Today, we will:

- Discuss the BL equations and the BL procedure
- Do a BL example, boundary layer on a flat plate aligned with the flow

F. The Boundary Layer Approximation (continued)

1. Introduction
2. The BL coordinate system
3. The BL equations – see textbook for derivation

These BL equations are valid for steady, laminar, incompressible, 2-D flow in the *x*-*y* plane, and the Reynolds number, $\mathrm{Re}_L = \rho V L/\mu$, must be high enough for the approximations to be valid, but small enough for the flow to remain laminar:

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

***x*-momentum:**

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

Or, using Bernoulli in the outer flow:

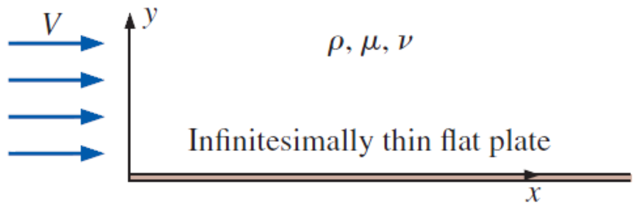
$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \nu\frac{\partial^2 u}{\partial y^2}$$

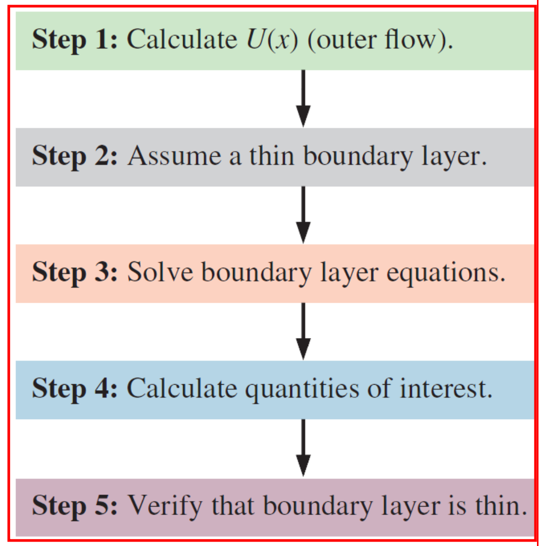
***y*-momentum:**

$$\frac{\partial P}{\partial y} \approx 0 \text{ through the BL} \rightarrow P = P(x) \text{ only}$$

4. The Boundary Layer Procedure

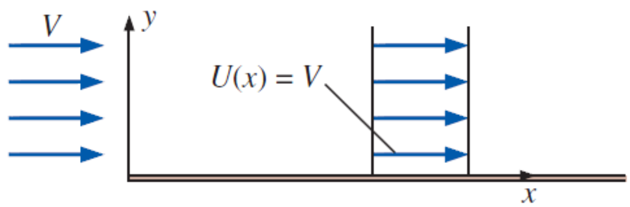
Example: The Laminar Flat Plate Boundary Layer





We go through the steps of the boundary layer procedure:

- **Step 1**: The outer flow is ***U*(*x*) = *U* = *V* = constant**. In other words, the outer flow is simply a uniform stream of constant velocity.
- **Step 2**: A very thin boundary layer is assumed (so thin that it does not affect the outer flow). In other words, the outer flow does not even know that the boundary layer is there.



- **Step 3**: The boundary layer equations must be solved; they reduce to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

There are four required boundary conditions,

$$u = 0 \quad \text{at } y = 0 \qquad u = U \quad \text{as } y \to \infty$$

$$v = 0 \quad \text{at } y = 0 \qquad u = U \quad \text{for all } y \text{ at } x = 0$$

This equation set was first solved by **P. R. H. Blasius** in 1908 – numerically, but *by hand*!

Blasius introduced a **similarity variable** η that combines independent variables *x* and *y* into one nondimensional independent variable,

Similarity variable

$$\eta = y\sqrt{\frac{U}{\nu x}}$$

and he solved for a nondimensionalized form of the *x*-component of velocity,

$$f' = \frac{u}{U} = \text{function of } \eta$$

The similarity solution is f' as a function of η.

The key here is that ***one single similarity velocity profile holds for any x-location along the flat plate***. In other words, the velocity profile shape is the same ("similar") at any location, but it is merely *stretched vertically* as the boundary layer grows down the plate. This is illustrated in Fig. 10-98 in the text.

