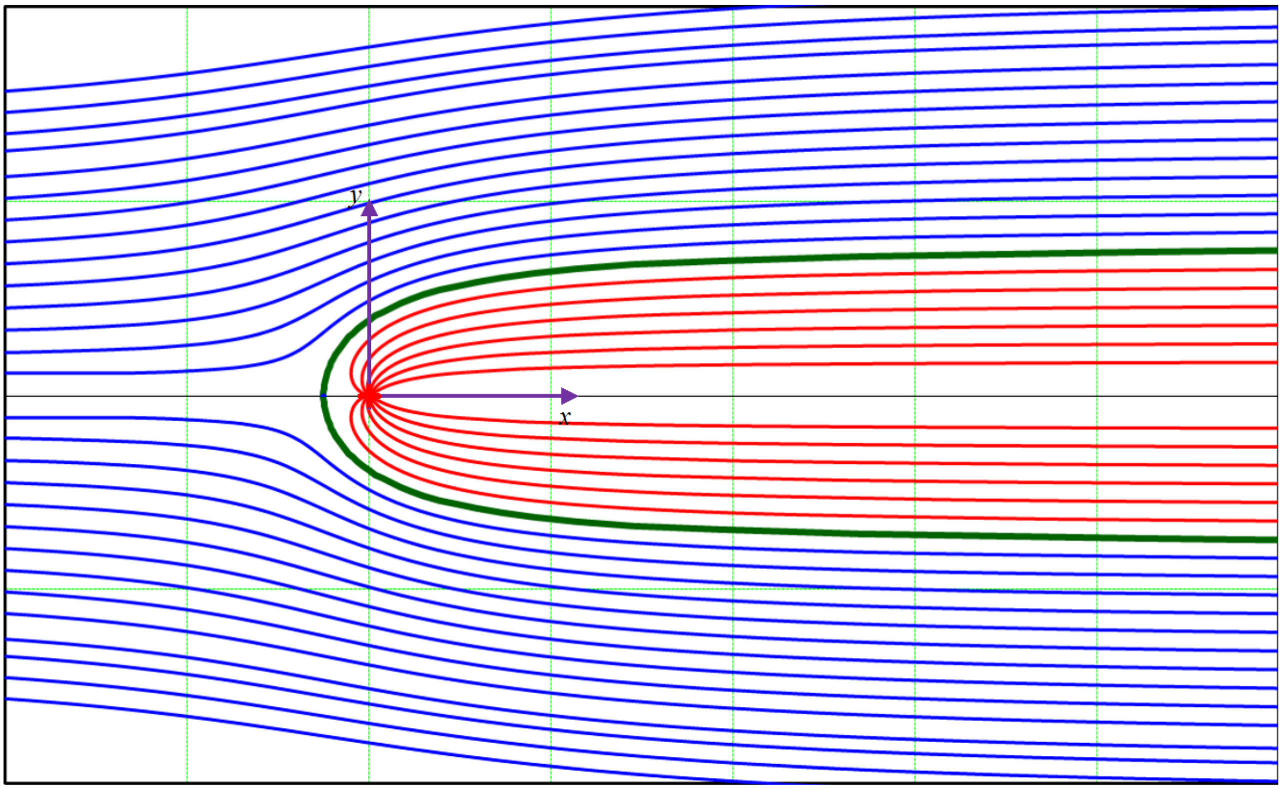
Today, we will:

- Continue examples of superposition of irrotational flows – flow over a circular cylinder
- Start discussing the last approximation of Chapter 10: **The Boundary Layer Approx.**

Recall, the Rankine half-body:



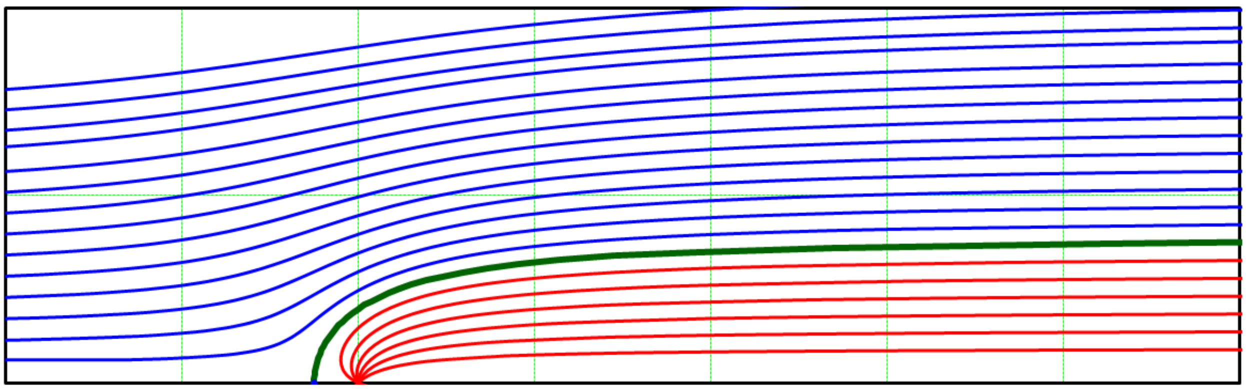
Example: Rankine Half-Body

Given: A Rankine half-body is constructed using a horizontal freestream of velocity V = 5.0 m/s and line source at the origin of strength $\frac{\dot{V}}{L} = 2.5\pi \ \frac{\text{m}^2}{\text{s}}$. The stream function is

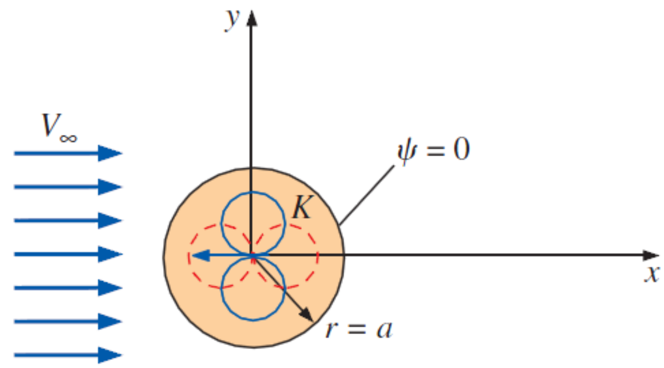
$$\psi = Vr\sin\theta + \frac{1}{2\pi}\frac{\dot{V}}{L}\theta$$

To do: Generate expressions for u_r and u_θ, and calculate the distance a (the distance between the origin and the stagnation point).

Solution:



b. Example of superposition: Flow over a circular cylinder

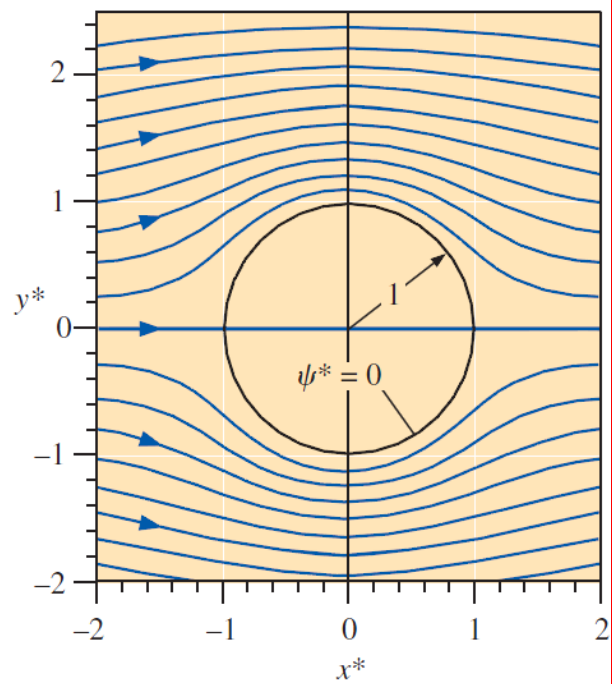


Given: Superpose a uniform stream of velocity V_∞ and a doublet of strength K at the origin.

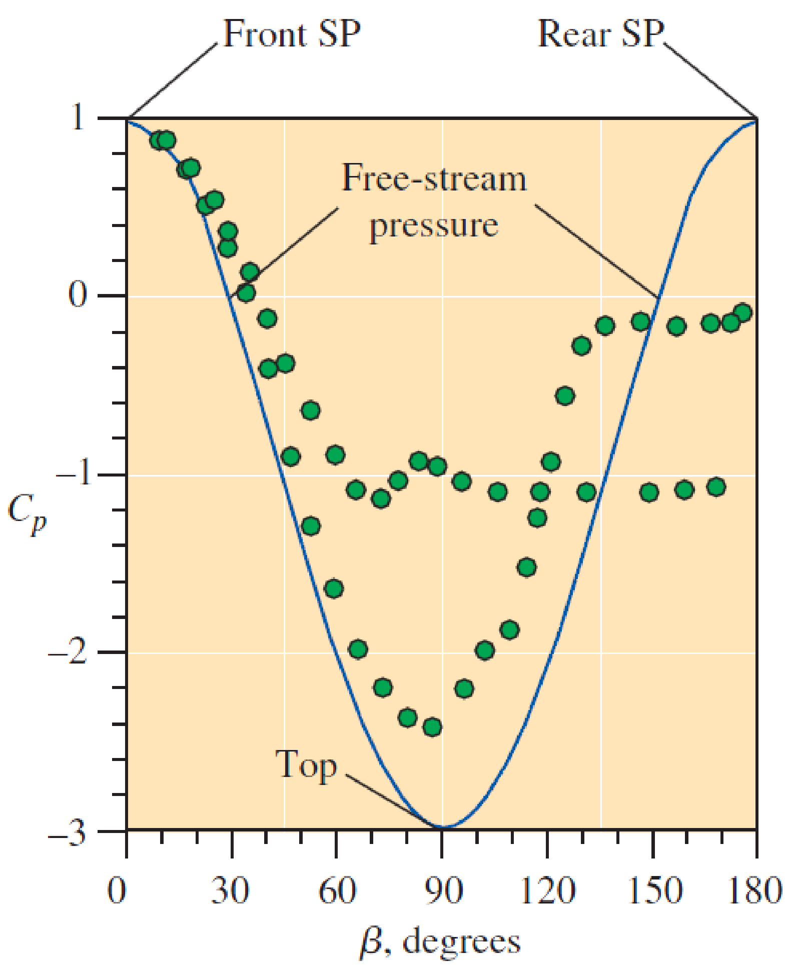
To do: Plot streamlines, and discuss the flow that results from this superposition.

Solution:

- We simply add up the stream functions for the two building block flows: $\psi = \psi_{\text{freestream}} + \psi_{\text{doublet}} = V_\infty y - K\dfrac{\sin\theta}{r}$.
- But we know that $\boxed{y = r\sin\theta}$, thus, $\boxed{\psi = V_\infty r\sin\theta - K\dfrac{\sin\theta}{r}}$.
- For "convenience", and with hindsight, we choose to set $\psi = 0$ at $r = a$.
  [It turns out that radius a is a special radius that becomes the radius of the circle.]
- Set $r = a$ in our equation for the stream function:
  $$0 = V_\infty a\sin\theta - K\frac{\sin\theta}{a} \quad\rightarrow\quad \boxed{K = V_\infty a^2}.$$
- Then our final expression for ψ becomes
  $$\boxed{\psi = V_\infty \sin\theta\left(r - \frac{a^2}{r}\right)}.$$



- Plot streamlines: [we plot nondimensionally, setting $x^*=x/a$ and $y^*=y/a$]
- From our equation for ψ above, we can calculate the velocity field from the definition of ψ, i.e., $u_r = \dfrac{1}{r}\dfrac{\partial\psi}{\partial\theta}$ $\quad u_\theta = -\dfrac{\partial\psi}{\partial r}$. See text for details. On the cylinder ($r = a$),
  $\boxed{u_r = 0 \quad u_\theta = -2V_\infty\sin\theta}$
- We can also define the **pressure coefficient**, $\boxed{C_p = \dfrac{P - P_\infty}{\frac{1}{2}\rho V_\infty^{\,2}} = 1 - \dfrac{V^2}{V_\infty^{\,2}}}$.
- *On the cylinder*, it turns out that $\boxed{C_p = 1 - 4\sin^2\beta}$, where β is the angle from the nose.



Front SP
Rear SP
Free-stream pressure
Top
1
0
−1
−2
−3
C_p
0
30
60
90
120
150
180
β, degrees

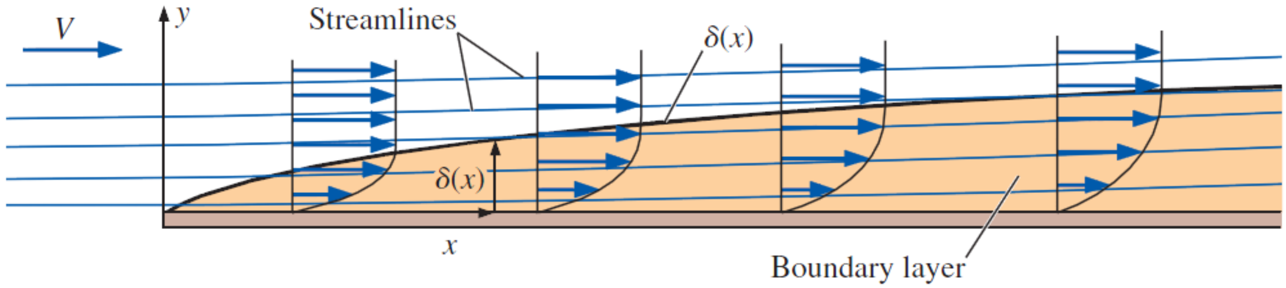
F. The Boundary Layer Approximation

1. Introduction

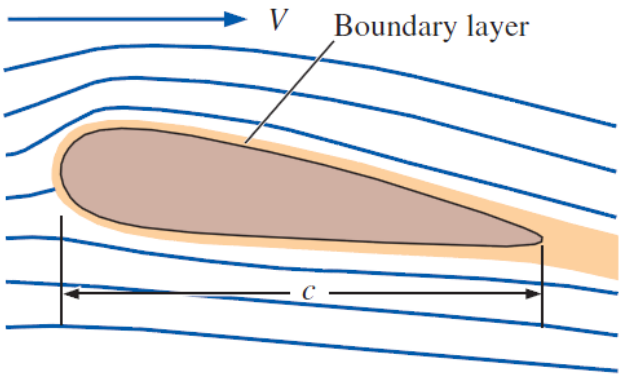
Definition: A ***boundary layer*** is a thin layer in which viscous effects and vorticity are significant, and cannot be ignored.

Examples

- BL on a flat plate aligned with the freestream flow (we show top side only):



- BL on an airfoil:



2. The Boundary Layer Coordinate System

In a 2-D flow, we let x = distance along the wall, and y = distance normal to the wall.