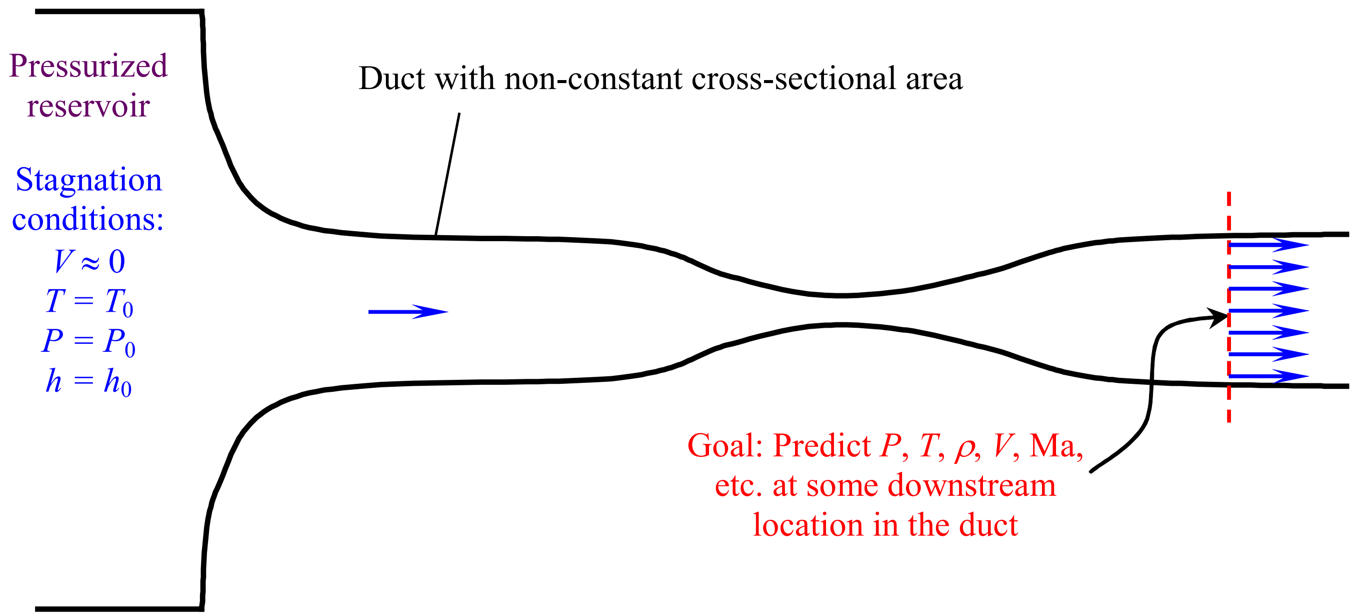
Today, we will:

- Discuss isentropic, compressible, adiabatic flow in ducts
- Discuss converging-diverging ducts, and introduce choking and shock waves

2. One-Dimensional Isentropic Adiabatic Flow in Ducts

a. Setup and equations



For simplicity, we approximate the flow as ***isentropic*** (negligible friction and other irreversibilities) and ***adiabatic*** (no heat transfer from the air to the surroundings – insulated duct walls).

Equations for isentropic, compressible, adiabatic flow of an ideal gas:

- For any ideal gas: $\boxed{\dfrac{T_0}{T} = 1 + \dfrac{k-1}{2}\mathrm{Ma}^2}$ $\boxed{\dfrac{\rho_0}{\rho} = \left(1 + \dfrac{k-1}{2}\mathrm{Ma}^2\right)^{\frac{1}{k-1}}}$ $\boxed{\dfrac{P_0}{P} = \left(1 + \dfrac{k-1}{2}\mathrm{Ma}^2\right)^{\frac{k}{k-1}}}$
- For air ($k = 1.4$): $\boxed{\dfrac{T_0}{T} = 1 + 0.2\mathrm{Ma}^2}$ $\boxed{\dfrac{\rho_0}{\rho} = \left(1 + 0.2\mathrm{Ma}^2\right)^{2.5}}$ $\boxed{\dfrac{P_0}{P} = \left(1 + 0.2\mathrm{Ma}^2\right)^{3.5}}$
- Or, for air in terms of temperature, $\boxed{\dfrac{\rho_0}{\rho} = \left(\dfrac{T_0}{T}\right)^{2.5}}$ $\boxed{\dfrac{P_0}{P} = \left(\dfrac{T_0}{T}\right)^{3.5}}$ also $\boxed{\dfrac{c_0}{c} = \left(\dfrac{T_0}{T}\right)^{1/2}}$

Subsonic water nozzle

Supersonic rocket nozzle

For air: $\boxed{\dfrac{A}{A^*} = \dfrac{1}{\text{Ma}} \dfrac{\left(1+0.2\text{Ma}^2\right)^3}{1.728}}$ and $\boxed{\dfrac{T_0}{T^*} = \dfrac{k+1}{2} = 1.2}$ from which we get

$$\boxed{\frac{\rho_0}{\rho^*} = \left(\frac{T_0}{T^*}\right)^{2.5} = 1.2^{2.5} = 1.577} \quad \boxed{\frac{P_0}{P^*} = \left(\frac{T_0}{T^*}\right)^{3.5} = 1.2^{3.5} = 1.893} \quad \boxed{\frac{c_0}{c^*} = \left(\frac{T_0}{T^*}\right)^{1/2} = 1.2^{1/2} = 1.095}$$

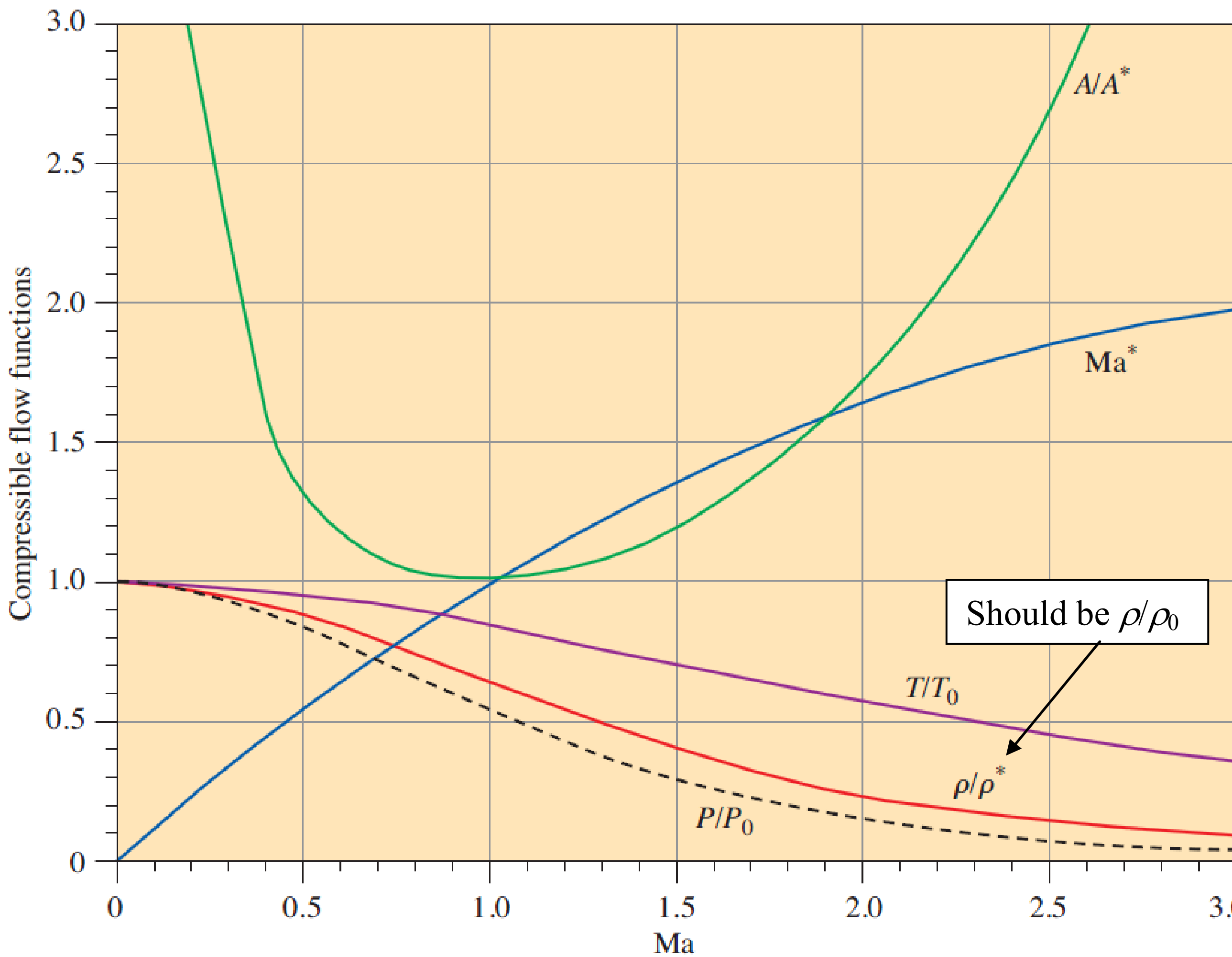
But these are usually listed in terms of their inverses, i.e.,

$\boxed{\dfrac{T^*}{T_0} = \dfrac{2}{k+1} = 0.8333}$ $\boxed{\dfrac{\rho^*}{\rho_0} = 0.6339}$ $\boxed{\dfrac{P^*}{P_0} = 0.5283}$ $\boxed{\dfrac{c^*}{c_0} = 0.9129}$ = **sonic** or **critical** values

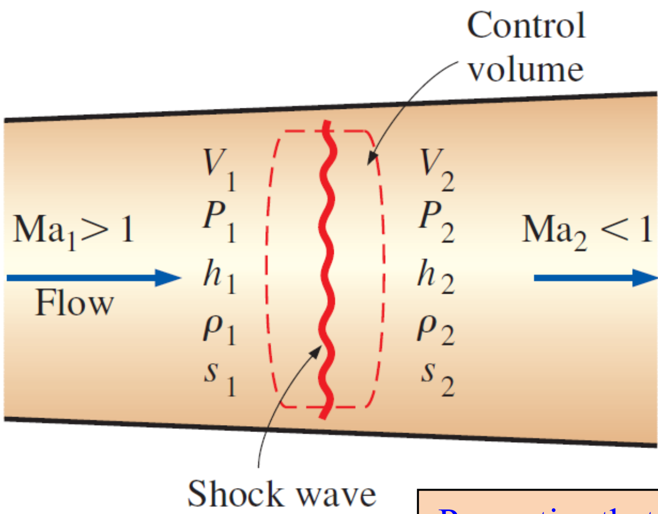
We also define a **critical Mach number**, Ma^*, as

$$\boxed{\text{Ma}^* = \frac{V}{c^*} = \frac{V}{c}\frac{c}{c^*} = \frac{\text{Ma}\cdot c}{c^*} = \frac{\text{Ma}\cdot\sqrt{kRT}}{\sqrt{kRT^*}} = \text{Ma}\sqrt{\frac{T}{T^*}}}$$

We plot all these as functions of Mach number in Appendix A-13 (for air):



Consider a *stationary* normal shock wave (as in a supersonic wind tunnel)



Properties that ***increase*** across the shock:

- $P_2 > P_1$
- $T_2 > T_1$, thus:
  - $c_2 > c_1$
  - $h_2 > h_1$
- $\rho_2 > \rho_1$
- $s_2 > s_1$
- $A_2^* > A_1^*$

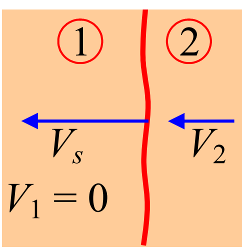
Properties that ***decrease*** across the shock:

- $Ma_2 < Ma_1$
- $P_{02} < P_{01}$
- $\rho_{02} > \rho_{02}$
- $V_2 < V_1$

Properties that ***stay the same*** across the shock:

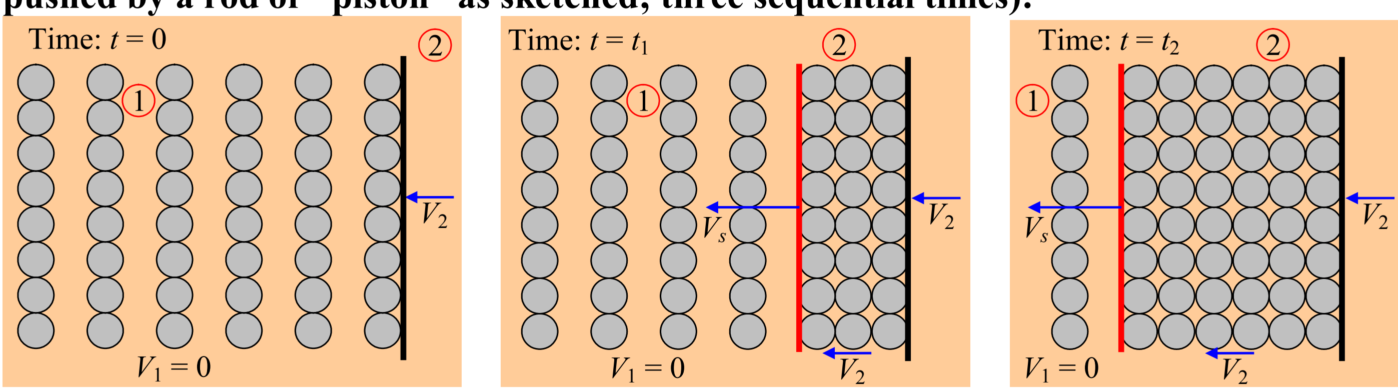
- $T_{02} = T_{01}$
- $h_{02} = h_{01}$

Consider instead a *moving* normal shock wave (as in a blast wave from an explosion)



- The shock is moving into quiescent air (region 1)
- In this frame of reference we define $Ma_1 = V_s/c_1$
- The shock wave travels into region 1 at supersonic speed ($Ma_1 > 1$)
- The air behind the shock (region 2) follows at a slower speed

The "dime analogy" (model the moving shock as rows of dimes that pile up when pushed by a rod or "piston" as sketched; three sequential times):



Comments:

- The vertical red line is analogous to a shock wave: $V_1 = 0$, $V_s > V_2$, $\rho_2 > \rho_1$ (there is sudden increase in density, and the "wave front" of dimes moves faster than the piston).
- The dimes in region 1 don't "know" anything is happening until the shock hits them.
- Similarly in a shock wave in air, the air in region 1 does not "know" anything is happening until the shock wave hits it.