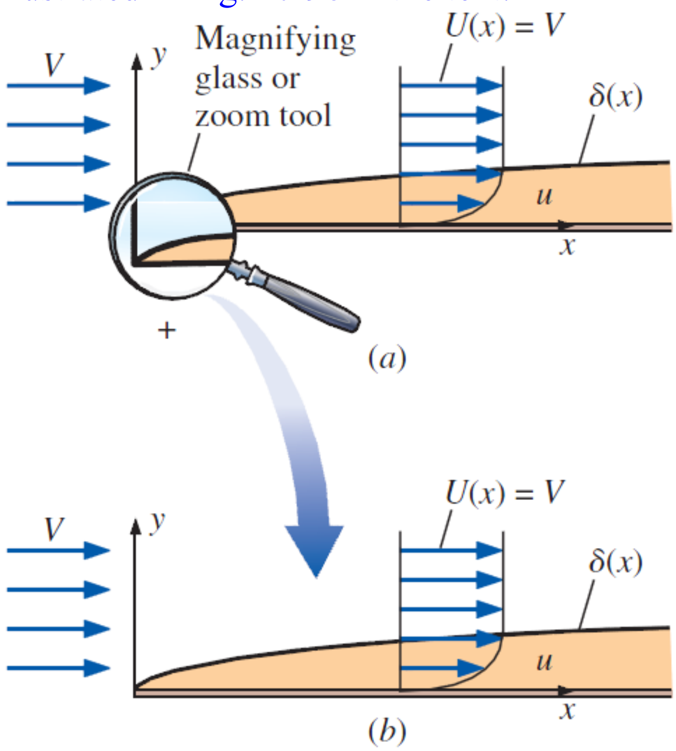
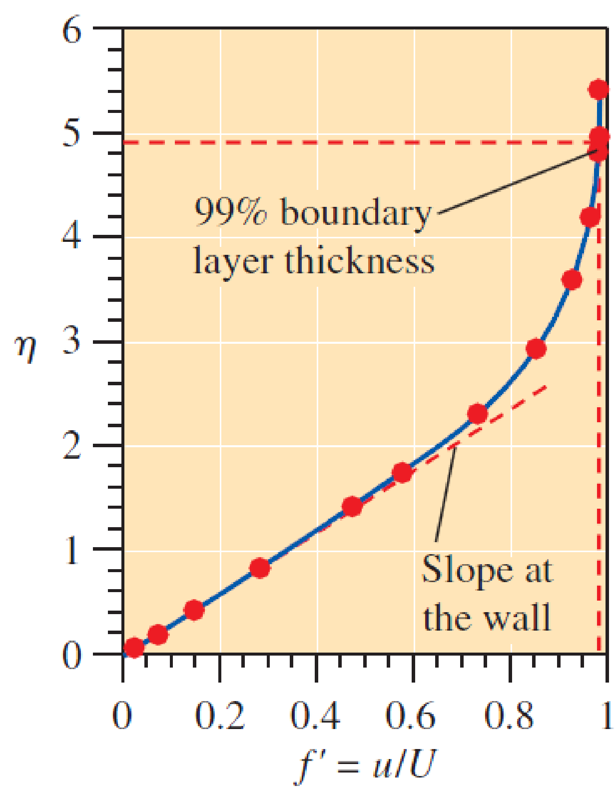


FIGURE 10–98
A useful result of the similarity assumption is that the flow looks the same (is *similar*) regardless of how far we zoom in or out; (*a*) view from a distance, as a person might see, (*b*) close-up view, as an ant might see.

The similarity solution itself is tabulated in Table 10-3, and is plotted in Fig. 10-99.

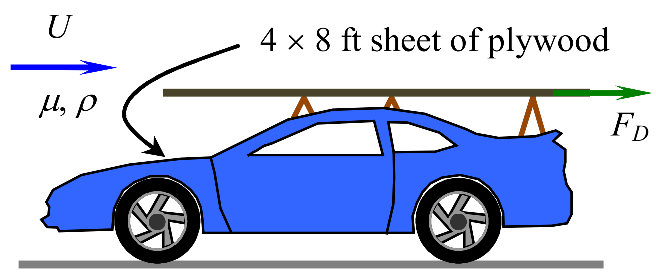
FIGURE 10–99
The Blasius profile in similarity variables for the boundary layer growing on a semi-infinite flat plate. Experimental data (circles) are at $Re_x = 3.64 \times 10^5$.
From Panton (1996).



This one velocity profile, plotted in nondimensional form as above, applies at *any* x- location in the boundary layer.

Example: Drag on a sheet of plywood

Given: Craig buys a 4 × 8 ft sheet of plywood at Lowe's and mounts it on the top of his car. He drives (carefully) at 35 mph ≈ 51.3 ft/s. The air density and kinematic viscosity in English units are $\rho = 0.07518$ lbm/ft^3 and $\nu = 1.632\times10^{-4}$ ft^2/s, respectively.



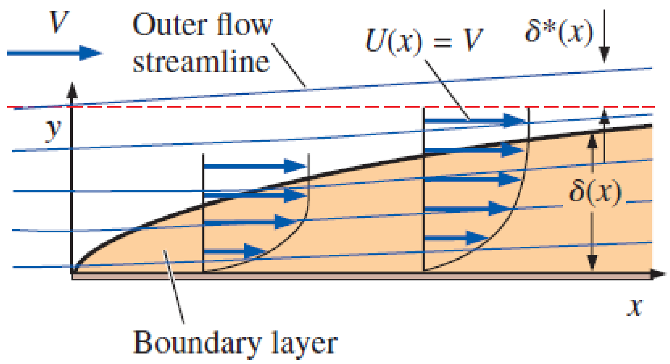
To do: Estimate δ at the end of the plate ($x = L$) and the drag force on the plate.

Solution:

d. Displacement thickness, δ^*

Definition:

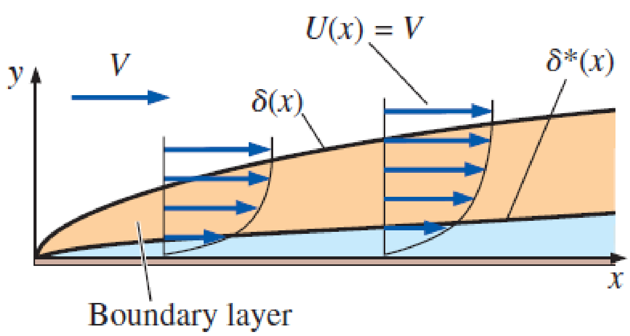
δ^* = the distance that a streamline just outside the BL is deflected away from the wall due to the effect of the BL; the distance in which the outer flow is "displaced" away from the wall.



Alternate Definition:

δ^* = the imaginary increase in wall thickness seen by the outer flow, due to the presence of the BL. [The outer flow "feels" like the wall is thicker than it actually is.]

Actual wall case:



Apparent wall case: